2011 International Symposium on Nonlinear Theory and its Applications
NOLTA2011, Kobe, Japan, September 4-7, 2011

Analysis of a Controlled Piecewise-Constant Circuit with Time-Delay Feedback

Daichi Manabe[‡] and Tadashi Tsubone[†]

†Nagaoka University of Technology 1603–1, Kamitomioka, Nagaoka, Nigata, 940–2188, Japan
Email: s103215@stn.nagaokaut.ac.jp

Abstract—In this study, we consider a piecewise-constant chaotic circuit with time-delayed feedback. The system exhibits some stabilized Unstable Periodic Orbits which are embedded on chaos attractor of the chaotic system as Delayed Feedback Controlled system. We consider providing the theoretical analysis for the condition of stability of the system and the domain of attraction. In general, it is relatively hard to treat a dynamical system with time-delayed components in theoretical sense. However, a dynamics of our proposed system is governed by piecewise-constant vector field, so it can be analyzed based on the simple geometry of the phase space. The fact makes us approach to rigorous analysis of the system behavior. In this paper, some experimental results are demonstrated in the real circuit.

1. Introduction

Since chaotic phenomena is characterized as sensitivity to initial conditions and perceived random behavior, it is difficult to predict the behavior in distant future. Hence chaos is often treated as objects that should be controlled and there are many works to stabilize Unstable periodic orbits (abbr. UPOs) embedded on chaos attractor which nonlinear dynamical systems exhibit. The approach is called as control of chaos in general [1].

As an example of methods of control of chaos, Delayed Feedback Control (abbr. DFC) proposed by Pyragas is well known [2]. DFC scheme stabilises UPOs by using the feedback of the difference between the current state and the delayed one. DFC has a significant advantage which requiring no preliminary calculation of the target periodic orbit or equilibrium point. However, DFC has a limitation such as any hyperbolic unstable periodic orbit with odd number condition can not be stabilized. The odd number condition means UPOs which have an odd-number of real characteristic multipliers greater than unity [3]. For continuous time systems, the odd number condition have been studied in detail by Nakajima [4] and Just at. al [5]. And some works to expand the range of application have been considered with maintaining the advantage of DFC [6][7].

In general, systems with time-delayed feedback are described by differential-difference equations and the state space has infinite dimension. The fact means that it is intrinsically hard to analyze the controlled system by DFC method.

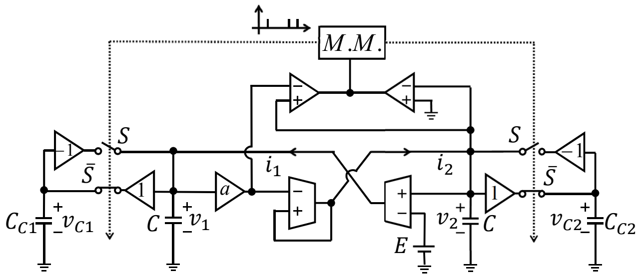

Figure 1: Circuit diagram of the piecewise-constant system

In this study, we focus on a piecewise-constant chaotic circuit with time-delayed feedback and consider the mechanism of stabilization of UPO by using time-delayed state feedback. The trajectories generated by the chaotic system is piecewise-linear rigorously, so the return map also must be piecewise-linear. By using the return map, we can analyze the basic system behavior theoretically [8]. In this paper, we propose a controlled system which has time-delayed feedback and maintains piecewise-constant characteristic. The system behavior with time-delayed feedback is governed by piecewise-constant vector field and can be treated on the view point of geometric sense. The fact suggests that the controlled system can be analyzed with respect to the stability and the domain of attraction, theoretically. In this paper, we show some experimental results from the real circuit and verify the stability.

2. A chaotic spiking oscillator with piecewise-constant vector field

Figure 1 shows the circuit diagram of the piecewise-constant circuit. The triangle labelled 1 (-1, respectively) is a linear amplifier with gain 1 (-1, respectively). The triangles labelled "$+\ -$" are comparators. These amplifiers and comparators are realized by an operational amplifier with sufficiently large input impedance. Trapezoids are differential voltage-controlled transconductance amplifiers and their output currents are i_1 and i_2, respectively. They are characterized by

$$\begin{aligned} i_1 &= I_a \cdot \mathrm{sgn}(v_2 - E), \\ i_2 &= I_a \cdot \mathrm{sgn}(v_2 - a v_1), \end{aligned} \quad (1)$$
$$\left(\mathrm{sgn}(x) = \begin{cases} 1 & \text{for } x \geq 0, \\ -1 & \text{for } x < 0. \end{cases} \right)$$

where v_1 and v_2 are voltages across the capacitors C. I_a is constant which is controlled by a bias current of transconductance amplifiers. Connecting two capacitors to both output terminals of the transconductance amplifiers, we obtain a two dimensional nonlinear system. When S is opened, the circuit dynamics is described by

$$\begin{aligned} \dot{x} &= \mathrm{sgn}(y-1), \\ \dot{y} &= \mathrm{sgn}(y-ax), \end{aligned} \tag{2}$$

where " $\cdot$ " represents the derivative of τ and the following dimensionless variable and parameters are used.

$$\tau = \frac{I_a}{CE}t, \quad x = \frac{1}{E}v_1, \quad y = \frac{1}{E}v_2. \tag{3}$$

Here, we assume the following parameter condition:

$$a > \frac{\sqrt{2}+1}{\sqrt{2}-1}. \tag{4}$$

In this parameter range, Equation (2) has unstable rect-spiral trajectories as shown in Fig. 2.

In this circuit as shown in Fig. 1, $M.M.$ is a monostable multivibrator which outputs pulse signals to close the switch S and to open $\bar{S}$ instantaneously. Two comparators detect impulsive switching condition. If $v_2 \le av_1$ or $v_2 \ge 0$, the switch S is opened and $\bar{S}$ is closed. For the meantime, the voltage v_1 and v_2 is stored to C_{C1} and C_{C2}, respectively. If $v_2 > av_1$ and $v_2 < 0$, then $M.M.$ is triggered by the pair of comparators, and the switch S is closed and $\bar{S}$ is opened instantaneously. At that time, the voltage v_1 and v_2 is reset instantaneously to the inverse voltage $-v_1$ and $-v_2$, respectively. That is

$$\begin{aligned} [v_1(t^+), v_2(t^+)]^T &= [-v_1(t), -v_2(t)]^T, \\ &\text{if } v_2(t) > av_1(t) \text{ and } v_2(t) < 0, \end{aligned} \tag{5}$$

where $t^+ \equiv \lim_{\epsilon \to 0}\{t+\epsilon\}$.

Because the parameter condition (4), the trajectory must reach to $\{(v_1, v_2)|v_2 > av_1, v_2 < 0\}$. Namely, any trajectories must hit l_{th}=$\{(x,y)|y = ax, y = 0\}$ and jumps from $(\frac{y(\tau)}{a}, y(\tau))$ to $(-\frac{y(\tau^+)}{a}, -y(\tau^+))$ as shown in the left figure of Fig. 2, where τ is the n-th switching moments.

Consequently, Eqn. (2) and (5) with the condition (4) are transformed into

$$\begin{aligned} &\left\{ \begin{array}{l} \dot{x} = \mathrm{sgn}(y-1), \\ \dot{y} = \mathrm{sgn}(y-ax), \end{array} \right. \quad \text{for } S = \text{off}, \\ &[x(\tau^+), y(\tau^+)]^T = [-x(\tau), -y(\tau)]^T, \\ &\quad \text{if } y(\tau) > ax(\tau) \text{ and } y(\tau) < 0, \\ &\quad \left(a > \frac{\sqrt{2}+1}{\sqrt{2}-1} \right). \end{aligned} \tag{6}$$

Now the system is characterized by only parameter a. The right figure of Fig. 2 shows a typical chaotic attractor with $a = 5.84$.

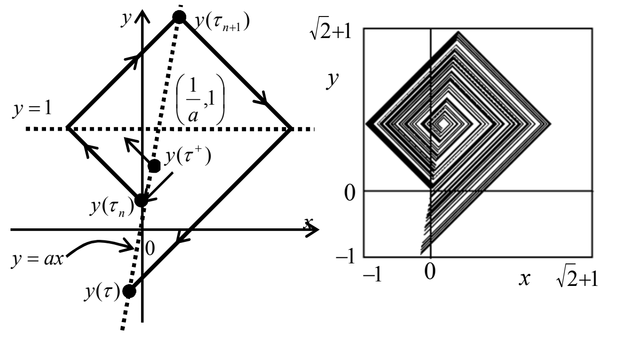

Figure 2: Behavior of Trajectories on the phase space and a typical chaos attractor. ($a \simeq 5.84$)

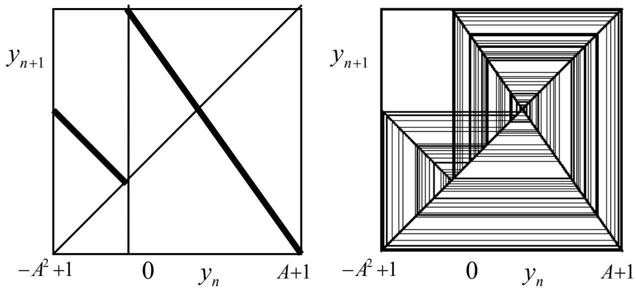

Figure 3: A typical 1-D return map

3. 1-D return map

In order to analyze the dynamics of the chaotic spiking oscillator, we derive a 1-D return map. Trajectory rotates divergently around the singular point $(\frac{1}{a}$,1) and reaches $l \equiv \{(x,y)|y = ax\}$ of piecewise constant vector field within a finite time. We define τ_n as the n-th reaching time to l. The trajectory starting from $(x(\tau_n), y(\tau_n))$ must return to $(x(\tau_{n+1}), y(\tau_{n+1}))$ on l. Letting $(x(\tau_n), y(\tau_n))$ on l be represented by its y-coordinate, we can define one dimensional return map f from l to itself and f is represented as

$$f : l \mapsto l, \quad y_{n+1} = f(y_n), \tag{7}$$

where we rewrite $y_n = y(\tau_n)$. By using piecewise-constant trajectory and linear algebraic procedure, we obtain an explicit expression for the function f:

$$f(y_n) = \left\{ \begin{array}{ll} -A(y_n - 1) + 1 & \text{for } y_n \ge 0, \\ -y_n & \text{for } y_n < 0, \end{array} \right. \tag{8}$$

where $A = \dfrac{a+1}{a-1}$. For the conditions $1 < A < 2$, Eqn. (8) exhibits a chaotic strictly[8]. Typical map f are shown in Fig. 3. Henceforth, this parameter condition is considered.

4. A piecewise-constant chaotic circuit with delayed feedback control

4.1. Delayed feedback control

DFC proposed by K. Pyragas is practically useful for control of chaos. Consider the following general dynamic system

$$\dot{x} = f(x) \qquad (x \in R^d). \tag{9}$$

DFC stabilizes UPO with period τ_d embedded in a chaotic attractor. The control input is given by

$$u(t) = K[y(\tau_d - t) - y(t)]. \tag{10}$$

Here τ_d is also a delay time. If this time coincides with the period of UPO, then the perturbation becomes zero on the stabilized periodic orbit. The controlled system is the followings

$$\begin{aligned} \dot{y} &= f(x) + u(t) \\ &= f(x) + K(y(\tau_d - t) - y(t)). \end{aligned} \tag{11}$$

K is an control gain. Choosing an appropriate K can achieve the stabilization. DFC has an advantage such that no preliminary calculation of the UPO is not required. However, it is difficult to analyze since the closed loop system is described by a differential-difference equation with infinite dimension.

4.2. Proposed method

We consider delayed time feedback is applying to the piecewise-constant circuit. We propose a following method which includes the control term in the signum function.

$$\begin{cases} \dot{x} = \text{sgn}(y - 1 + u_x(\tau)), \\ \dot{y} = \text{sgn}(y - ax), \end{cases} \tag{12}$$

$$u_x(\tau) = K(x(T - \tau) - x(\tau)). \tag{13}$$

The method is different from the typical DFC and maintains the piecewise-constant vector field. So the system can be analyzed based on the simple geometry of the phase space.

4.3. Computational simulation

The effectiveness of our method is confirmed by computational simulations. The system dynamics is described by

$$\begin{cases} \dot{x} = \text{sgn}(y - 1 + u_x(\tau)), \\ \dot{y} = \text{sgn}(y - ax), \end{cases} \tag{14}$$

$$u_x(\tau) = K(x(T - \tau) - x(\tau)), \tag{15}$$

$$(x(\tau^+), y(\tau^+)) \quad = \quad (-x(\tau), -y(\tau)), \tag{16}$$

if $y(\tau) > ax(\tau)$ and $y(\tau) < 0$,

where the following parameters are used,

$$T = 3.886, \quad a = \frac{\sqrt{2}+1}{\sqrt{2}-1} \simeq 5.84, \quad K = 0.2. \tag{17}$$

The goal is to stabilize the orbit of one cycle UPO. Since the system has piecewise constant characteristic, the moved distance one cycle UPO is equal to elapsed time. In fact, T can be led to 3.883 from 1-D return map easily. We simulate the system on the following conditions.
(Case 1) not controlled,
(Case 2) Initial value is $(x_0, y_0) = (-0.061, 1.199)$,
(Case 3) Initial value is $(x_0, y_0) = (0.221, -0.457)$,

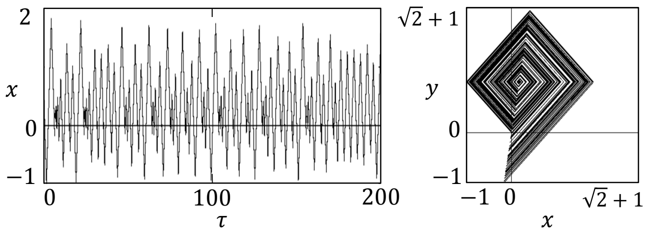

Figure 4: Chaos attractor with $K = 0$ (case 1)

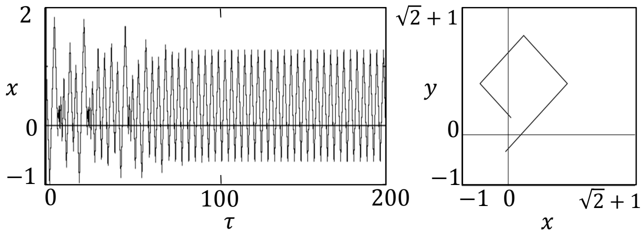

Figure 5: Stabilized periodic attractor with $K = 0.2$ (case 2)

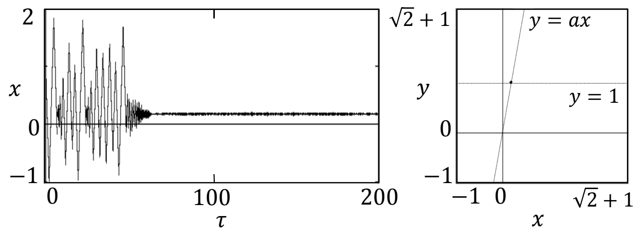

Figure 6: Converged trajectory to singular point with $K = 0.2$ (case 3)

Figure 4 shows the original chaos attractor of the system not controlled. Figure 5 shows the case of applying control scheme after $\tau = 50$, in this Case 2, stabilized periodic orbit with one cycle is confirmed. Also Fig. 6 shows the case of applying control scheme after $\tau = 48$, in this Case 3, the trajectory converges to singular point $(\frac{1}{a}, 1)$. That is, depending on the initial condition at the moment of starting the control, some co-existence of attractors are observed.

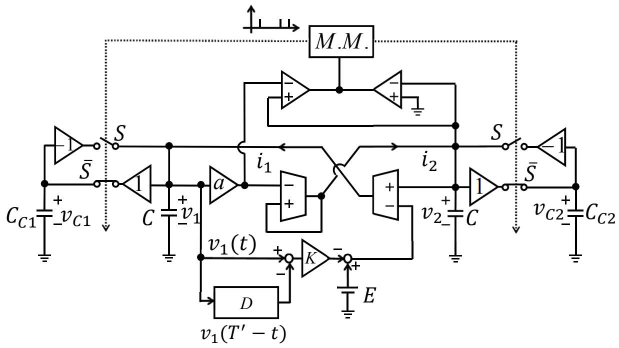

Figure 7: Experimental circuit with delayed feedbacks.

5. The experimental circuit

5.1. Implement circuit and dynamics

Figure 7 shows the controlled circuit. We use OTA: LM13700 for transconductance amplifier, LM339 for comparator, 4538 for monostable multivibrator and 4066 for analog switch. In order to generate time-delayed states, Digital signal processor 6713 DSK is used. The circuit dynamics is represented by

$$\begin{cases} C\frac{dv_1}{dt} = \text{sgn}(v_2 - E + KD_1), \\ C\frac{dv_2}{dt} = \text{sgn}(v_2 - av_1), \\ D_1 = v_1(T' - t) - v_1(t), \end{cases} \quad (18)$$

$$(v_1(t^+), v_2(t^+)) \quad = \quad (-v_1(t), -v_2(t)), \quad (19)$$

if $v_2 > av_1$ and $v_2 < 0$,

$$\tau = \frac{I_a}{CE}t, \;\; x = \frac{1}{E}v_1, \;\; y = \frac{1}{E}v_2, \quad (20)$$

where parameter values are followings: $C = 10$[nF], $a \simeq 5.9$, $E = 1.0$[V]. Figure 8 and 9 shows chaos attractor and stabilized periodic orbit, respectively.

6. Conclusion

We considered a piecewise-constant chaotic circuit with time-delayed feedback. From the system, some stabilized UPO can be confirmed by using time-delayed feedback state. The dynamics of our proposed system was governed by piecewise-constant vector field, namely it was suggested that the simple analysis based on geometry approach of the phase space is possible. Some experimental results were demonstrated in the real circuit. In the future, we will provide analytical result for the stability and the domain of attraction.

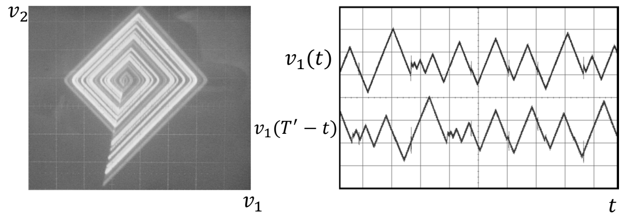

Figure 8: Chaos attractor and delayed states (horizontal axis: v_1[500mV/div],vertical axis: v_2[500mV/div] for left column. horizontal axis: $t[\mu$sec/div], vertical axis: $v_1(t), v_1(T' - t)$[1V/div] for right column. $K = 0$)

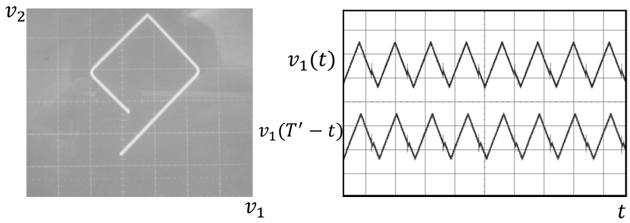

Figure 9: A stabilized periodic orbit and delayed states (horizontal axis: v_1[500mV/div],vertical axis: v_2[500mV/div] for left column. horizontal axis: $t[\mu$sec/div], vertical axis: $v_1(t), v_1(T'-t)$[1V/div] for right column. $K = 0.13$)

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