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Stochastic models for climate reconstructions – how wrong is too wrong?

Johannes P. Werner[†,∗], Andrea Toreti[‡] and Juerg Luterbacher[∗]

†Bjerknes Centre for Climate Research and Department of Earth Science, University of Bergen
Postboks 7800, NO 5020 Bergen, Norway
‡Institute for Environment and Sustainability, Joint Research Centre, European Commission
Via E. Fermi 2749 I 21027 Ispra (VA), Italy
∗ Justus Liebig University Giessen, Department of Geography Climatology, Climate Dynamics and Climate Change
Senckenbergstrasse 1, D 35390 Giessen, Germany
Email: johannes.werner@geo.uib.no, andrea.toreti@jrc.ec.europa.eu, juerg.luterbacher@geogr.uni-giessen.de

Abstract—In the last few years, new methods for climate field reconstructions have been proposed. Apart from some stationarity assumptions they usually rely on having an i.i.d. temperature process from year to year. In this contribution we investigate how a mismatch between the used model and the observed process parameters can introduce additional errors into the reconstruction. We conclude that the error introduced by the model mismatch is usually less severe than the one caused by the rather noisy data recording process in the climate archives used.

1. Introduction

While in past centuries climate has been considered to be stable, we have learned in the last decades that this is not the case. Clearly, marked transitions took place with ice ages and warm periods, but also on shorter time scales changes can be observed. The projections of possible future climate are done using dynamical models driven by changes in solar irradiance, volcanic aerosols, and green house gases. In order to test whether these models can indeed correctly describe Earth's climate they need to be compared to observations – which unfortunately are only available for the last one or two centuries at most. Thus, the climate of the past needs to be inferred from climate archives (proxies). These proxies can be seen as noisy recorders of climate information: trees close to an ecotone mostly react to one single stressor, like summer temperatures for the Alpine tree line, thus records of past tree growth can be used to reconstruct summer temperatures. To this end, a (usually statistical) model is used to model temperature evolution and the dependence of tree growth on it. In the simplest, the summer temperatures of subsequent years are independent from each other and a linear reaction of tree growth on climate is proposed [1, 2]. There are two major points for improvement: tree growth is a nonlinear function and both temperature and moisture can play an important role [3]; secondly, there exists a spatio-temporal process linking the state of climate variables of subsequent years. Here we focus on this latter part because little has been done to make improvements. For instance, the proposed reconstruction method of [4], employed for reconstruction of Arctic temperatures [5] but also tested over Europe [6], only assumes i.i.d. normal innovations. Here, we use the Kramers-Moyal-Expansion (KME) [7, 8, 9] on European temperature data to see whether this simplification actually holds and what the influence of observed vs. modelled climate parameters is. We show how reconstruction skill deteriorates as a function of model mismatch, but conclude that for most purposes the influence of non-ideal climate archives is likely to play a more important role.

2. The Kramers-Moyal-Expansion (KME)

We now briefly describe the underlying assumptions; for a more detailed discussion see for example [10, 7, 11] or textbooks such as [12, 13]. While the method can in principle be extended to more than one dimension [14], the amount of available data is insufficient in this application, as there are at most 160 years of observations. The system behaviour is assumed to be relatively continuous in the sense of the Central Limit Theorem and we argue that large jumps are absent in the year to year evolution of temperature anomalies. Thus, on the considered time scales the process can be approximated to be continuous in the limit of infinitely small step size.

Such a stochastic system can be represented by a stochastic differential equation (SDE) of the Langevin type $\mathrm{d}x_t = f(x_t)\,\mathrm{d}t + g(x_t)\,\mathrm{d}W_t$, where $f(x_t)$ and $g(x_t)$ are the deterministic and the stochastic part of the dynamics and $\mathrm{d}W_t$ is a Wiener process. In general the SDE has time varying parts f, g. Here, estimates from time slices of a 2000 year long climate model simulation indicate minor changes that are compatible with stationary functions. It can also be more useful to look at the probabilities and probability currents through the corresponding Fokker-Planck-equation

$$\partial_t p(x,t) = -\partial_x \left[D^{(1)}(x) - \partial_x D^{(2)}(x) \right] p(x,t).$$

The two functions $D^{(1,2)}(x)$ are the deterministic drift and the stochastic diffusion coefficient respectively. Using the Kramers-Moyal expansion we can estimate them:

$$D^{(1)}(X) \quad = \lim_{\tau\to 0} \tfrac{1}{\tau} \left\langle x(t+\tau) - x(t) \right\rangle_{x(t)=X} \qquad (1)$$

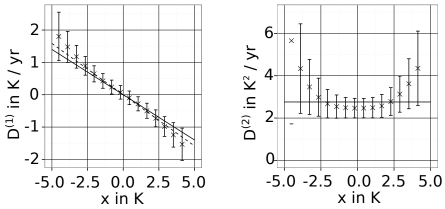

Figure 1: KME of annually averaged temperature data at 47.5° N, 7.7° E, drift and diffusion estimates.

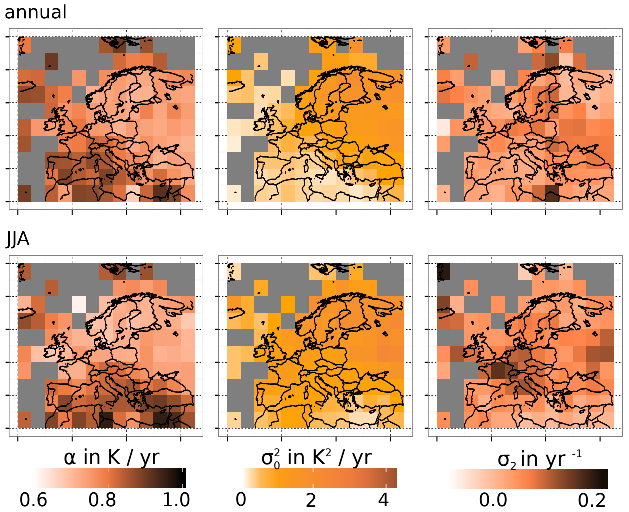

Figure 2: KME of CRU temperature data for annual averages (top) and summer (bottom row) temperatures. Shown are results of fitting a linear function to the drift and a quadratic function to the diffusion.

$$D^{(2)}(X) \quad = \lim_{\tau\to 0} \frac{1}{2\tau} \left\langle (x(t+\tau) - x(t))^2 \right\rangle_{x(t)=X} \tag{2}$$

In most texts, the limit $\lim_{\tau\to 0}$ is taken. The effect of finite sampling time on the conditional moments and thus on the form of the KME coefficients has been investigated by [11]. Since we deal with seasonal/annual mean values, we follow the approach of [8]. Similar to their study we are not interested in the evolution for infinitesimal times, but the averaged effect of a fast varying variable. We therefore evaluate (1,2) for finite times. The shape of the resulting drift and diffusion coefficients are then approximated by low order functions whose coefficients are determined through optimisation. In contrast to [8] we stay in the Itô interpretation of the SDE.

3. Application of the KME to European temperature data

We apply the KME to the gridded instrumental CRUTemp4v [15] data on a 5° × 5° grid. This dataset has been used in recent climate field reconstructions [16, 5] and it provides anomalies w.r.t. the 1961-90 mean. The estimated deterministic drift term at 47.5° N, 7.7° E is displayed in figure 1a (crosses with 95% confidence interval). A linear function is fitted to the data (solid line). The linear function is related to the postulated [4, 6] AR(1) local temperature process

$$T_{t+1} - \mu_T = \alpha\,(T_t - \mu_T) + \epsilon_{T,t}, \tag{3}$$

where α, μ_T are, respectively, the AR(1) coefficient and the process mean, $\epsilon_{T,t} \sim N(0, \sigma_T^2)$ is the noise term, modelled to be purely additive in [4] and [6]. Using from now on temperature anomalies $x_t = T_t - \mu_T$, we can see that the corresponding deterministic part of the Langevin equation would read $f(x_t) = (\alpha - 1)\,x_t$.

Next we consider the diffusion term and the resulting type of noise in the Langevin equation. Looking a the experimental diffusion coefficient (crosses and error bars in figure 1b), we can see that the results indicate a non-zero curvature to the diffusion. We assess the deviation from the idealised case (3) by fitting $D^{(2)}(x) = \sigma_0^2 + \sigma_2 x^2$ to the estimated diffusion.

The spatial distribution of the parameters is shown in figure 2. The first column displays the linear coefficient α. It is mostly between 0.7-0.9, except for some grid cells over the Atlantic and the Mediterranean. In the latter region as much as two thirds of the data are missing, which leads to more uncertain estimates. The second and third column of figure 2 show the estimated additive noise strength σ_0^2 and the curvature of the diffusion σ_2. Interannual variability is lower in the South East and higher in the North West. The curvature of the diffusion term (third column in 2) is mostly smaller than 0.2 everywhere. While clearly being present in the instrumental data, the non-additive term of the interannual variability seems relatively small. However, the question remains whether a stochastic description using an additive noise term as in (3) is justifiable.

In principle, using Bayesian inference on a hierarchy of models [5, 17] permits inclusion of arbitrary stochastic models. However, complexity comes with a trade-off: parameters and variables are usually estimated using a Gibbs sampler, iteratively updating each value using the analytically derived expressions for the posterior conditional probability densities. For simple models, draws are taken from distributions for which fast and stable implementations exist. For more complicated posteriors the draws are sampled using e.g. Metropolis-Hastings like algorithms. This leads to additional computational steps when employing Bayesian inference. We will discuss the necessary extensions to the model of [18] and discuss the additional computational challenges. Afterwards, we will show estimates of the reconstruction error introduced by a too simple stochastic model.

4. Influence of model mismatch in climate reconstructions

The quadratic temperature dependence of the diffusion can be seen as an increased variability of next year's (or season's) temperatures following an extremely cold (or

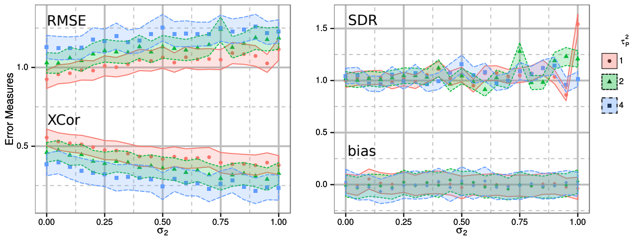

Figure 3: Reconstruction errors for three proxy noise levels , $\tau_P^2 = \{1, 2, 4\}$. Left: RMSE and cross correlation coefficient, right: SDR and bias.

warm) year. While the numerical value seems to indicate only a small correction, we nevertheless should test its influence. Thus, we need to extend (3) with a corresponding term:

$$x_{t+1} = \alpha x_t + \sqrt{\sigma_0^2 + \sigma_2 \cdot x_t^2}\, \epsilon_{T,t}, \tag{4}$$

where $\epsilon_{T,t}$ is still a Gaussian process. The extension to temperature anomalies in space could be through a covariance matrix $\Sigma = \Sigma_0 + x_t^T \Sigma_2 x_t$. While this extension does not seem like a major rewrite, it has dire consequences for the estimation process used by [6]. The relatively simple posterior density distributions derived by [4] need to be modified with the temperature anomalies now showing up in the variance of the posterior probability density for the temperature anomalies. The additional costs of this modification are twofold: first, inclusion of a second spatial covariance structure Σ_2, either to be parametrised or estimated during the inference. As noted by [19], estimating such a spatial covariance matrix is difficult and convergence cannot be guaranteed. Most importantly, an expensive Metropolis-Hastings step is needed to actually estimate the conditional PDF for each temperature in each time step.

Now, we estimate the errors caused by assuming the simple model in (3) when the true data follows a process as in (4). As the main goal of [4] and [6] is to reconstruct past climate from long climate archives, we need to know how big the actual impact of using a too simple temporal model is on the reconstruction quality compared to errors from the input data noise. Long proxy time series can have large uncertainties attached to the data itself and the exact proxy response function. The transfer function describing the response of the proxy on the climate field variables also is often unknown.

To assess the additional error introduced by the model mismatch we construct a 1000 time step long artificial data set following the procedure outlined by [4]. We vary the quadratic term σ_2, using the noise term as in (4). When constructing the time series one has to take care that for finitely big time steps the Euler-Maruyama method does not necessarily converge for this drift and diffusion [20]. Additionally the stationary density degenerates asymptotically to to a delta function around the origin as the curvature σ_2 increases. The resulting temperature anomaly data (created with zero offset, $\mu = 0$) are then standardised to unit variance. From this data, pseudo proxies are built using a linear proxy response function. We also include a set of proxies with auto correlation in its transfer function $p_t = \beta_0 + \beta_1 x_t + \beta_2 p_{t-1} + \epsilon_{p,t}$. The proxy noise remains a Gaussian distributed i.i.d. random variable $\epsilon_{p,t} \sim \mathrm{N}(0, \tau_p^2)$. The proxy noise strength is varied over $\tau_P \in \{1, 2, 4\}$, which cover the range of real world values [21]. The interval is then split in two parts, a calibration interval (200 time steps) with instrumental data being available, the true data being distorted by low noise, and a reconstruction interval (800 time steps), where only the artificial proxy data are present. The resulting reconstructions are then evaluated using four different error measures: the mean bias, the cross correlation function, the root mean square error (RMSE) and the standard deviation ratio (SDR) of reconstruction and target data. In figure 3 we display these results for a range of $\sigma_2 \in [0, 1]$, which is even larger than the observed ratios in section 3. The bias remains close to zero, and the SDR remains relatively constant close to one. The cross correlation between reconstruction and true target (XCor) is around 0.5 for medium noise strength and no curvature and decreases slowly with increased σ_2. The RMSE is around 1.0 for medium noise, increasing in accordance with the model mismatch. The shading shows the 95% confidence bands around the mean values of 5000 realisations of the Gibbs sampler.

The added curvature can indeed have a substantial effect on the fidelity of the reconstruction, if it is sufficiently big. To rank the effect of the model mismatch we compare the values at the maximum observed curvature $\sigma_2 = 0.2$ to those at zero curvature. We can see from figure 3 that increasing the curvature has roughly the same effect as doubling the noise variance. As many of the proxies used e.g. for the Arctic region in [16] have noise variance exceeding the values used in this experiment, the largest contribution of uncertainty comes first from the proxy noise, then from the spatial sparseness – one cannot expect to get a meaningful reconstruction far away from the data sources. Only after tackling these issues, further complicated models should be included in the reconstruction method of [4].

5. Conclusion

In the ongoing quest on reconstructing past climate fields, new tools as Bayesian inference are currently being adopted. New methods introduce new challenges, every tool chain being only as strong as its weakest link. While Bayesian inference does have many advantages over simple linear regression schemes, the underlying stochastic models need to be either derived from dynamical analyses of the underlying processes, statistical properties of the data or thoroughly checked for consistency with them. The Kramers-Moyal-Expansion can be a tool for such checks, although precautions have to be taken. Stochastic modelling usually requires long time series, which are not available in climate research: data go back a few centuries at maximum. The results obtained from analyses of instrumental data indicate that for European temperatures a simple autoregressive model can be considerd sufficient to describe the local temporal evolution of the temperature field. Our analysis shows that while improving the model over the most simple assumptions could give better results, the errors introduced by sub-par input data are at least of comparable size. Thus, it has to be carefully considered if the cost of including e.g. a small non-additive term into the interannual variability does not heavily outweigh the possible benefits.

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