2015 International Symposium on Nonlinear Theory and its Applications
NOLTA2015, Kowloon, Hong Kong, China, December 1-4, 2015

# An Emulator of Mutual Meminductors

D.S. Yu †, Y. Liang †, H.H.C. Iu‡ and T. Fernando‡

†School of Information and Electrical Engineering, China University of Mining and Technology,
No.1 University Road, Xuzhou, 221116, P. R. China
‡ School of Electrical, Electronic and Computer Engineering, the University of Western Australia,
Perth, 6009, Australia
Email: dongsiee@163.com, herbert.iu@uwa.edu.au

**Abstract–** In this paper, a practical circuit is introduced for the first time which can portray both the self and mutual characteristics of two flux coupled meminductors. The induced power transferred from one meminductor to the other is analyzed in accordance with the mutual action. Agreement between PSPICE simulation and theoretical analysis verifies the practicability and flexibility of this new emulator.

## 1. Introduction

Meminductor (MI) and memcapacitor (MC) are newly generalized energy storage elements from the conception of memristor (MR) [1]-[2]. The MI links the charge and the time integral of the flux (TIF), and will open up a new area of possibilities and functionalities in electronics. Since MI is still commercially unavailable, significant motivation in the research community has emerged on analyzing the dynamic behavior of MIs, by means of its emulators. D. Biolek and his research team have established several behavioral models for meminductive systems based on the constitutive relations, and they also proposed mutators for mimicking MI by the grounded MR [3]-[5]. An effective interface circuit constructed by only two current conveyors which can provide six connecting combinations for transforming a MR into meminductive and memcapacitive systems is designed in [6]. A universal mutator for bidirectional transformations among MR, MC, and MI by making use of only three off-the-shelf active devices is reported in [7]. By establishing a MI emulator using off-the-shelf components, the response of the MI under DC, sinusoidal, and periodic current signals is discussed by simulation and a general closed-form expression for meminductance is also derived in [8]. In [9], a mutator-based MI emulator with inductance varied along with an external current source is proposed. The simulation analysis shows that multiple MI circuits can be built in various configurations with MRs combined within this proposed mutator. With regard to hardware implementation of MI, in [10], a practical flux-controlled MI emulator with floating terminals and variable structure is designed without using a MR. These emulators have assisted researchers commendably in discovering the dynamic behaviors and potential applications of MIs in circuit design. The investigation presented in [11] demonstrates pinched hysteretic magnetic flux–current signals at room temperature based on the spin Hall magnetoresistance effect in several-nanometer-thick thin films, exhibiting the nonvolatile memorizing property and magnetic energy storage ability of the MI. This research result could open an avenue for nanoscale MI design and manufacture.

The existence of mutual inductance of regular inductors via magnetic field is well known. Many electrical devices operate on the basis of mutual inductance. Recently, definition of a flux controlled mutual meminductive system is interestingly proposed in [12] based on the constitutive relation of MI. However, in addition to the self meminductance not being included in this definition, no corresponding emulators are provided in [12]. In this paper, we propose an emulation circuit which can portray both the self and mutual characteristics of two coupled MIs and the induced power transferred from one MI to the other is also analyzed for the first time.

## 2. The Mutual MI Emulator

The symbol proposed in [11] for indicating mutual meminductive system is enclosed by dotted frame as shown in Fig.1. By referring to [2], a TIF controlled MI without mutual effects can be described by

$$i_{\mathrm{MI}}(t) = L_{\mathrm{M}}^{-1}(\rho(t))\varphi(t)\,, \tag{1}$$

where $L_{\mathrm{M}}^{-1}$ is the inverse self meminductance, $\rho(t)$ is the time integral of flux $\varphi(t)$ and $i_{\mathrm{MI}}$ is the meminductive current.

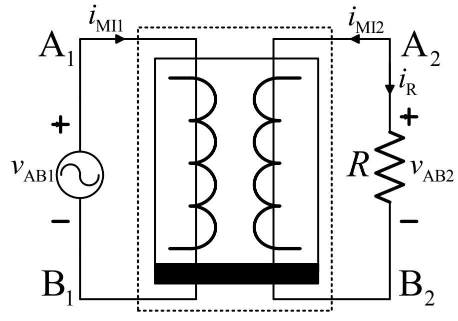


**Fig. 1** Mutual MIs

The expression of inverse meminductance $L_{\mathrm{M}}^{-1}$ could possess very high complexity according to the inside structure of MI [11]. Here, we consider ideal MIs with very simple constitutive relations for the sake of simplicity. $L_M^{-1}$ is defined by a linear relation as

$$L_M^{-1} = L_{M0}^{-1} + \alpha\rho(t)\,. \tag{2}$$

$L_{M0}^{-1}$ is the initial value of inverse meminductance. It can be seen that in this case, the instantaneous inverse meminductance is proportional to $\rho(t)$ with coefficient $\alpha$. Hence, the experimental implementation can be simplified as well by adopting ideal MIs with the controllable meminductance of (2).

The proposed schematic of mutual MI emulators is shown in Fig. 2. Without considering the mutual effects, the two circuits (MI1 and MI2) enclosed by dotted frame can be independently operated as the TIF controlled MIs [9].

The upper MI, enclosed by red dotted frame, is taken for demonstration first without considering the mutual effect by connecting the left terminals of $R_{c1}$ and $R_{c2}$ to the ground. Four transimpedance operational amplifiers (AD844), three amplifiers (one TL084 chip), and one multiplier (AD633) are employed and tagged as U11, U21, U31, U41, U51, U61, U81 and U71, respectively. In accordance with the inherent input-output performance of AD844, there is no current flowing into terminal $y$, the voltage of terminal $x$ equals to the voltage of terminal $y$, and the current flowing into terminal $x$ equals to the input current of terminal $z$. Hence, the output voltages at terminal $p$ of U11 can be written as

$$v_{c11} = \frac{1}{R_{11}C_{11}}\varphi_{A1B1}\,. \tag{3}$$

Equation (3) reveals that the output voltage of U11 is in proportion to $\varphi_{A1BA}$. The value of $v_{c11}$ can be easily altered by tuning the values of $R_{11}$ and $C_{11}$. The output voltage $v_{u51}$ of the integrator constructed by amplifier U51, as well as a DC offset voltage $v_{s1}$, are delivered to an inverting adder, whose output is then sampled by multiplier U71 as the input. According to the input-output performance of AD633, we have

$$v_{w1} = \frac{v_{u61}\cdot v_{c11}}{10}\,. \tag{4}$$

where,

$$v_{u51} = -\frac{1}{R_{11}R_{21}C_{11}C_{21}}\rho_{A1B1}\,, \tag{5}$$

$$v_{u61} = \frac{R_{51}}{R_{11}R_{21}R_{31}C_{11}C_{21}}\rho_{A1B1} - \frac{R_{51}}{R_{41}}v_{s1}\,. \tag{6}$$

The output voltage $v_{w1}$ is delivered to U21 via an inverting proportional amplifier to control the current going through $R_{81}$. Note that, due to the action of AD844s, the current flowing through $R_{81}$ is equivalent to the input current of the upper MI emulator,

$$i_{MI1} = -\frac{R_{71}v_{w1}}{R_{61}R_{81}}\,. \tag{7}$$

By combining (3) with (6), we have

$$v_{u61} = \frac{10R_{11}C_{11}R_{61}R_{81}}{R_{71}}\cdot\frac{i_{MR1}}{\varphi_{A1B1}}\,. \tag{8}$$

Hence, the flux-controlled $W_1$ can be derived as,

$$L_{M1}^{-1}(\rho_{A1B1}) = \frac{i_{MR1}}{v_{A1B1}} = \alpha_1\rho_{A1B1} + L_{M01}^{-1}\,, \tag{9}$$

where $\alpha_1$ reflects the variation rate of meminductance. Parameter $\alpha_1$ and initial value $\beta_1$ can be computed as

$$\alpha_1 = \frac{R_{71}R_{51}}{10R_{11}^2R_{21}R_{31}R_{61}R_{81}C_{11}^2C_{21}}\,, \tag{10}$$

$$L_{M01}^{-1} = -\frac{R_{71}R_{51}}{10R_{11}R_{41}R_{61}R_{81}C_{11}}v_{s1}\,. \tag{11}$$

Equation (11) reveals that the initial value of meminductance can be adjusted by resistor $R_{41}$ and DC voltage supply $v_{s1}$.

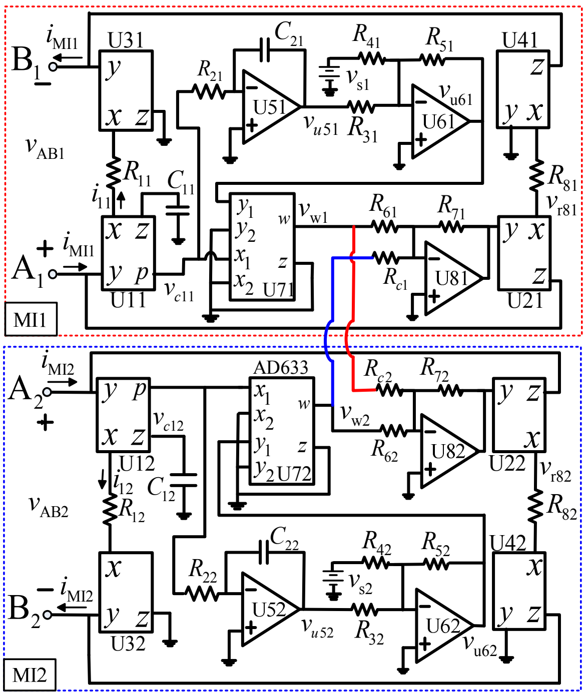


**Fig. 2** The proposed mutual MI emulator

For the mutual MIs of Fig.1, two extra inputs of inverting adders U81 and U82 are introduced to achieve the mutual operation, as exhibited by Fig.2. The left terminal of $R_{c1}$ is connected to the output terminal of U72, while the left terminal of $R_{c2}$ is shorten to the output terminal of U71. Two op amps U51 and U52 are operated as integrators in order to obtain equivalent TIF $\rho(t)$. The two DC voltage sources $v_{s1}$ and $v_{s2}$ are for configuring the initial value of each meminductance. By considering the TIF controlled mutual meminductance, the set of system equations can be written as

$$\begin{cases} i_{\mathrm{MI1}} = (L_{\mathrm{M01}}^{-1} + \alpha_1\rho_1)\varphi_1 + \beta_1(L_{\mathrm{M02}}^{-1} + \alpha_2\rho_2)\varphi_2 \\ i_{\mathrm{MI2}} = (L_{\mathrm{M02}}^{-1} + \alpha_2\rho_2)\varphi_2 + \beta_2(L_{\mathrm{M01}}^{-1} + \alpha_1\rho_1)\varphi_1 \end{cases}, \quad (12)$$

where $\beta_1(L_{M02}{}^{-1}+\alpha_2\rho_2)$ and $\beta_2(L_{M01}{}^{-1}+\alpha_1\rho_1)$ are the inverse mutual meminductance and both $\beta_1$, $\beta_2$ are the coefficients which decide the mutual strength. Equation (12) demonstrates that the current going through each MI is in fact aroused by self and mutual fluxes.

According to operational functions of the active chips and Kirchhoff's Voltage and Current Laws, we can obtain the parameters for describing the lower MI emulator (enclosed by blue dotted frame) and the mutual strength,

$$L_{\mathrm{M02}}^{-1} = -\frac{R_{52}R_{72}v_{s2}}{10R_{12}R_{42}R_{62}R_{82}C_{12}}, \quad (13)$$

$$\alpha_2 = \frac{R_{52}R_{72}}{10R_{12}^2R_{22}R_{32}R_{62}R_{82}C_{12}^2C_{22}}, \quad (14)$$

$$\beta_1 = \frac{R_{71}R_{62}R_{82}}{R_{c1}R_{81}R_{72}}, \beta_2 = \frac{R_{72}R_{61}R_{81}}{R_{c2}R_{82}R_{71}}. \quad (15)$$

Equations (10)-(15) reflect that $L_{M01}{}^{-1}$, $L_{M02}{}^{-1}$, $\alpha_1$, $\alpha_2$, $\beta_1$ and $\beta_2$ can be conveniently adjusted by tuning the values of resistors and capacitors.

## 3. Experimental Analysis

The experimental parameters for testing the mutual MI emulators are given as: $R_{11}$=20kΩ, $R_{12}$=10kΩ, $R_{21}$=80kΩ, $R_{22}$=100kΩ, $R_{31}$=80kΩ, $R_{32}$=50kΩ, $R_{41}$=100kΩ, $R_{42}$=200kΩ, $R_{51}$=30kΩ, $R_{52}$=80kΩ, $R_{61}$=300kΩ, $R_{62}$=15kΩ, $R_{71}$=750kΩ, $R_{72}$=100kΩ, $R_{81}$=2kΩ, $R_{82}$=2kΩ, $C_{11}$=300nF, $C_{12}$=200nF $C_{21}$=400nF, $C_{22}$=200nF, $v_{s1}$=$v_s$=-15V, $R_{c1}$=17kΩ, $R_{c2}$=300kΩ. On the basis of this parameter configuration it can be calculated that the initial inverse meminductance values satisfy $L_{M01}{}^{-1}$=0.09375mH$^{-1}$ and $L_{M02}{}^{-1}$=0.5 mH$^{-1}$.

In order to perform a comparative investigation with theoretical and simulation analysis, the simulation data are transferred from PSPICE into OriginPro8.0 software to obtain the dictated curves. Firstly, a sinusoidal voltage of $v_{A1B1}$=3sin($2\pi ft$) is imposed on $A_1B_1$ to verify the single MI emulator without considering the mutual effects. In this case, the inverting adder structured by U61 is in fact operated as an inverting proportional amplifier. Note that, since voltage -$v_{r81}$ is in proportion to the current going through MI1 and hence is sampled for describing $i_{MI1}$.The relations between current and terminal flux are portrayed in Fig.3 using -$v_{r81}$ and $v_{c11}$, with the excitation frequency $f$ taking values of 8Hz, 10Hz and 15Hz, respectively. It can be seen that the flux versus current loops are passing through the origin and evidently shrunk as the frequency $f$ increases, which is a typical feature of MI called the frequency dependence of pinched hysteresis loops.

Then, a voltage source and a resistor $R$ are connected to the primary side ($A_1B_1$) and the secondary side ($A_2B_2$), respectively, to verify the mutual actions of the proposed emulator, as shown in Fig.1. By configuring $v_{A1B1}$=3sin($20\pi t$) and $R$=1kΩ, the sampled voltage $v_{A1B1}$, equivalent current -$v_{r81}$ and flux $v_{c11}$ in time domain are shown in Fig.4. It can be seen that the excitation voltage $v_{A1B1}$ is a 10Hz standard sinusoidal waveform with the phase ahead of $v_{c11}$ and -$v_{r81}$. The phase difference between $v_{A1B1}$ and -$v_{r81}$ ($i_{MI1}$) is less than π/2. Also, it is worth noting that $v_{c11}$ and -$v_{r81}$ are no longer simultaneously passing through zero, but with a noticeable time difference Δ$t$=1.93ms. This time difference is in fact caused by the mutual action from the secondary MI, and can be decreased by attenuating the mutual strength (increasing $R_{c1}$).

Referring to the well-known mutual inductors, it can be deduced that the mutual action from the primary MI will result in induced current in the secondary MI. On the basis of Kirchhoff's Current Law, the sum of currents $i_{MI2}$ and $i_R$ is equal to zero, hence $v_{r82}$ is captured to represent current $i_R$.

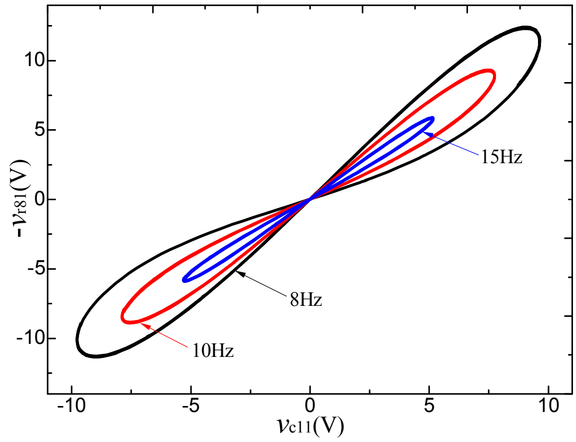


**Fig. 3** Curves of the pinched hysteresis loops

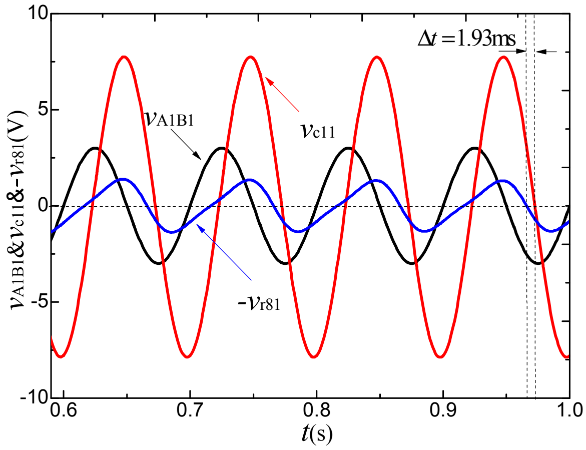


**Fig. 4** Waveforms of $v_{A1B1}$, -$v_{r81}$ and $v_{c11}$

The induced voltage and current of the secondary MIs, $v_{A2B2}$ and $v_{r82}$, are shown in Fig.5. It can be observed that the voltage and current are in linear proportion and simultaneously passing through zero. The flux is evidently lagging off voltage and current with a phase angle of $\pi/2$. Note that current, voltage and flux are no longer sinusoidal waveforms due to the action of nonlinear self and mutual meminductance. The power dissipated $P(t)$ on the load resistor $R$ is also displayed in Fig.5 by dotted line. The maximal value of $P(t)$ is 3.312uW, which appears with the peak value of voltage $v_{AB2}$ instead of $v_{AB1}$. These simulation results reveal that this emulator is capable of demonstrating the dynamic complexities of energy delivering process of two mutual MIs.

Inspired by the preceding analysis, it can be deduced that, in similarity with normal mutual inductors, two flux coupled MIs could transform electrical energy into magnetic energy, then back into electrical energy via the mutual action. Hence, the mutual MIs can be redefined as a transformer with variable inductance and called as "memtransformer". Since its operation depends on the electromagnetic induction between two stationary MIs and the variable magnetic flux with changing magnitude and polarity, the so-called memtransformer is supposed to be used as AC device.

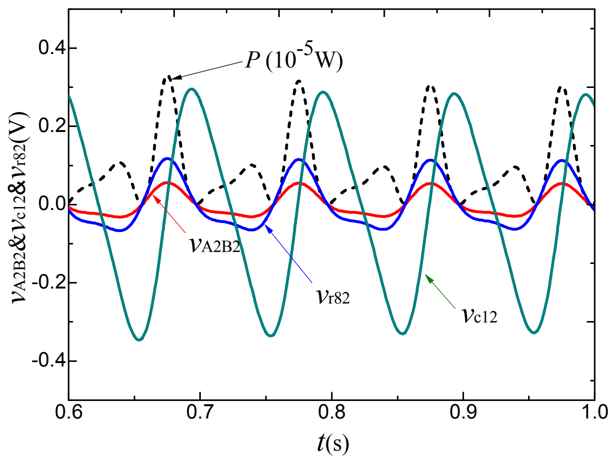


**Fig. 5** Waveforms of $v_{A2B2}$, $v_{r82}$ and $v_{c12}$

## 4. Conclusions

This paper proposes a novel circuit emulator which can be flexibly modified and used for analyzing the dynamic behaviors of two mutual MIs. The PSPICE results unfold that induced power can be generated in the secondary MI due to the mutual flux action. Although the system equations (12) are specific, the work presented in this letter can provide references for future research related to mutual MIs. The practical implementation of this mutual MI will be carried out in our future research.

## Acknowledgments

This work was supported in part by China Postdoctoral Science Foundation (2013M531423), National Natural Science Foundation of China (51307174), Fundamental Research Funds for the Central Universities (2013QNB28).

## References

[1] L. O. Chua, "Memristor: the missing circuit element," *IEEE Trans. Circuit Theory,* vol.**18**, no.5, pp. 507-519, 1971.

[2] M. Di Ventra, Y.V. Pershin and L.O. Chua, "Circuit elements with memory: memristors, memcapacitors and meminductors," *Proc. IEEE,* vol.**97**, pp. 1717-1724,2009.

[3] D. Biolek, Z. Biolek, and V. Biolková, "SPICE modeling of memristive, memcapacitative and meminductive systems," *European Conf. on Circuit Theory and Design*, pp.249-252, Aug. 2009.

[4] D. Biolek, Z. Biolek, and V. Biolkova, "PSPICE modeling of meminductor," *Analog Integr. Circ. Sig. Process*, vol. 66, no. 1, pp. 129-137, Jul. 2011.

[5] D. Biolek, V.Biolkova, Z.Kolka, "Mutators simulating memcapacitors and meminductors," *IEEE Asia-Pacific Conf. on Circuit. Syst.*, pp.800-803, Dec. 2010.

[6] D.S. Yu, Y. Liang Yan, H.H.C. Iu and Y.H.Hu, "Mutator for transferring a memristor emulator into meminductive and memcapacitive circuits," *Chinese Phys. B*, vol.23, pp. 070702, 2014.

[7] D. S. Yu, Y. Liang, H.H.C. Iu and L.O. Chua, "A universal mutator for transformations among memristor, memcapacitor, and meminductor," *IEEE Trans. on Circuits and Systems II-Briefs*, vol. **61**, no.10, pp.758-762, 2014.

[8] M.E. Fouda, M. E. Fouda, "Meminductor Response Under Periodic Current Excitations," *Circuit., Syst., Sig. Proc.*, vol.33, no.5, pp.1573-1583,May. 2014.

[9] M. Pd. Sah, R. K. Budhathoki, C. J. Yang and H. Kim, "Mutator-Based Meminductor Emulator for Circuit Applications," *Circuit., Syst., Sig. Proc.*, vol.33, no.8, pp 2363-2383, Aug. 2014.

[10] Y. Liang, H. Chen and D.S. Yu, "A practical implementation of a floating memristor-less meminductor emulator," *IEEE Trans. on Circuit Syst. II-Briefs*, vol. **61**, no.5, pp. 299-303, 2014.

[11] J.H. Han, C. Song, S. Gao , Y.Y. Wang , C. Chen and F. Pan, "Realization of the Meminductor," *ACS Nano*,vol.8, no.10, pp.10043-10047, Sep. 2014.

[12] G. Cohen, Y. Pershin and M. Di Ventra, "Lagrange formalism of memory circuit elements: classical and quantum formulations," *Phys. Review B*, vol. 85, no.16, pp. 165428, 2012.