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Evaluation of individual differences in injection locking of relaxation oscillators

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Abstract—When a common non-periodic signal injects to non-linear oscillators, it exhibits synchronization phenomena. This phenomena is called as an injection locking for some oscillators against the external force. In general, it supposes that all oscillators are identical. Therefore, the discussion of the case where the oscillators are not identical is not sufficient. Then, we investigate a relationship between individual difference and synchronization. We experiment with the relaxation oscillator circuit having individual differences.

1. Introduction

When a common noise is injected into oscillators, the oscillators exhibit synchronization phenomena[1]. Such phenomenon is called a noise-induced synchronization. Teramae and Tanaka analyzed the noise induced synchronization phenomena numerically and theoretically when a white noise is induced into the oscillators[1]. Also, the synchronization phenomena are observed too in the case where the colored noise is induced[2][3]. The noise induced synchronization is confirmed by an implementation circuit[4][5].

The noise induced synchronization phenomenon is realized that the oscillators are driven by the common noise signal. In other words, the oscillators are not dependent on the initial value. In the precedence research, the synchronization state of the oscillators is measured by Lyapunov exponents[1][6][5]. However, if each oscillator has some differences, the discussion of the synchronization is not sufficient. Such differences prevent the oscillators synchronization. To develop engineering applications by the noise synchronization, we have to clarify the relationship between the oscillator mismatching and the synchronization.

In this article, we consider the effect of individual difference of oscillators. Our objective oscillator is a piecewise linear relaxation oscillator that can be analyzed by exact solution. Also, an implementation of the relaxation oscillator is easily. By using such relaxation oscillator, we investigate the noise synchronization phenomena in the case where each oscillator has parameter mismatching. By using the implementation relaxation oscillators, we observe the oscillator state depending on the individual difference. We develop the noise generator to apply the external force to the oscillators. The generator can change the range of the values that is a uniform distributed noise. By using this generator, we investigate the effect of the variation of the external force noise signal.

2. Relaxation oscillator with a time variant threshold

In order to clarify the synchronization phenomena of the relaxation oscillators, when the common non-periodic external force is injected, we measure the synchronization status. Figure 1 shows a circuit diagram of the relaxation oscillator. The oscillator consists of a piecewise linear bipolar hysteresis element. The threshold of hysteresis is driven by a binary optical signal. The system injects the external force by the optical signal in order to insulate between each oscillator circuit. This circuit is regarded as a electric firefly (EFF)[7]. Therefore, this relaxation oscillator circuit is called an EFF in this article.

The circuit equations of the EFF are shown in the following equations.

$$\begin{cases} C(R_1+R_2)\frac{\mathrm{d}}{\mathrm{d}t}v_c(t) &= -v_c(t)+v_o(t), \\ v_o(t) &= \begin{cases} +E & v_c(t) < v_s(t) \\ -E & v_c(t) > v_w(t) \end{cases}, \end{cases} \quad (1)$$

$$\begin{cases} v_s(t) &= \begin{cases} Ea_1 & u(t) \text{ is } 1 \\ Ea_2 & u(t) \text{ is } 0 \end{cases}, \\ v_w(t) &= Eb, \end{cases} \quad (2)$$

$$\begin{cases} a_1 &= -b = (VR_4+R_5)/(R_3+R_4+R_5) \\ a_2 &= (VR_4+VR_5)/(R_3+R_4+VR_5) \end{cases}, \quad (3)$$

where, $VR_4 \in [0, 1\text{k}]$, $VR_5 \in [0, 100\text{k}]$ is the variable resistor. $v_c(t)$ is a capacitor voltage, $v_o(t)$ denotes a binary output voltage. These $v_c(t)$ and $v_o(t)$ are internal state variable of the EFF. $v_s(t)$ and $v_w(t)$ are the upper threshold and the lower threshold of the bipolar hysteresis, respectively. $v_s(t)$ is changed by the external force $u(t)$.

Here, we consider the following conversion.

$$\tau = \frac{t}{RC},\ x = \frac{v_c}{E},\ y = \frac{v_o}{E},\ S = \frac{v_s}{E},\ W = \frac{v_w}{E} \quad (4)$$

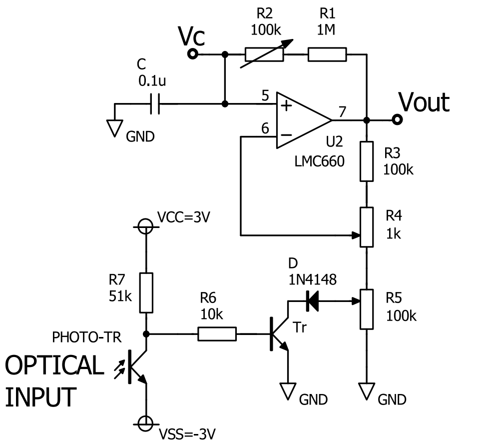

Figure 1: EFF circuit diagram.

By using the above conversion, we derive the normalized equation. The normalized equation is described by

$$\begin{cases} \frac{\mathrm{d}}{\mathrm{d}\tau}x(\tau) &= -x(\tau) + y(\tau), \\ y(\tau) &= \begin{cases} 1 & x(\tau) < S(\tau) \\ -1 & x(\tau) > W(\tau) \end{cases}, \end{cases} \tag{5}$$

$$\begin{cases} S(\tau) &= \begin{cases} a_1 & u(\tau) \text{ is } 1 \\ a_2 & u(\tau) \text{ is } 0 \end{cases}, \\ W(\tau) &= b, \end{cases} \tag{6}$$

where, $x(t)$ denotes a state variable of the relaxation oscillator, $y(t)$ denotes a bipolar hysteresis output. $S(t)$ and $W(t)$ are threshold of the binary hysteresis. $S(t)$ is time-variant that is driven by a external force of $u(t)$. EFF has three threshold parameters a_1, a_2, and b.

$u(t)$ is described by the following equation.

$$u(\tau) = \begin{cases} 0 & \tau_n \le \tau < \tau_n + r_n h_n \\ 1 & \tau_n + r_n h_n \le \tau < \tau_{n+1} \end{cases}, \ n \in \mathbb{N}, \tag{7}$$

$$\tau_{n+1} = h_n + \tau_n, \tag{8}$$

where, $u(\tau) = 1$ means the light injected state, and $u(\tau) = 0$ is the light non-injected state. τ_n denotes a start timing of the n-th pulse, $h_n > 0$ denotes an interval of the n-th pulse, and $r_n \in (0, 1)$ denotes a duty rate of the n-th pulse.

This oscillator has a long-period oscillation mode and a short-period oscillation mode. These oscillation modes are switched by the external force.

3. The response to non-periodic external force

In this section, we investigate the response of the relaxation oscillator, when a common non-periodic external force is injected. The non-periodic external force is a pulse waveform that has random period, and the duty r_n of the pulse is fixed to 0.5. The distribution of the random periods corresponds to the frequency distribution in the external force.

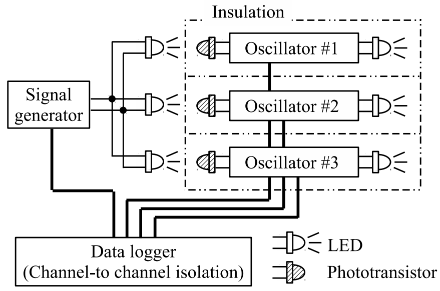

Figure 2: Experimental system.

Figure 2 shows the experimental system. Our developed signal generation circuit generates the common external force. The signal generator is configured by the Arduino Uno. The signal generator derives a light signal into the EFF. The data logger (GRAPHTEC GL900-4) records the oscillation states of EFFs and the external force signal. For accurate measurements, each recording channel of the data logger must be insulated each other. The used data logger satisfies this requirement.

3.1. EFFs of small individual differences

In this section, we investigate the synchronization phenomena when EFFs have small individual differences.

First, we measure the characteristics of EFFs where the external force is a constant. In other words, $u(\tau) = 0$ or $u(\tau) = 1$. Table 1 shows the measured oscillation period of each EFF. In this table, 'on' denotes the oscillation period of long-period oscillation mode, 'off' denotes the oscillation period of short-period oscillation mode. The oscillation period of each EFF will almost match.

We measure synchronization phenomena when a non-periodic pulse is injected into EFFs. The pulse interval of the non-periodic external force is driven by uniform distribution random noise. The pulse interval τ_n is fluctuated as the following equation.

$$h_n = T + \mathrm{Uniform}(-\Delta T,\ \Delta T), \tag{9}$$

h_n is a uniform distribution random number in the range form $T - \Delta T$ to $T + \Delta T$, and the expected value is T. ΔT is varied from 0[ms] to 30[ms] in interval at 10[ms]. The excepted period T is varied form 150[ms] to 300[ms] in interval at 1[ms]. The data logger records the measurement data. The sampling interval is 1[ms], and the measurement time is 3[min].

Next, we calculate the temporal correlation coefficients of the outputs to investigate the synchronization state. Figure 3 shows the temporal correlation coefficients between each EFF and the external force. Figure 4 shows the temporal correlation coefficient between each EFF. $cc_{i,j}$ $i, j \in \{1, 2, 3, EF\}$ denotes the temporal correlation coefficient between i and j. If two signals are in-phase synchronization state, the temporal correlation coefficient is 1. We consider the relationship between the external force and each

Table 1: Oscillation period (individual difference is small)

	Oscillation period[ms]	
	on	off
EFF1	250.1	200.2
EFF2	250.1	200.1
EFF3	250.0	200.0

Table 2: Oscillation period (individual difference is large)

	Oscillation period[ms]	
	on	off
EFF1	250.1	200.1
EFF2	255.2	205.1
EFF3	245.0	195.0

EFF. Figure 3a is a case that the periodic external force is injected. Each EFF and the external force are synchronized when the period of the external force is from 200[ms] to 300[ms]. The synchronization range is narrow when ΔT is large. We investigate the synchronization between each EFF. Figure 4a shows the case where a periodic external force is injected Each EFF is synchronized when the period of the external force is from 200[ms] to 300[ms]; This is consistent with the synchronization range of the external force and each EFF. In addition, the synchronization observed in the other range. The cause is that the EFF driven by the periodic external force has multiple stable states. Therefore, if each EFF has the same stable state, EFF archives a synchronization. In addition, each EFF has a stable state when the period of external force is from 200[ms] to 250[ms]. Therefore each EFF is synchronized with each other in this case. If ΔT is large, the temporal correlation coefficient increased. That is, when the variation of the external force is large, the synchronization can be easily induced.

Results of the average period indicate that the EFF has the same response to the external force. Almost synchronization in this system is in-phase synchronization. The in-phase synchronization range of EFF is large, when the variance of the external force is large. In the case where the individual difference of EFF is small, these observed results indicate that the variation in external force is affected to induce in-phase synchronization.

3.2. EFFs of large individual differences

Next, we consider the case where the individual difference is large. First, we measure the characteristics of EFFs where external force is constant. Table 2 shows the oscillation period of each EFF. Each EFF has a different oscillation period.

We inject the uniform distribution signal into the EFF that has a large individual difference. ΔT is varied from 0[ms] to 30[ms] in interval at 10[ms]. The excepted period T is varied form 150[ms] to 300[ms] in interval at 1[ms]. The data logger records the measurement data. The sampling interval is 1[ms], and the measurement time is 3[min].

To investigate the synchronous state, we calculate the temporal correlation coefficients of the output time series. Figure 5 shows the temporal correlation coefficients between each EFF and the external force. Figure 6 shows the temporal correlation coefficients between each EFF. The correlation coefficients between each EFF and the external force are the same tendency as if the individual difference is small. However, in-phase synchronization range of the external force and the EFF is different for each EFF. The synchronization range between each EFF is the only overlapping area of the synchronization range between each EFF and the external force. If ΔT is large, the trend is similar. In addition, when ΔT is large, the synchronization range between each EFF becomes narrow. In other words, the variation of the external forces interferes with the synchronization. This situation is quite different from the case where the individual difference is small.

The synchronization range is given by the overlap range of all EFF and the external force in the case where the individual difference is large. In addition, the variation of the external force interferes with the synchronization. Therefore, the synchronization range becomes narrow.

4. Conclusions

In this article, we observed the synchronization phenomena in the EFF whose threshold driven by the common non-periodic external force. As a result, we have been confirmed the synchronization phenomena between the EFF with the common non-periodic external force. In the case where the individual difference is small, we confirmed that the synchronization range is wide. In this case, the synchronization range has been extended by the variation of the external force. If the individual difference is large, the synchronization range is narrow. In addition, the variation of the external force affects to narrow the synchronization range. Thus, the influence of the variations in the external force is different from the individual differences of the oscillators.

We will derive the synchronization condition theoretically in the future.

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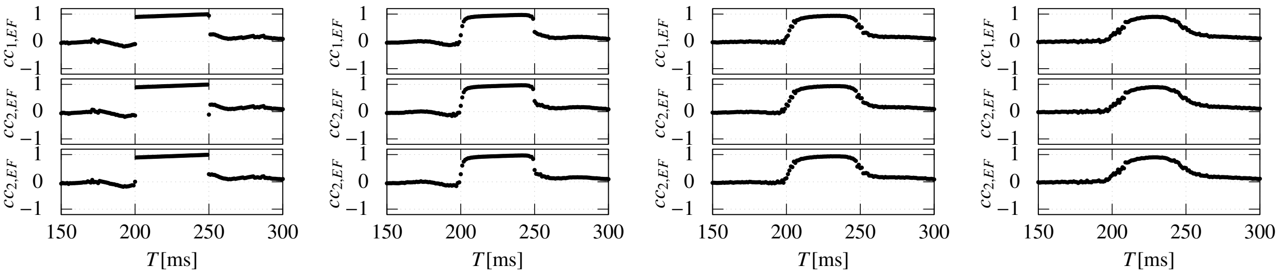
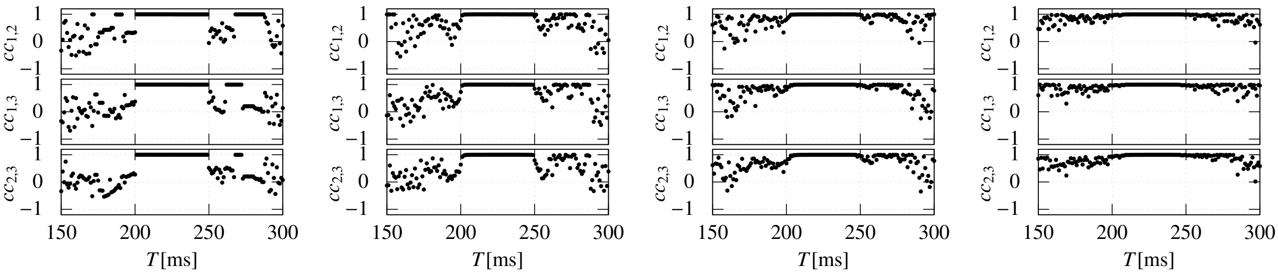


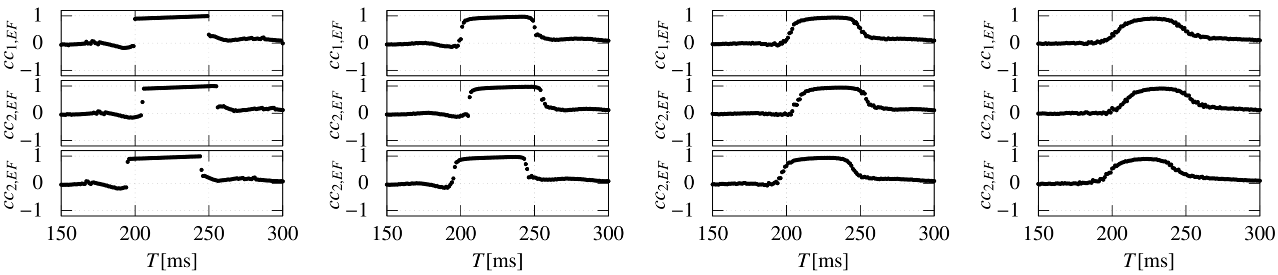
(a) ΔT = 0[ms] (b) ΔT = 10[ms] (c) ΔT = 20[ms] (d) ΔT = 30[ms]

Figure 3: Temporal correlation coefficient between each EFF and external force (individual difference is small).



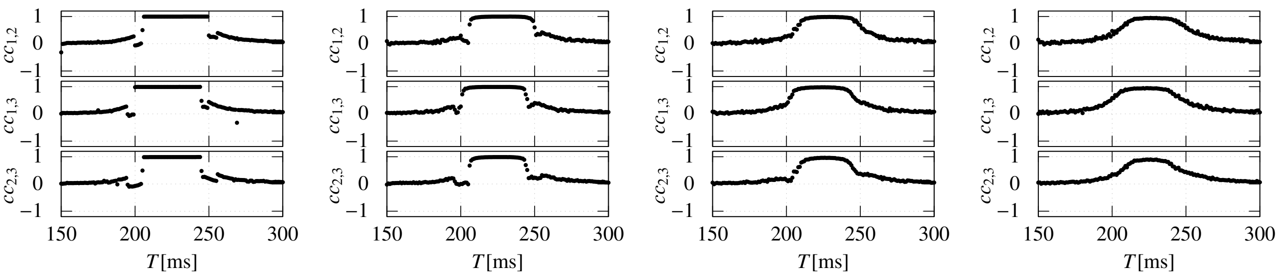
(a) ΔT = 0[ms] (b) ΔT = 10[ms] (c) ΔT = 20[ms] (d) ΔT = 30[ms]

Figure 4: Temporal correlation coefficient between EFFs (individual difference is small).



(a) ΔT = 0[ms] (b) ΔT = 10[ms] (c) ΔT = 20[ms] (d) ΔT = 30[ms]

Figure 5: Temporal correlation coefficient between each EFF and external force (individual difference is small).



(a) ΔT = 0[ms] (b) ΔT = 10[ms] (c) ΔT = 20[ms] (d) ΔT = 30[ms]

Figure 6: Temporal correlation coefficient between EFFs (individual difference is small).

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