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Pulse Wave Propagation Observed in a Ring of a Large Number of Inductor - Coupled Bistable Oscillators

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Abstract—In this paper, we investigate pulse wave propagation phenomenon in a ring of a large number of coupled bistable oscillators. In particular, we focus our attention to such a practical condition that each oscillator has a slightly different intrinsic oscillation frequency in addition to noise. We elucidate numerically that the propagating pulse can be observed by increasing the coupling strength even for such a practical condition. Moreover, we present the interaction phenomena between two pulses and confirm pulse unification, vanishing, repulsion and passing through phenomena.

1. Introduction

The study of systems of coupled oscillators has attracted constant interest in various areas of engineering, physics, and mathematics[1]. Examples include beam-scanning control system, Josephson junction arrays, model of information processing in the brain, etc. The rich behaviors of these systems, such as mutual entrainment, self-synchronization, wave propagation and so on, are observed[2, 3, 4, 5].

In our previous work, we investigated pulse wave propagation phenomenon in a ring of coupled identical bistable oscillators[6]. We confirmed that the main body of the propagating pulse wave consisted of several adjacent oscillators with large amplitude propagating with constant speed. Although such continuous propagating pulse wave has been investigated especially for reaction-diffusion systems[7, 8], there seems few studies on pulse wave propagation phenomenon observed in simple oscillatory array.

In this paper, we investigate the pulse wave propagation phenomenon in a ring of a large number of coupled bistable oscillators. In particular, we focus our attention to such a practical condition that each oscillator has a slightly different intrinsic oscillation frequency in addition to noise. This assumption is very natural and realistic, because there exists unavoidable error in each oscillator in addition to noise in practical circuit. We elucidate numerically that propagating pulse wave exists under both fluctuation and noise if the coupling strength is sufficiently large.

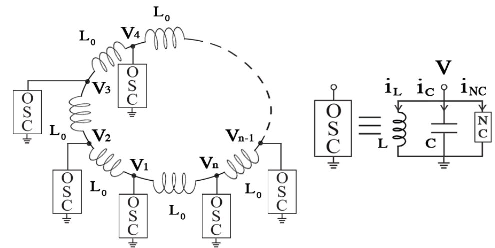


Figure 1: A ring of inductor-coupled bistable oscillators

2. Circuit model

Figure 1 presents a ring of inductor-coupled bistable oscillators. We assume the hard-type nonlinearity for NC, i.e., $i_{NC} = g_1V - g_3V^3 + g_5V^5$, $g_1, g_3, g_5 > 0$. Namely, each isolated oscillator has two steady-states: no oscillation and periodic oscillation depending on the initial condition. Assuming also a certain amount of noise , the circuit equation can be written by the following autonomous system after normalization[9]:

$$\begin{aligned}
\dot{x}_i &= y_i + A \cdot n_i(t)\\
\dot{y}_i &= -\varepsilon\omega_i(1-\beta x_i^2 + x_i^4)y_i\\
&\qquad -\omega_i^2(1+\alpha)x_i + \omega_i^2\alpha(x_{i-1} + x_{i+1})\\
&, \ i = 1, 2, \cdots, N, \quad x_0 = x_N, x_{N+1} = x_1,\\
&(\cdot = d/dt)
\end{aligned} \tag{1}$$

where N is the number of coupled oscillators. The x_i denotes the normalized output voltage of the i-th oscillator, y_i denotes its derivative and ω_i denotes the intrinsic angular frequency of the i-th oscillator. Parameter ε (> 0) shows the degree of nonlinearity. Parameter α ($0 \leqq \alpha \leqq 1$) is a coupling factor; namely $\alpha = 1$ means maximum coupling, and $\alpha = 0$ means no coupling. Parameter β controls amplitude of oscillation. The $n_i(t)$ is a time-varying uniform random number distributing from -1 to $+1$, which is introduced at each iteration of numerical integration (hence, this term can be regarded as a noise to this system).

Although we take $N = 100$ throughout this paper, pulse wave propagation phenomenon can be observed in an arbi-

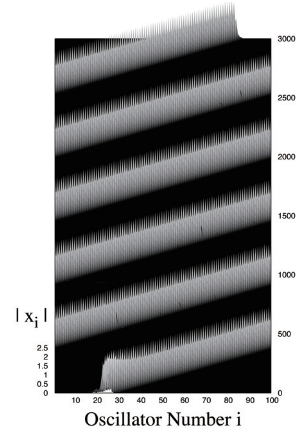


Figure 2: A bird's-eye view plot of typical single propagating pulse observed in coupled identical oscillators without noise for $\alpha = 0.3, \beta = 3.2$ and $\varepsilon = 0.36$. Initial condition is $x_{22} = -0.9$, $x_{23} = -0.7$, $x_{24} = 1.6$, $x_{25} = 1.4$, $y_{22} = 0.9$, $y_{23} = -1.6$, $y_{24} = -1.4$, $y_{25} = 1.2$ and all other variables are zero.

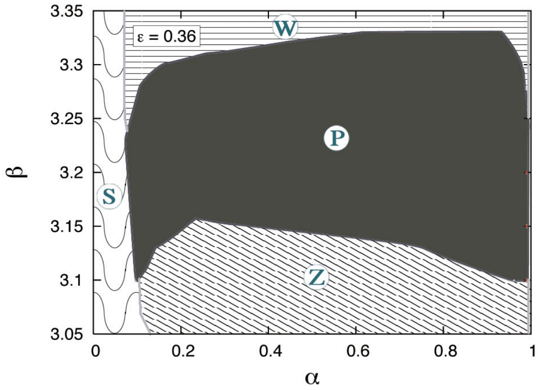


Figure 3: Existence regions of propagating pulse and other phenomena in coupled identical oscillators without noise for $\varepsilon = 0.36$. The "S", "W" and "Z" are existence regions of standing pulse, whole oscillation and no oscillation respectively. This figure is obtained from the same initial condition as Fig. 2.

trary number of coupled oscillators.[1] The reason why we adopt the ring structure as a manner of coupling, is that the effect of both ends can be neglected. For a large number of coupled oscillators, the dynamics of inside arrays are less influenced by the boundary condition. Therefore, we can observe almost qualitatively the same phenomena in the coupled lattice case for instance.

3. Pulse wave propagation phenomenon

3.1. A single propagating pulse wave

First of all, we will show a typical wave propagation phenomenon observed in coupled identical oscillator case without noise for review. Consequently, we put $\omega_i = 1$, $A = 0$ in Eq.(1). Figure 2 presents a bird's eye view plot of this phenomenon for $\varepsilon = 0.36, \alpha = 0.3$ and $\beta = 3.2$, where absolute value of x_i is plotted.[2] It is clearly seen that a pulse wave propagates with a constant speed which corresponds to the slant line in Fig.2. The main body of the propagating pulse wave consists of several adjacent oscillators with large amplitude[6]. Such a propagating pulse exists in a wide regime in the parameter space. Figure 3 presents the existence region of such a propagating pulse for $\varepsilon = 0.36$.[3] It is noted that if we set β to an appropriate value (nearly from 3.15 to 3.30), there exists the propagating pulse in a wide range of α (the "P" regime in Fig. 3). In the left-hand side of the propagating pulse regime (the "S" regime in Fig. 3), in which α is smaller than 0.1 approximately, there exists a standing pulse[4] instead of the propagating pulse. Around the region above the propagating pulse regime (the "W" regime in Fig. 3), whole oscillation such as all oscillators oscillate with large amplitude can be observed. In the region below the propagating pulse regime (the "Z" regime in Fig. 3), no oscillation exists. These three regions seem to overlap to some extent with the propagating pulse regime.

[1]We observe this phenomenon even for the $N = 6$ case.

[2]All numerical integrations are carried out by the fourth-order Runge-Kutta method with step size = 0.01.

[3]This result can be derived by computer simulation. we gradually vary α and β with step size = 0.01, and then we check whether propagating pulse exists or not in each point.

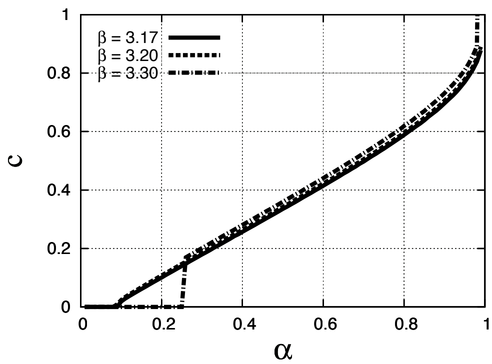


Figure 4: The propagation speed c in terms of α for three values of β for $\varepsilon = 0.36$.

Next, we investigate the propagation speed of a pulse wave. Figure 4 presents the propagation speed ($= c$) in terms of α (= coupling constant) with three values of β. The propagation speed is calculated by $c = N/T_N$, where T_N is the time for a pulse to go around the ring and where N is the number of oscillators. Namely, we numerically check how long does it elapse between a pulse wave pass through a certain oscillator and the moment it is gone back to the oscillator, and we define such time as T_N. Comparing the result for each β in Fig.4, the characteristics for $\alpha > 0.3$

[4]We call the pulses (oscillations) which are stationary in space as "standing pulse". They can be periodic, quasi-periodic, and chaotic.

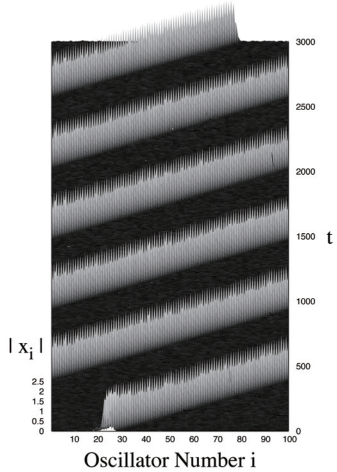


Figure 5: A single propagating pulse under both frequency fluctuation and noise for $\sigma = 0.02$, $A = 0.2$, $\alpha = 0.3$, $\beta = 3.2$ and $\varepsilon = 0.36$. Initial condition is same as Fig. 2

seems almost the same, but that for $\alpha < 0.3$ differs to some extent. Namely, there exists no propagating pulse for $\beta = 3.3$ for $\alpha < 0.3$, there exists propagating pulse for $\beta = 3.17$ and 3.20. Namely, the propagation speed much depends on α, though it is slightly influenced by the value of β.[5] When α is sufficiently small, propagation speed becomes zero, since there is no propagating pulse wave which is clearly shown as "S" and "W" regime in Fig.3. It is noted that the propagation speed becomes faster with larger α. For example, c for $\alpha = 0.9$ is approximately seven times as large as that for $\alpha = 0.2$ in case of $\beta = 3.2$. The actual values for each case is the following: $c = 0.706$ for $\alpha = 0.9$, and $c = 0.108$ for $\alpha = 0.2$.

Next, we assume that each oscillator has a slightly different intrinsic oscillation frequency in addition to noise. We distribute the intrinsic angular frequencies according to Gaussian distribution with average ($\equiv a$) equal to 1.0, and with various standard deviation ($\equiv \sigma$). We also give time-varying noise to this system by adding a uniform random number $A \cdot n_i(t)$ distributing from $-A$ to $+A$ to the first equation of Eq.(1) at each iteration of numerical integration. Figure 5 presents a bird's eye view plot for $\sigma = 0.02$ and $A = 0.2$ with the same initial condition and parameters as in Fig.2. It is clear that propagating pulse wave exist in spite of both fluctuation and noise. In this case, since the observed phenomenon is probabilistic, other phenomena such as the standing pulse may be observed with the same parameters and initial condition.

As we increase the coupling factor α gradually, the probability of emergence of the propagating pulse increases compared to other phenomena such as standing pulse, no oscillation and whole oscillation. Here, we make 30 trials for fixed α in order to check whether the propagating pulse exists or not. In each trial, the intrinsic angular frequencies change every time according to Gaussian distribution. The A is set to 0.2 which can be regarded as sufficiently large noise. Then, we calculate the probability (P = p / q), where p is the number of emergence of the propagating pulse and q is the trial number. We plot α versus P in Fig. 6 for $\beta = 3.2$ and $\varepsilon = 0.36$ by changing α. Comparing the result for $\sigma = 0.01$ with that for $\sigma = 0.02$, qualitatively the same characteristics can be observed, though the critical point of two curves (the point at which the probability becomes positive) are different. Namely, it is noted that when α is small, no propagating pulse exists. In particular, when α is nearly smaller than 0.1, the standing pulse exists for all trials. When α becomes larger, the propagating pulse exists in several trials out of 30 trials which include other phenomena. As α becomes larger, the probability tends to 1.0, which means that the propagating pulse exists for all trials. Therefore, we can say that the immunity of the propagating pulse against frequency fluctuation and noise becomes stronger as α is increased.

[5] In the results, the value of ε is fixed to 0.36 for simplicity. In case of other value of ε, the propagation speed much depends on α in the same way.

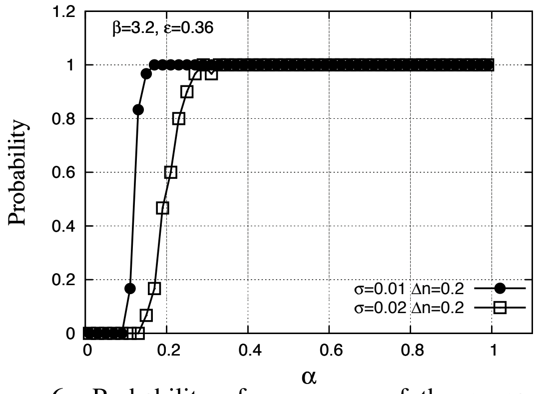
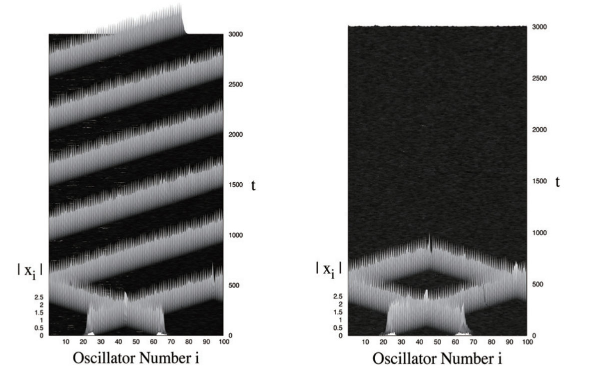


Figure 6: Probability of emergence of the propagating pulse in terms of coupling strength α for $\sigma = 0.01$ (0.02), $A = 0.2, \beta = 3.2$ and $\varepsilon = 0.36$.

3.2. Interaction phenomena

When we give large initial values to more than one place at the same time, multiple propagating pulses emerge. They collide at a certain time and interact with each other. In this section, we will investigate various pulse interaction phenomena under frequency fluctuation and noise. For simplicity, we show interaction among two pulses.

In coupled identical oscillators without fluctuation and noise, there exist repulsion and passing through phenomena depending mainly on initial condition[6]. Namely, if we give the reverse-phase initial condition to two places on the ring array, two pulse waves propagating in the opposite direction collide and repel. Similarly, when we give the same-phase initial condition, two pulse waves collide and pass through. However, under frequency fluctuation and noise, the above phenomenon can't be observed eas-



(a) pulse unification (b) pulse vanishing

Figure 7: A bird's-eye view plots for interaction among two pulses for $\alpha = 0.3, \beta = 3.2$ and $\varepsilon = 0.36$. In case (a), two propagating pulses pass through two times before only one pulse survives. In case (b), two propagating pulses pass through two times before they vanish. Initial conditions of case (a) and (b) is x_{22} (x_{67}) = −0.9, x_{23} (x_{66}) = −0.7, x_{24} (x_{65}) = 1.6, x_{25} (x_{64}) = 1.4, y_{22} (y_{67}) = 0.9, y_{23} (y_{66}) = −1.6, y_{24} (y_{65}) = −1.4, y_{25} (y_{64}) = 1.2 and all other variables are zero.

ily. For example, for both the same-phase and the reverse-phase initial conditions, in 30 trials in which the intrinsic angular frequencies distribute randomly obeying Gaussian distribution ($\sigma = 0.02$) in addition to noise ($A = 0.2$) as in the previous section, we observe a few samples of repulsive and passing through phenomena. However we observe other phenomena such as pulse unification and pulse vanishing phenomena in almost all other trials. Figures 7 (a) and (b) demonstrate two typical phenomena under frequency fluctuation and noise for the same-phase initial condition. Figure 7(a) shows that two propagating pulses pass through a few times before only one pulse survives (the other pulse may disappear or merge). Figure 7(b) shows that two propagating pulses pass through a few times before they vanish. It should be noted that as α is increased, the vanishing phenomenon is more easily observed for the same-phase initial condition. For example, the pulse vanishing phenomenon can be observed in 25 samples out of 30 trials (≑ 83%) for $\alpha = 0.9$, though it can be observed only in 8 samples out of 30 trials (≑ 27%) for $\alpha = 0.3$.

Next we will explain the phenomena observed for the reverse-phase initial condition. After all, the typical phenomena we observe are those of Fig. 7 (a) and (b); namely pulse unification or vanishing. In this case, the pulse unification phenomenon can be observed more than the pulse vanishing phenomenon as α is increased. In identical coupled oscillators without frequency fluctuation and noise, there also exists "the standing wave slip" phenomenon[6]. This phenomenon can not be observed under frequency fluctuation and noise. This is because the standing pulse exists only for small α in which case the immunity of the propagating pulse against frequency fluctuation and noise seems to be weak.

4. Conclusion

It is confirmed numerically that there exists the propagating pulse even under both frequency fluctuation and noise for comparatively large coupling strength (we observe mainly the standing pulses for small coupling strength). Moreover, we present the interaction phenomena among two pulses under such practical condition. These interactions present the interesting behaviors such as pulse repulsion, passing through, unification and vanishing, etc. Based on the obtained results, it is possible to assume that the pulse wave may be observed in practical circuit. As a future problem, we will investigate this system theoretically and also implement this system in actual circuit.

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