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Time-varying delay-connection induced amplitude death in a pair of double-scroll circuits

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Abstract—Time-delay connection induced amplitude death has been considered as an attractive phenomenon in the field of nonlinear science. Our previous study [Konishi, Kokame, Hara, 2010] analytically revealed that a time-varying delay connection can induce amplitude death and derived a systematic procedure to design the connection. The present paper investigates amplitude death in a pair of the well-known double-scroll circuits. The amplitude death region in a connection parameter space is obtained using a linear stability analysis. The amplitude death is experimentally observed in the coupled circuits. It is confirmed that the region in the connection parameter space on our circuit experiments agrees well with that on the analytical results.

1. Introduction

Various interesting phenomena in coupled oscillators have attracted a growing interest [1]. It is well accepted that amplitude death, the connection induced stabilization of coupled oscillators, is an important phenomenon in the field of nonlinear science [2, 3]. Although amplitude death never occurs in identical coupled oscillators [3, 4], a transmission delay in connections can induce it [5]. The time-delay induced amplitude death has been the subject of many research papers [6, 7, 8, 9, 10, 11, 12, 13].

Amplitude death has significant potential for practical applications because it is the stabilization of unstable steady states in coupled nonlinear oscillators. For the practical situations where the connection delay is long due to long-distance signal transmission, it is impossible to induce the amplitude death. A few researchers proposed the ideas which can induce death even by long connection delays. Atay reported that long distributed delay connections facilitate amplitude death [14]. Konishi *et al.* showed that the multiple long delay connections can induce it [15]. These reports provided useful ideas; however, it would be difficult to realize the distributed delay connections and the cost of the multiple delay connections would be higher.

In recent years, we analytically showed [16] that a time-varying delay connection, which overcomes the disadvantages reported in the studies [14, 15], can induce amplitude death. Although our previous paper analyzed the stability of amplitude death and provided a systematic procedure to design connections, there is no experimental verification of the analytical results.

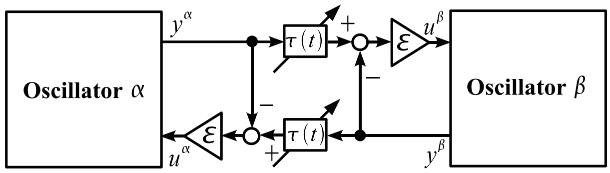


Figure 1: Block diagram of a pair of oscillators (1) coupled by connections (2).

The present paper experimentally shows that the time-varying delay connection can induce amplitude death in a pair of the well-known double-scroll circuits. The circuits are easily implemented by popular-priced circuit devices; the time-varying delay signals are realized by the peripheral interface controllers (PICs) which are the microcontrollers made by Microchip Technology. We confirm that the amplitude death region in a connection parameter space on the circuit experiments agrees well with that on the analytical results.

2. Coupled oscillators [16]

Let us consider m-dimensional oscillator α and oscillator β, as illustrated in Fig. 1,

$$\begin{aligned} \dot{\boldsymbol{x}}^{\alpha,\beta} &= \boldsymbol{F}\left(\boldsymbol{x}^{\alpha,\beta}\right) + \boldsymbol{b}u^{\alpha,\beta}, \\ y^{\alpha,\beta} &= \boldsymbol{c}\boldsymbol{x}^{\alpha,\beta}, \end{aligned} \tag{1}$$

where $\boldsymbol{x}^{\alpha,\beta} \in \boldsymbol{R}^m$ are the state variables and $u^{\alpha,\beta} \in \boldsymbol{R}$ are the coupling signals. $y^{\alpha,\beta} \in \boldsymbol{R}$ are the output signals. $\boldsymbol{b} \in \boldsymbol{R}^m$ and $\boldsymbol{c} \in \boldsymbol{R}^{1\times m}$ are the input and output vectors. The fixed point of each oscillator without coupling (i.e., $u^{\alpha,\beta} \equiv 0$) is written as $\bar{\boldsymbol{x}} : \boldsymbol{F}(\bar{\boldsymbol{x}}) = \boldsymbol{0}$. The coupling signals are described by

$$u^{\alpha,\beta} = \varepsilon \left\{ y^{\beta,\alpha}_{\tau(t)} - y^{\alpha,\beta} \right\}, \tag{2}$$

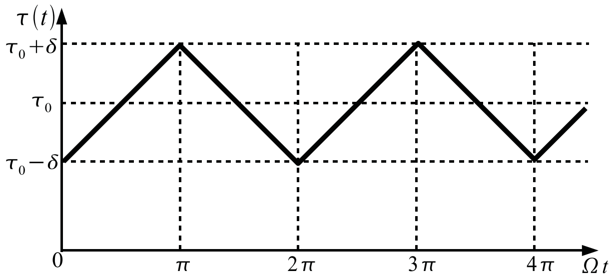


Figure 2: Sketch of time-varying delay $\tau(t)$: periodic sawtooth type function.

where $y^{\alpha,\beta}_{\tau(t)} := y^{\alpha,\beta}(t-\tau(t))$ are the delayed output signals and $\varepsilon > 0$ is the coupling strength. The time delay $\tau(t) \geq 0$ varies around a nominal delay $\tau_0 > 0$ with amplitude $\delta \in [0, \tau_0]$ as follows (see Fig. 2):

$$\tau(t) := \tau_0 + \delta f(\Omega t).$$

$\Omega > 0$ is the frequency of variation. $f(x)$ is the periodic sawtooth type function,

$$f(x) := \begin{cases} +\dfrac{2}{\pi}\left(x - \dfrac{\pi}{2} - 2n\pi\right) & \text{if } x \in [2n\pi, (2n+1)\pi), \\ -\dfrac{2}{\pi}\left(x - \dfrac{3\pi}{2} - 2n\pi\right) & \text{if } x \in [(2n+1)\pi, 2(n+1)\pi), \end{cases}$$

for $n = 0, 1, \ldots$. Oscillators (1) with connections (2) have the homogeneous steady state: $\begin{bmatrix}\boldsymbol{x}^{\alpha T} & \boldsymbol{x}^{\beta T}\end{bmatrix}^T = \begin{bmatrix}\bar{\boldsymbol{x}}^T & \bar{\boldsymbol{x}}^T\end{bmatrix}^T$. Here, $\boldsymbol{x}^{\alpha,\beta} = \bar{\boldsymbol{x}} + \Delta\boldsymbol{x}^{\alpha,\beta}$ are substituted into the coupled oscillators. The linearized systems at the homogeneous state are described by

$$\Delta\dot{\boldsymbol{x}}^{\alpha,\beta} \;=\; \boldsymbol{A}\Delta\boldsymbol{x}^{\alpha,\beta} + \varepsilon\boldsymbol{b}\boldsymbol{c}\left\{\Delta\boldsymbol{x}^{\beta,\alpha}_{\tau(t)} - \Delta\boldsymbol{x}^{\alpha,\beta}\right\}, \quad (3)$$

where $\Delta\boldsymbol{x}^{\beta,\alpha}_{\tau(t)} := \Delta\boldsymbol{x}^{\beta,\alpha}(t-\tau(t))$ and $\boldsymbol{A} := \{\partial\boldsymbol{F}(\boldsymbol{x})/\partial\boldsymbol{x}\}_{\boldsymbol{x}=\bar{\boldsymbol{x}}}$. Linear systems (3) can be rewritten as

$$\dot{\mathsf{x}}(t) = \mathsf{A}\mathsf{x}(t) + \mathsf{B}\mathsf{x}(t-\tau(t)), \quad (4)$$

where $\mathsf{x}(t) := \begin{bmatrix}\Delta\boldsymbol{x}^{\alpha T} & \Delta\boldsymbol{x}^{\beta T}\end{bmatrix}^T$,

$$\mathsf{A} := \begin{bmatrix}\boldsymbol{A} - \varepsilon\boldsymbol{b}\boldsymbol{c} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{A} - \varepsilon\boldsymbol{b}\boldsymbol{c}\end{bmatrix},\ \mathsf{B} := \begin{bmatrix}\boldsymbol{0} & \varepsilon\boldsymbol{b}\boldsymbol{c} \\ \varepsilon\boldsymbol{b}\boldsymbol{c} & \boldsymbol{0}\end{bmatrix}.$$

According to study [17], we notice that if a time-invariant comparison system,

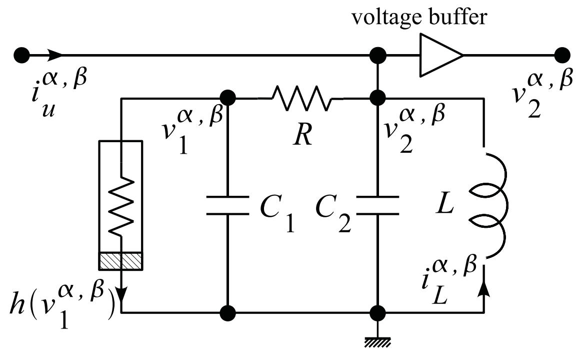
$$\dot{\mathsf{x}}(t) = \mathsf{A}\mathsf{x}(t) + \frac{1}{2\delta}\mathsf{B}\int_{t-\tau_0-\delta}^{t-\tau_0+\delta}\mathsf{x}(\theta)\mathrm{d}\theta, \quad (5)$$

is asymptotically stable, then linear system (4) is stable for large Ω. The stability of system (5) is governed by the roots of its characteristic equation,

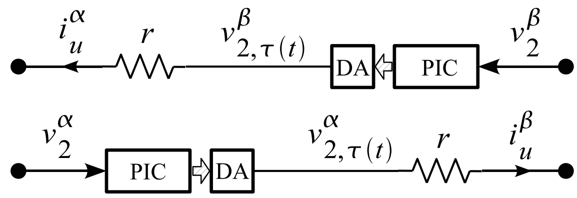
$$g(\lambda) := \det\left[\lambda\boldsymbol{I} - \mathsf{A} - \mathsf{B}e^{-\lambda\tau_0}H(\lambda\delta)\right] = 0, \quad (6)$$

where

$$H(x) := \begin{cases}(\sinh x)/x & \text{if } x \neq 0, \\ 1 & \text{if } x = 0.\end{cases}$$



(a)



(b)

Figure 3: Circuit diagrams: (a) double-scroll circuits; (b) time-varying connections.

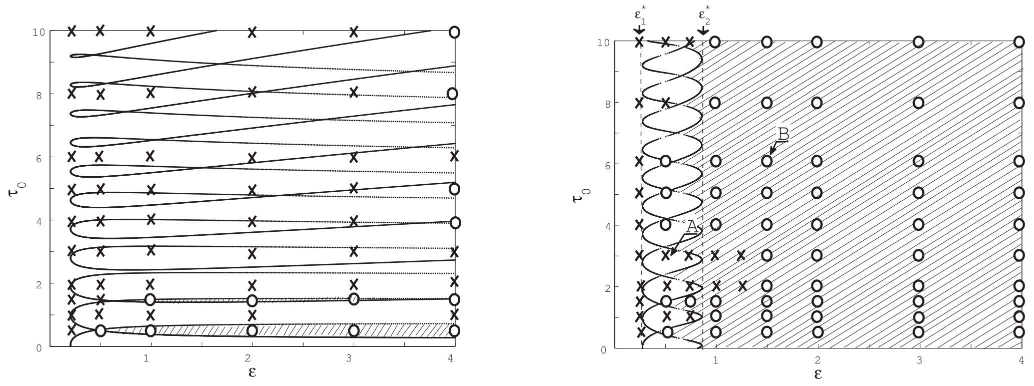
3. Coupled double-scroll circuits

As illustrated in Fig. 3(a), the double scroll circuits [18] are governed by

$$\begin{cases} C_1\dfrac{\mathrm{d}v_1^{\alpha,\beta}}{\mathrm{d}t} &=& \dfrac{1}{R}\left(v_2^{\alpha,\beta} - v_1^{\alpha,\beta}\right) - h\left(v_1^{\alpha,\beta}\right) \\ C_2\dfrac{\mathrm{d}v_2^{\alpha,\beta}}{\mathrm{d}t} &=& \dfrac{1}{R}\left(v_1^{\alpha,\beta} - v_2^{\alpha,\beta}\right) + i_{\mathrm{L}}^{\alpha,\beta} + i_u^{\alpha,\beta} \\ L\dfrac{\mathrm{d}i_{\mathrm{L}}^{\alpha,\beta}}{\mathrm{d}t} &=& -v_2^{\alpha,\beta} \end{cases}. \quad (7)$$

$v_1^{\alpha,\beta}$ [V], $v_2^{\alpha,\beta}$ [V], and $i_{\mathrm{L}}^{\alpha,\beta}$ [A] denote the voltages across C_1 [F], C_2 [F], and the current through L [H] respectively. Currents $h\left(v_1^{\alpha,\beta}\right)$ [A], where

$$h(v) := m_0 v + \frac{1}{2}(m_1 - m_0)\left|v + B_{\mathrm{p}}\right| + \frac{1}{2}(m_0 - m_1)\left|v - B_{\mathrm{p}}\right|,$$



(a) (b)

Figure 4: Stability regions in ε-τ_0 plane: curves and shaded area are the stability boundaries and the stability regions, which are analytically estimated by Eq. (6); symbol ◯ ($\times$) denotes the occurrence (non occurrence) of stabilization experimentally. (a) time-constant delay connection ($\delta = 0$), (b) time-varying delay connection ($\delta = 0.56, \Omega = 28$).

flow through the nonlinear resistors.

The circuit diagram of the time-varying connections is illustrated in Fig. 3(b). The voltages $v_2^{\alpha,\beta}$ are applied to PIC (PIC18F2550) devices. These devices read the voltages via their own analog to digital converters, and output the delayed digitized signals. These signals are transformed into the delayed voltages $v_{2,\tau(t)}^{\alpha,\beta} := v_2^{\alpha,\beta}(t-\tau(t))$ by the digital to analog converters (DAs) using R/2R resistor network. The currents $i_u^{\alpha,\beta}$ through resistors r are described by

$$i_u^{\alpha,\beta} = \frac{1}{r}\left(v_{2,\tau(t)}^{\beta,\alpha} - v_2^{\alpha,\beta}\right).$$

Circuits (7) are described by the dimensionless form Eq. (1) with

$$\boldsymbol{F}(\boldsymbol{x}) := \begin{bmatrix} \eta\left\{x_2 - x_1 - g(x_1)\right\} \\ x_1 - x_2 + x_3 \\ -\gamma x_2 \end{bmatrix},\ \boldsymbol{b} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},\ \boldsymbol{c} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}^T,$$

where the dimensionless time $t/(RC_2)$ is used instead of the real time t. The state variables, the parameters, and the nonlinear function are rewritten as

$$\begin{aligned} & x_1^{\alpha,\beta} := \frac{v_1^{\alpha,\beta}}{B_{\mathrm{p}}},\ x_2^{\alpha,\beta} := \frac{v_2^{\alpha,\beta}}{B_{\mathrm{p}}},\ x_3^{\alpha,\beta} := \frac{i_L^{\alpha,\beta} R}{B_{\mathrm{p}}}, \\ & a := m_1 R,\ b := m_0 R,\ \eta := C_2/C_1,\ \gamma := R^2 C_2/L, \\ & g(x) := bx + (b-a)\left\{|x-1| - |x+1|\right\}/2. \end{aligned}$$

Each oscillator without connection (i.e., $u^{\alpha,\beta} \equiv 0$) has three fixed points: $\bar{\boldsymbol{x}}_\pm := \begin{bmatrix} \pm p & 0 & \mp p \end{bmatrix}^T$ and $\bar{\boldsymbol{x}}_0 := \boldsymbol{0}$, where $p := (b-a)/(b+1)$. For simplicity, the present paper considers the stabilization of $\bar{\boldsymbol{x}}_+$. The dynamics of oscillators (1) coupled by connections (2) around $\bar{\boldsymbol{x}}_+$ is described by Eq. (4), where

$$\boldsymbol{A} = \begin{bmatrix} -\eta(b+1) & \eta & 0 \\ 1 & -1 & 1 \\ 0 & -\gamma & 0 \end{bmatrix},\ \varepsilon = \frac{R}{r}.$$

4. Circuit experiments

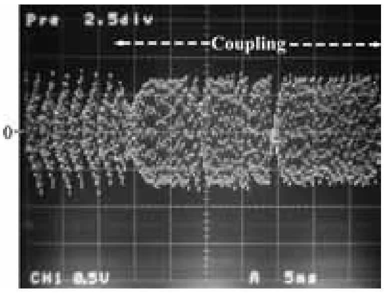
In this paper, the parameters are fixed at

$$\begin{aligned} & C_1 = 0.01 \times 10^{-6}\mathrm{F},\ C_2 = 0.1 \times 10^{-6}\mathrm{F}, \\ & L = 18 \times 10^{-3}\mathrm{H},\ B_p = 1.0\mathrm{V},\ R = 1800\Omega, \\ & m_0 = -0.4 \times 10^{-3},\ m_1 = -0.8 \times 10^{-3}, \end{aligned}$$

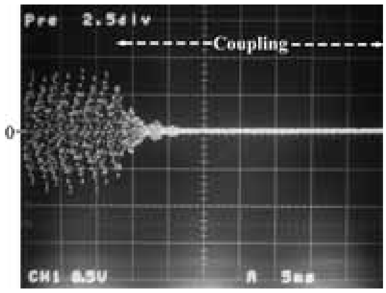
where the double-scroll attractor exists in each oscillator without connection. We derive the stability boundary curves of Eq. (6) by using the procedure proposed in our previous study [16], as shown in Fig. 4. From these curves, the stability region (i.e., shaded area in Fig. 4) can be obtained.

Figure 4(a) shows the curves and the regions for the time-constant delay connection (i.e., $\delta = 0$). It can be seen that there exist a few small stability regions on ε-τ_0 plane. In contrast, for the time-varying delay connection with $\delta = 0.56$ and $\Omega = 28$, as shown in Fig. 4(b), there exists no curve in the wide range $\varepsilon \in (\varepsilon_2^*, +\infty)$ on ε-τ_0 plane. This fact implies that there is no upper limit of τ_0 in the range: the fixed point is stabilized by the arbitrarily long nominal delay τ_0 when ε is chosen from the range.

We fill in symbol ◯ ($\times$) in ε-τ_0 plane if the stabilization (non-stabilization) is experimentally observed.



(a)



(b)

Figure 5: Time series data of the circuit voltage v_2^α: (a) point A ($\varepsilon = 0.5$, $\tau_0 = 3.0$), (b) point B ($\varepsilon = 1.5$, $\tau_0 = 6.0$) in Fig. 4. Horizontal axis: t (5 ms/div); vertical axis: v_2^α (0.5 V/div).

The time series data of the voltage v_2^α is shown in Fig. 5(a). At time $t = 12.5 \times 10^{-3}$ s, the oscillators are coupled by the connections with parameters $(\varepsilon = 0.5, \tau_0 = 3.0)$ corresponding to point A in Fig. 4(a). It can be seen that the voltage does not converge on the fixed point. Meanwhile, Fig. 5(b) shows that for the connections corresponding to point B, the voltage converges on the fixed point. The stability region on the analytical estimation roughly agrees with that on the circuit experiments.

5. Conclusion

The present paper experimentally showed that the time-varying delay connections induce the amplitude death in the coupled double-scroll circuits. The stability analysis agrees with the results of our experiments.

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