2008 International Symposium on Nonlinear Theory and its Applications
NOLTA'08, Budapest, Hungary, September 7-10, 2008

ID model with Higher-Order Connections for the Traveling Salesman Problem

Takahiro SOTA† , Yoshihiro HAYAKAWA† and Koji NAKAJIMA†

†Laboratory for Brainware/Laboratory for Nanoelectronics and Spintronics
Research Institute of Electrical Communication, Tohoku University
2-1-1 Katahira, Aoba-ku, Sendai-shi, 980-8577, Japan
Email: sota@nakajima.riec.tohoku.ac.jp, asaka@riec.tohoku.ac.jp, hello@nakajima.riec.tohoku.ac.jp

Abstract—We can solve the N-Queen problem, which belongs to a particular kind of combinatorial optimization problems, by using the Inverse function Delayed (ID) network with normal connections and the success rate achieves 100%. In such a case, if and only if the state of the ID network is stationary, it always shows an optimal solution. This feature is very useful because of an easy judgment. However, it is hard to solve the Traveling Salesman Problem (TSP) with this model. The problem is that the network states between an optimal solution and the others are not always separable.

In this paper, we propose the ID model with higher-order synapse connections (HC-ID) to solve this problem. We demonstrate that the HC-ID model works to separate between the optimal solutions and the others very well. In addition, in computer simulations, it is shown that optimal solutions of the TSP are also only stationary states.

1. Introduction

We have proposed the Inverse function Delayed model (ID model) as one of neural networks [1]. One important feature of the ID model is to have negative resistance in its dynamics. We consider that the ID model is a powerful tool for avoiding the local minima problem because the negative resistance destabilizes local minima state. We have pursued research on solving combinatorial optimization problems [2], associative memory [3], and so on. In particular, the ID model can destabilize only local minima in some kinds of the combinatorial optimization problems[2].

In such cases, the equilibrium point of global minima converges on adjacent to $x = 0$ or $x = 1$, and the local minima is distributed away from $x = 0$ or $x = 1$. We can selectively cover only local minima by the negative resistance of the ID model, and then we can destabilize them. Additionally, we can estimate the distribution of the equilibrium point without solving it [4], hence we can set the suitable negative resistance region previously. The N-Queen problem and 4-color problem, which belong to this kinds of problems, have been solved at 100% success rate. In the Traveling Salesman Problem (TSP), however, the equilibrium points of global minima and local minima are distributed disorderly. Consequently we can't destabilize only local minima by using the negative resistance.

Meanwhile, in order to solve the TSP, the higher-order Hopfield network has been proposed [5]. This model includes the higher-order synapse connections, and we can converge the global minima to the vertexes of the output space by using this model [6]. However the local minima remain away from the vertex yet.

In this paper, we introduce the higher-order synapse connections to the ID model. First, we show separable distribution of local minima and global minima by introducing the higher-order synapse connections to the ID model. Then, we consider selectively destabilizing only local minima by using negative resistance. Moreover, we confirm it by solving a 4-city TSP as preliminary tests.

2. The Higher-Order Connection ID Model

The ID model, which has normal synapse connection, has negative resistance in its dynamics, and has an energy function similar to Lyapunov Function [2]. From these characteristics, this model can avoid local minimum problem of some kinds of combinatorial optimization problems. In this section, we propose the higher-order connection ID model (HC-ID model). The higher-order synapse is the multiple connection which links from multiple neuron to one neuron, and it has been used to solve the 3-SAT problem [7].

2.1. Basic Equations

In case of the TSP, we can design the HC-ID network with third-order synapse connection. The ID model with third-order connection is described as follows:

$$\tau_u \frac{du_i}{dt} = \sum_j \sum_k \sum_l w_{ijkl} x_j x_k x_l + \sum_j \sum_k w_{ijk} x_j x_k + \sum_j w_{ij} x_j + h_i - u_i, \quad (1)$$

$$\tau_x \frac{dx_i}{dt} = u_i - g(x), \quad (2)$$

where u_i, x_i and h_i are the internal state, the output and the bias of neuron i, respectively. w_{ij} is the synaptic weight

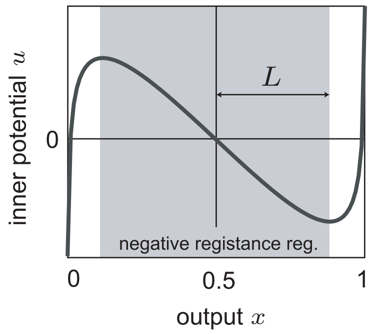

Figure 1: The g-function and negative resistance region.

from neuron j to neuron i, and $w_{ijk\cdots}$ is the higher-order synaptic weight from neurons $j, k, \cdots$ to neuron i, τ_u is the time constant of the internal state, $\tau_x (\ll \tau_u)$ is the time constant of the output. $g(x)$ is the inverse output function of the Hopfield network (see later).

The following equations are derived from Eqs.(1) and (2) as well as the ID model:

$$\tau_x \frac{d^2 x_i}{dt^2} + \eta(x_i) \frac{dx_i}{dt} = -\frac{\partial U}{\partial x_i}, \tag{3}$$

where

$$\eta(x_i) = \frac{\partial g(x_i)}{\partial x_i} + \frac{\tau_x}{\tau_u}, \tag{4}$$

$$\frac{\partial U}{\partial x_i} = \frac{1}{\tau_u} \left(g(x) - \sum_j \sum_k \sum_l w_{ijkl} x_j x_k x_l \right. \\ \left. - \sum_j \sum_k w_{ijk} x_j x_k - \sum_j w_{ij} x_j - h_i \right). \tag{5}$$

From Eq.(3), it is shown that the HC-ID model is updated by dynamics of particles subjected to inertia force and friction $\eta(x_i)$ in potential U. This dynamics are similar to the ordinary ID model.

$g(x)$ in Eq.(4) is

$$g(x) = f^{-1}(x) - \alpha \left(x - \frac{1}{2} \right), \tag{6}$$

where

$$f^{-1}(x) = \left[\frac{1}{2} \left(1 + \tanh \left(\frac{\beta x}{2} \right) \right) \right]^{-1} = \frac{1}{\beta} \ln \left(\frac{x}{1-x} \right). \tag{7}$$

If $g(x)$ has an N shape as shown in Figure 1, $\eta(x_i)$ has a negative value [2]. This region $\eta(x_i) < 0$ is called the negative resistance. From Eqs.(6) and (7), α and β is a control parameter of the range and the gain of the negative resistance, respectively.

2.2. The Energy Function

When the synaptic weights are symmetric ($w_{ijkl\cdots} = w_{jikl\cdots} = w_{jilk\cdots} = \cdots$), the energy function of HC-ID can be defined as the Hopfield network or the ID model.

$$E_{\text{hi-ID}} = -\frac{1}{4\tau_u} \sum_i \sum_j \sum_k \sum_l w_{ijkl} x_i x_j x_k x_l \\ -\frac{1}{3\tau_u} \sum_i \sum_j \sum_k w_{ijk} x_i x_j x_k \\ -\frac{1}{2\tau_u} \sum_i \sum_j w_{ij} x_i x_j - \frac{1}{\tau_u} \sum_i h_i x_i \\ +\frac{1}{\tau_u} \sum_i \int_{\frac{1}{2}}^{x_i} g(x) dx + \frac{\tau_x}{2} \sum_i \left(\frac{dx_i}{dt} \right)^2 \tag{8}$$

Moreover, from Eqs. (1) and (2), its time derivative is

$$\frac{dE_{\text{hi-ID}}}{dt} = -\sum_i \eta(x_i) \left(\frac{dx_i}{dt} \right)^2. \tag{9}$$

If $\eta(x_i) < 0$, $E_{\text{hi-ID}}$ increases with time like the normal connection ID model. Hence the HC-ID model can also destabilize only local minima if and only if local minima are in the negative resistance region.

3. Design the HC-ID network for the TSP

3.1. The Energy Function for the TSP

When solving the combinatorial optimization problem by using the neural network, we have to describe the problem as the energy function E_{TH}. The optimal solutions are assigned to the global minima of E_{TH}.

Hence we obtain E_{TH} of the TSP as [6]:

$$E_{TH} = \frac{A}{2} \sum_i \left(\sum_x x_{xi} - 1 \right)^2 + \frac{A}{2} \sum_x \left(\sum_i x_{xi} - 1 \right)^2 \\ + \frac{B}{2} \sum_x \sum_y \sum_i d_{xy} x_{xi} x_{yi+1} (1 - x_{xi} x_{yi+1}) \\ + \frac{C}{2} \left(\sum_x \sum_y \sum_i d_{xy} x_{xi} (x_{yi-1} + x_{yi+1}) \right)^2, \tag{10}$$

where A, B, and C are positive values, d_{xy} is distance between city x and city y.

The first and second terms of Eq.(10) are conditions to express the cyclic tours. The forth term expresses a squared value of tour length subject to the cyclic tours. Since this term is a squared value, the difference of distance is amplified. The third term is added purposely to separate the equilibrium point of global minima and local minima (detail will be discussed later). If the network shows cyclic tours, this term has to be zero. Therefore it is satisfied that the optimal solution is the minimum points of E_{TH} subject to the cyclic tours.

3.2. Correction of Self Weight and Bias

Since the synaptic connections are symmetric, we can use Eq.(10) as the Eq.(8) except for the fifth and sixth terms. The shape of the potential depends on $g(x)$, and $g(x)$ is a function of α. In order to make the effects of α negligible in the potential, the following corrections to the self weight and bias should be made [2]:

$$W = w_{i,i} - \alpha, \tag{11}$$
$$H = h_i + \alpha/2. \tag{12}$$

On the other hand, we want to read the stationary state as the answer representation. Thus the equilibrium point states that are being considered, are those where dx_i/dt is zero. Therefore, the sixth term is zero.

3.3. Distribution of the Equilibrium Point

From Eqs.(1), (8) and (10), the dynamics of the HC-ID network for the TSP are described as follow:

$$\begin{aligned}\tau_u \frac{du_{ak}}{dt} &+ u_{ak} \\ = &- A\left(\sum_x x_{xk} - 1\right) - A\left(\sum_i x_{ai} - 1\right) \\ &- B\,(1/2 - x_{ak}) \sum_x d_{ax}(x_{xk-1} + x_{xk+1}) \\ &- 2C \sum_x \sum_y \sum_i d_{xy} x_{xi}(x_{yi-1} + x_{yi+1}) \\ &\quad \times \sum_x d_{ax}(x_{xk-1} + x_{xk+1}). \end{aligned} \tag{13}$$

Since we can make undesirable states unstable in the HC-ID model by using the negative resistance, in order to make all local minima unstable, we have to estimate the equilibrium points of global minima and local minima. Meanwhile the ID network with the high-gain limit model ($\beta \to \infty$) has achieved the high performance of searching the optimal solution [2]. Hence, we use the high-gain limit model in this section. In such a case, the negative resistance region is $0 < x_{ak} < 1$, so that the output of stationary state is only 0 or 1.

If we require that stationary states always show valid tours as well as the higher-order Hopfield network, A has to be satisfied with

$$A > \max\left\{(N-1)Bd_M, d_M\,(4(N-2)Cd_M + B/2)\right\}, \tag{14}$$

where N is the number of cities, d_M is maximum distance between two cities. In later discussion, we assume that A is satisfied with Eq.(14).

The equilibrium point of the inner potential can be approximated as follow:

$$u_{ak} \simeq \begin{cases} \left\{-4Cd(\vec{x}) - \frac{B}{2}\right\}(d_{ak-1} + d_{ak+1}) & (x_{ak} \sim 0) \\ \left\{-4Cd(\vec{x}) + \frac{B}{2}\right\}(d_{ak-1} + d_{ak+1}) & (x_{ak} \sim 1) \end{cases}, \tag{15}$$

where $d(\vec{x})$ is tour length depending on the route which is indicated by the output vector $\vec{x}$.

From Eq.(15), equation $u_{ak} < 0$ is always satisfied when $x_{ak} \sim 0$, thus this state is stable. On the other hand, when $x_{ak} \sim 1$, the sign of u_{ak} depends on $\{-4Cd(\vec{x}) + B/2\}$. We require the positive sign to stabilize this unit.

Therefore, the HC-ID network has a possibility that only the equilibrium point of global minima is stabilized when we can set coefficient B and C to satisfy the equation [6]:

$$d(\vec{x_0}) < \frac{B}{8C} < d(\vec{x_1}), \tag{16}$$

where $d(\vec{x_0})$ and $d(\vec{x_1})$ is the tour length of global minima and local minima, respectively.

4. Numerical Experiments

In this section, in order to demonstrate the idea of the previous section, we compare the HC-ID network with the higher-order Hopfield model for solving 4-city TSP by computer simulation.

4.1. The Appearance Rate of the Network State

First, we investigated the converged states with various $B/8C$. Figure 2 shows the appearance rate of converged states with the HC-ID model (right) and the higher-order Hopfield model (left), respectively. Both results were tested by 100 trials with random initial values and the converged state of each trial was checked at $50 \times \tau_u$. The horizontal axis is the tour length, which is used to distinguish the converged states, and rightmost bar shows the appearance rate of the oscillation state.

In the case of the higher-order Hopfield network, as shown in Fig. 2, not only global minima but also local minima appear independently of $B/8C$ as the stationary states. In contrast, by using the HC-ID model, we can make only minima, which correspond to smaller tour length than $B/8C$, be stationary, moreover only the optimal solutions appear subject to Eq. (16). However, the HC-ID has the oscillation states which never appeared in the case of the higher-order Hopfield model.

Next, we investigated three maps for 4-city TSP. The results are shown on the Table 1. Each result is also measured by 100 trials. From results, if the network reaches a stationary state, the HC-ID network always shows an optimal solution in contrast with the higher-order Hopfield network.

4.2. Dependence of Success Rate on α

In the ID model with normal connections, it has been reported that the success rate depends strongly on α [4]. We also investigated α dependence of the HC-ID network with various β. Figure 3 shows the success rate as a function of α. The dependency of the HC-ID network on α is very similar to the normal ID network.

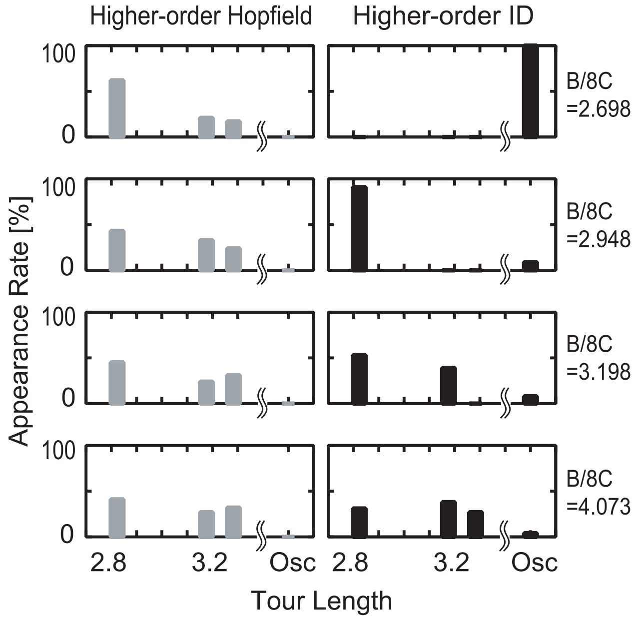

Figure 2: The histogram of appearance state

Table 1: The appearance rate of each network states [%]. In all maps, β = 8.0. 'H-Hop' means the higher-order Hopfield model.

	MapA(α = 3.0)		MapB(α = 3.0)		MapC(α = 4.0)	
	HC-ID	H-Hop	HC-ID	H-Hop	HC-ID	H-Hop
GM	91	46	86	53	98	57
LM	0	54	0	47	0	43
Osc	9	0	14	0	2	0

5. Conclusion

In this paper, we proposed the ID model with higher-order synapse connection (HC-ID) to deal with the TSP.

By using the HC-ID model for the TSP, if the β is large enough, the stationary state is on the vertex of the output space. In such a case, only the network state representing an optimal solution is consistent when $B/8C$ is set between the shortest tour length and the second shortest one. Meanwhile in case of the stationary states with the output of intermediate value, we can destabilize this state by the negative resistance of the HC-ID model. Therefore, the HC-ID model is expected to be a powerful tool for solving the TSP because it is always satisfied that the stationary state is an optimal solution. Moreover we confirmed it in 4-city TSP as preliminary tests by numerical experiments.

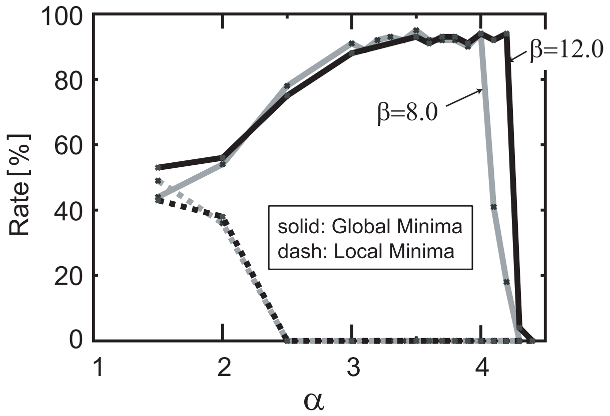

Figure 3: The rate of global minima and local minima in MapA.

References

[1] K. Nakajima and Y. Hayakawa, "Characteristics of inverse function delayed model for neural computation", *Proc. NOLTA'02*, pp.861–864, Xi'an, China, Oct. 2002.

[2] A. Sato, Y. Hayakawa, and K. Nakajima, "The Parameter Dependence of the Inverse Function Delayed Model on the Success Rate of Combinatorial Optimization Problems", *IEICE Trans. Fundamentals (JAPANESE EDITION)*, Vol.J89-A, No.11, pp.960–972, 2006.

[3] H. Li, Y. Hayakawa, and K. Nakajima, "Retrieval Property of Associative Memory Based on Inverse Function Delayed Neural Networks", *IEICE Trans. Fundamentals*, Vol.E88-A, No.8, pp.2192–2199, 2005.

[4] A. Sato, Y. Hayakawa, and K. Nakajima, "Avoidance of the Permanent Oscillating State in the Inverse Function Delayed Neural Network", *IEICE Trans. Fundamentals*, Vol.E90-A, No.10, pp.2775–2786, 2007.

[5] DE. Rumelhart, GE. Hinton, and JL. McClelland, " A general framework for parallel dustributed processing", *Paralel distributed processing*, Vol.1, pp45–76, 1986.

[6] S. Matsuda, " "Optimal" Neural representation of higher order for Traveling Salesman Problems", *Electronics and Communications in Japan*, Part 2, Vol.85, No.9, pp.32–42, 2002.

[7] M. Naganuma and T. Yanaru, "On the stability of Lagrange programming neural networks for satisfiability problems of prositional calculus", *Neurocomputing 13*, pp.119–133, 1996.

[8] Y. Hayakawa, T. Denda, and K. Nakajima, "Inverse function delayed model for optimization problems", *Proc. KES'04*, PART 1, Springer LNAI 3213, pp.981–987, Wellington, New Zealand, Sept. 2004.