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Spectral characteristics of consistency of a single-mode semiconductor laser injected with broadband random light

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Abstract—We numerically study characteristics of dynamical response of a semiconductor laser driven by broadband light with randomly fluctuating phase and amplitude. The output light waveform of the laser is thoroughly determined by the injected light when the laser exhibits a consistent response. We examine frequency components of the injected light that effectively affect the laser output. It is shown that components in a particular frequency band dominantly affects the laser output. We identify the center frequency and bandwidth of this band as functions of parameters of the laser system.

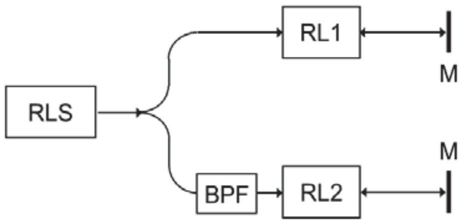

Figure 1: Illustration of the laser system configuration.

1. Introduction

In a variety of dynamical systems, it is often observed that a system reproduces a consistent output in response to a repeatedly applied external input signal: i.e., the output waveform dose not depend on the initial condition but is determined only by the external signal waveform. This property is called *consistency* [1]. It depends not only on the system but also on the type of external input signal whether the system has the consistency property.

The consistency property closely relates with synchronization between two independent dynamical systems which are subject to a common external input signal: two systems synchronize with each other if the system with a given input signal has consistency, and vice versa. This common-signal-induced synchronization (CSIS) has attracted much interest in particular for the case of random input signal, and it has been theoretically studied [2, 3, 4, 5, 6, 7].

The CSIS has been experimentally demonstrated in semiconductor lasers driven by common light, in which both the amplitude and phase fluctuate randomly [8, 9, 10] or only the phase fluctuates randomly with constant amplitude [11]. It has potential applications to secure communications. Recently, we have proposed a secure key distribution scheme using correlated randomness in lasers synchronized by injection of common random light with broad bandwidth, which has fast randomly fluctuating phase and/or amplitude [12, 13]. The security of this scheme relies on the difficulty of completely observing the broadband common random light with current technology. Such approach using the limits of observation technology is called *bounded observability* approach [14]. In order to achieve higher security in the above scheme, it is necessary to use a common random light with broader bandwidth, which is more difficult to completely observe. In addition, it is desirable that most part of the frequency components of the random light effectively affects the laser output, provided that the laser has consistency.

It has been shown that the CSIS of lasers is possible by random light with broad bandwidth up to the order of THz numerically [15] and experimentally [10]. These results show the consistency of a laser injected with random light of THz order broad bandwidth. In this paper, we consider a laser operated in such consistency regime and numerically investigate the frequency part of injected random light that effectively affects the laser output.

2. Model

Consider the laser system illustrated in Fig. 1, which is used in our simulation. A portion of light from a random light source (RLS) with broad bandwidth is injected into two semiconductor lasers, which we call response lasers (RL1, 2). The light has fast and randomly fluctuating phase and amplitude. The RLS can be implemented by using a super luminescent diode. The optical coupling is unidirectional from the RLS to the response lasers. A bandpass filter (BPF) is applied to the injected light for response laser 2. Each response laser has an external mirror (M) to form an optical self-feedback loop.

2.1. Laser

To model the semiconductor lasers in Fig. 1, we use the Lang-Kobayashi equation with optical injection [16]:

$$\frac{dE_j}{dt} = \left\{-i\Delta\omega_j + \frac{1+i\alpha}{2}G_N\left(N_j - N_{\mathrm{th}}\right)\right\}E_j + \frac{\kappa_{\mathrm{r}}}{\tau_{\mathrm{in}}}E_j(t-\tau)\exp[i\theta_j] + \frac{\kappa_{\mathrm{inj}}}{\tau_{\mathrm{in}}}E_{\mathrm{inj},j}(t), \quad (1)$$

$$\frac{dN_j}{dt} = J - \frac{1}{\tau_{\mathrm{s}}}N_j - G_N\left(N_j - N_0\right)|E_j|^2, \quad (2)$$

where $j = 1, 2$ indicate the response laser 1 and 2, respectively, E_j represents the complex electric field, N_j the carrier number density, α the linewidth enhancement factor, κ_{r} the optical feedback strength, τ the external-cavity delay time, $E_{\mathrm{inj},j}$ the complex electric field of the injected light to response laser j, and κ_{inj} the injection strength. Let $S(t)$ be the light generated by the RLS. The detuning parameter $\Delta\omega_j$ is defined by $\Delta\omega_j = \omega_0 - \omega_j$, where ω_0 is the center optical angular frequency of $S(t)$ and ω_j is that of the jth response laser. For later use, we define $f_0 = \omega_0/2\pi$.

2.2. Random light

We describe a model for the RLS output $S(t)$. Let $\rho(t)$ and $\phi(t)$ be fluctuations in the amplitude and phase of $S(t)$ defined by $S(t) = E_0|1+\varepsilon\rho(t)|\exp[i\phi(t)]$, respectively, where E_0 and ε are positive constants. We assume $\rho(t)$ and $\phi(t)$ are described by the stochastic differential equations

$$\frac{d\rho}{dt} = -\rho/\tau_{\mathrm{m}} + \sqrt{2/\tau_{\mathrm{m}}}\,\xi(t) \quad (3)$$

and

$$\frac{d\phi}{dt} = \sqrt{2/\tau_{\mathrm{m}}}\,\eta(t), \quad (4)$$

where τ_{m} is a positive constant. In Eqs. (3) and (4), $\xi(t)$ and $\eta(t)$ are the normalized white Gaussian noise with the properties $\langle\xi(t)\rangle = \langle\eta(t)\rangle = 0$, $\langle\xi(t)\eta(s)\rangle = 0$, and $\langle\xi(t)\xi(s)\rangle = \langle\eta(t)\eta(s)\rangle = \delta(t-s)$, where $\langle\cdot\rangle$ denotes the ensemble average and δ is Dirac's delta function. The amplitude $\rho(t)$ is the Ornstein-Uhlenbeck process, and it has the properties $\langle\rho(t)\rangle = 0$ and $\langle\rho(t)\rho(s)\rangle = \exp[-|t-s|/\tau_{\mathrm{m}}]$. This indicates that the correlation time of $\rho(t)$ is given by τ_{m}. On the other hand, $\phi(t)$ has the property $\langle[\phi(t)-\phi(s)]^2\rangle = 2\tau_{\mathrm{m}}^{-1}|t-s|$. Since $\phi(t)$ has the diffusion constant τ_{m}^{-1}, its characteristic time for correlation decay can be defined by τ_{m}. Therefore, τ_{m} gives the time scale of fluctuation of $S(t)$. This implies that the bandwidth of $S(t)$ is of the order of τ_{m}^{-1}.

The injected light is just given by $E_{\mathrm{inj},1} = S(t)$ for response laser 1. On the other hand, for response laser 2, it is given by the light obtained by passing $S(t)$ through a bandpass filter.

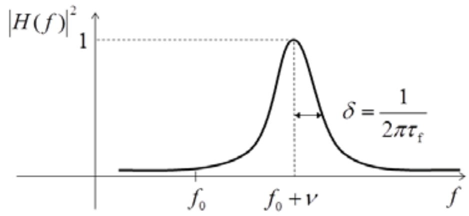

Figure 2: Illustration of $|H(f)|^2$ for the bandpass filter.

2.3. Bandpass filter

A bandpass filter is used for the injection to response laser 2. We describe its details. Let ν and τ_{f} be positive constants, and $S_\nu(t) = S(t)\exp[-i2\pi\nu t]$. Using this S_ν, we define a new variable Y by the equation

$$\tau_{\mathrm{f}}\frac{dY}{dt} = -Y + S_\nu(t). \quad (5)$$

Then, the injected light for response laser 2 is given by $E_{\mathrm{inj},2}(t) = Y(t)\exp[i2\pi\nu t]$. The frequency response function $H(f)$ of this filter, which transforms S to $E_{\mathrm{inj},2}$, is obtained as $H(f) = 1/\{1+i2\pi(f-\nu)\tau_{\mathrm{f}}\}$. Its modulus square is $|H(f)|^2 = 1/[1+\{2\pi\tau_{\mathrm{f}}(f-\nu)\}^2]$, which is illustrated in Fig. 2. The center frequency of $|H(f)|^2$ is given by $f_0+\nu$, and its half-maximum half width (HMHW) by $1/2\pi\tau_{\mathrm{f}}$, which we denote with δ.

2.4. Parameters

In our numerical simulations, the following parameter values were used: $G_N = 1.0\times10^{-12}\,\mathrm{m^3s^{-1}}$, $N_0 = 1.4\times10^{24}\,\mathrm{m^{-3}}$, $N_{\mathrm{th}} = 2.018\times10^{24}\,\mathrm{m^{-3}}$, $\tau_{\mathrm{in}} = 7.0\,\mathrm{ps}$, $\tau_{\mathrm{s}} = 2.04\,\mathrm{ns}$, $\tau = 0.3\,\mathrm{ns}$, $\theta_j = 0$, and $J = 1.09\,J_{\mathrm{th}}$, where $J_{\mathrm{th}} = N_{\mathrm{th}}/\tau_{\mathrm{s}}$ is the lasing threshold of injection current. For this value of J, the response lasers have the relaxation oscillation frequency 1.5 GHz. We assumed $\omega_1 = \omega_2$ and $\Delta\omega_j = 0$ for $j = 1, 2$. As for the injected light, we set $\tau_{\mathrm{m}} = 1.0\,\mathrm{ps}$, $\varepsilon = 0.3$, and $E_0 = [0.16J_{\mathrm{th}}/G_N(N_{\mathrm{th}}-N_0)]^{1/2}$. The other parameters α, κ_{inj}, and κ_{r} were varied in the simulations.

3. Numerical simulation

We are interested in frequency components of $S(t)$ that effectively affect the output of a response laser, provided that the laser has consistency. For this purpose, we compare the outputs of both response lasers. In order to measure the similarity of their outputs, we use the correlation between the output intensities $I_j(t)$ of the two response lasers, where $I_j(t) = |E_j(t)|^2$. The correlation between

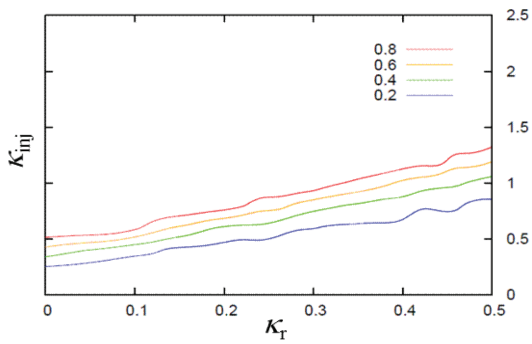

Figure 3: Region of consistency in $(\kappa_{\rm r}, \kappa_{\rm inj})$ plane. Contour lines of correlation C are shown, where $\alpha = 5$.

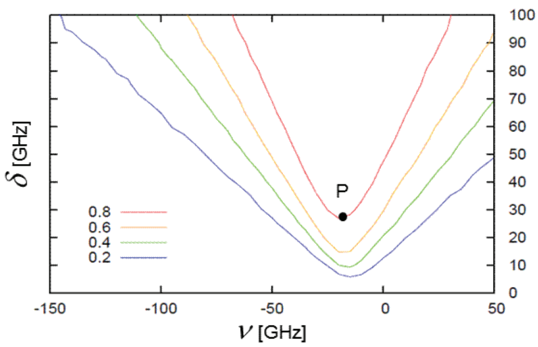

Figure 4: Output reproducibility of laser with filtered injection. Contour lines of correlation C are shown in (ν, δ) plane. Parameters are $\alpha = 5$, $\kappa_{\rm r} = 0$, and $\kappa_{\rm inj} = 1$.

$I_1(t)$ and $I_2(t)$ is defined as

$$C = \frac{\langle (I_1 - \mu_1)(I_2 - \mu_2) \rangle_T}{\sigma_1 \sigma_2}, \tag{6}$$

where μ_j and σ_j are the average and the standard deviation of I_j, respectively, and $\langle \cdot \rangle_T$ denotes the time average. By definition, C is in the range $-1 \leq C \leq 1$, and it takes the maximum $C = 1$ when the outputs are identical, i.e., $I_1(t) = I_2(t)$.

First, we identify a parameter region for the response lasers to have consistency. For this purpose, consider the case that an identical random light is injected into both response lasers, i.e., the case without bandpass filter. In this case, if two response lasers with different initial states generate the same output after a transient period, it indicates the consistency. Figure 3 shows contour plot of C as a function of $(\kappa_{\rm r}, \kappa_{\rm inj})$, where $\alpha = 5$. The condition $C > 0.8$ is satisfied in the region above red line, and this can be regarded as the region of consistency.

Next, we examine which frequency components of $S(t)$ dominantly affect the response laser output. Considering

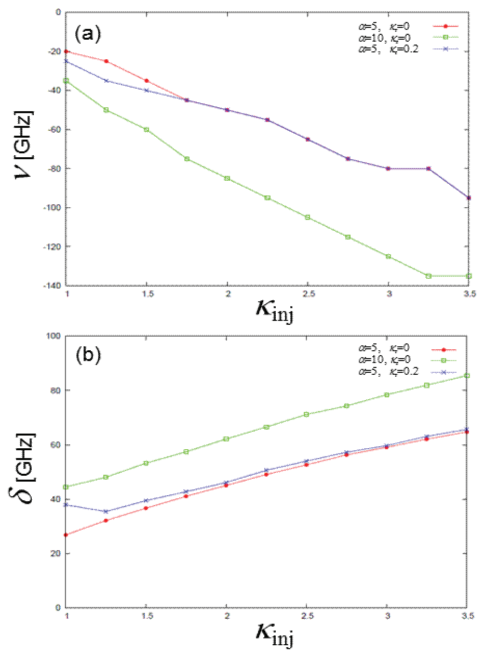

Figure 5: Dependence of PFB on injection strength $\kappa_{\rm inj}$. (a) $\nu_{\rm opt}$ vs. $\kappa_{\rm inj}$, and (b) $\delta_{\rm opt}$ vs. $\kappa_{\rm inj}$. Parameters are $(\alpha, \kappa_{\rm r}) = (5, 0)$ (red), $(10, 0)$ (green), and $(5, 0.2)$ (blue).

the case with a bandpass filter, we numerically calculated the correlation C between response laser 1 and 2 for different values of (ν, δ). Figure 4 shows contour plot of C in (ν, δ) plane. A value of C close to unity implies that response laser 2 can accurately reproduce the output of response laser 1 although their injection lights are not identical with each other. If we use $C = 0.8$ as a reference value, accurate output reproducibility with $C \geq 0.8$ is achieved in a wedge-shaped region above red line in Fig. 4. This region takes the minimum value of δ at point P located at $(\nu, \delta) \simeq (-20, 30)$: the minimum HMHW needed for achieving $C \geq 0.8$ is $\delta \simeq 30$ GHz, and the optimal center frequency of filtered band, which achieves $C \geq 0.8$ with this minimum HMHW, is given by $f_0 + \nu \simeq f_0 - 20$ GHz. This fact indicates that only the frequency components of $S(t)$ in the band of $\nu \simeq -20$ GHz and $\delta \simeq 30$ GHz almost determine the output of response laser 1. We call this frequency band a *principal frequency band* (PFB).

Figure 4 shows that larger values of δ are required to achieve $C \geq 0.8$ as ν deviates from its optimal value $\nu \simeq -20$ GHz. This increase in δ may be necessary to include the PFB in the filter-passed band, indicating that the PFB is essential in determining the response laser 1 output. Contour lines for different C values exhibit qualitatively the same behavior, and moreover their values of ν at the points of minimum δ are almost the same. This fact ensures that the center frequency of PFB is independent of a chosen reference value of C although the HMHW depends

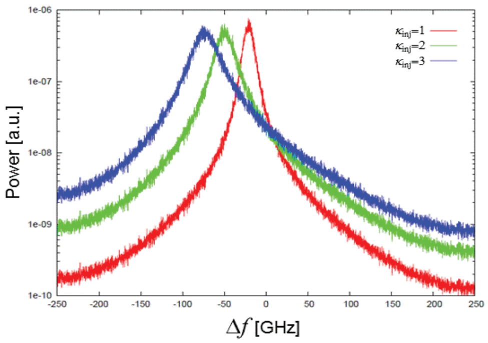

Figure 6: Power spectrum of response laser output. Parameters are $\alpha = 5$, $\kappa_{\rm r} = 0$, and $\kappa_{\rm inj} = 1$ (red), 2 (green), and 3 (blue).

on it. Hereafter, we fix the reference value as $C = 0.8$.

The PFB strongly depends on the injection strength $\kappa_{\rm inj}$. Let $\delta_{\rm opt}$ be a minimum value of δ, and $\nu_{\rm opt}$ be the value of ν at the point of minimum δ. Figures 5 (a) and (b) show $\nu_{\rm opt}$ and $\delta_{\rm opt}$ plotted as functions of $\kappa_{\rm inj}$, respectively, for different sets of $(\alpha, \kappa_{\rm r})$. They show that $\nu_{\rm opt}$ decreases while $\delta_{\rm opt}$ increases with increasing $\kappa_{\rm inj}$. It is clear that both $\nu_{\rm opt}$ and $\delta_{\rm opt}$ are almost independent of the feedback strength $\kappa_{\rm r}$. In contrast, the linewidth enhancement factor α significantly affects $\nu_{\rm opt}$ and $\delta_{\rm opt}$. It is desirable to make $\delta_{\rm opt}$ larger from the viewpoint of secure key distribution application. Therefore, a laser with larger α is suitable for this application.

We attempt to characterize the values of $\nu_{\rm opt}$ and $\delta_{\rm opt}$. In Fig. 6, we show the power spectrum of the real part of $E_1(t)$ for three different $\kappa_{\rm inj}$ values, where $\Delta f = f - f_0$ is the frequency measured from f_0. Comparing Fig. 6 with Fig. 5(a), we can see that $\nu_{\rm opt}$ coincides with the peak frequency of output of the response laser with injection. Figure 6 clearly shows that the HMHW of each spectrum increases with increasing $\kappa_{\rm inj}$. The HMHWs are 7, 12, and 16 GHz for $\kappa_{\rm inj} = 1, 2$, and 3, respectively. They are roughly in proportion to the $\delta_{\rm opt}$ values for $\kappa_{\rm inj} = 1, 2$, and 3, which are $\delta_{\rm opt} = 27, 45$, and 59 GHz.

4. Conclusions

We numerically studied spectral characteristics of the consistent response of a semiconductor laser injected with broadband random light. It has been found that the laser output is almost determined by partial frequency components of the injected light, which belong to a particular frequency band with bandwidth much narrower than that of the original random light, i.e., the PFB. The center frequency and bandwidth of the PFB depend on the injection strength $\kappa_{\rm inj}$, while they are insensitive to $\kappa_{\rm r}$. A broader bandwidth of the PFB is desired from the viewpoint of application to secure key distribution. We found that the PFB bandwidth increases as the linewidth enhancement factor α increases: the larger α the better for this application. It was shown that the center frequency and bandwidth of the PFB roughly coincide with the peak frequency and bandwidth of output of the response laser with injection, respectively.

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