2014 International Symposium on Nonlinear Theory and its Applications
NOLTA2014, Luzern, Switzerland, September 14-18, 2014

Effectivity of Randomness in Memory Patterns of Recurrent Neural Network Model

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Abstract—In this paper, we investigate effectivity of the randomness in memory patterns of a recurrent neural network model referred as RNN hereafter. We have shown that for the memory patterns with a certain structure, their basin volumes and furthermore visiting measures of the basins become smaller. In realizing a function of the memory search based on chaotic wandering in a chaotic neural network model referred as CNN, it is important to ensure that basin volumes of the memory patterns and visiting measures of the basins are sufficiently large. Therefore, we investigate how to construct the memory patterns which gives sufficiently large basin volumes of theirs in RNN, focusing on the randomness in the memory patterns. We apply 11 kinds of the memory patterns with changing the ratio of the randomness. As the randomness increases, basin volumes of the memory patterns increase. The basin volumes of the memory patterns without the randomness is quite smaller than those of pseudo memory patterns. Thus, the randomness in the memory patterns is practical in ensuring that their basin volumes are sufficiently large.

1. Introduction

Skarda and Freeman have shown that chaos could play the important roles in a learning process and a recalling process [1]. In addition, Nara and his colleagues have investigated chaotic memory dynamics for cycle memory patterns related with memory search functions [2, 3]. Chaos would play important roles in realizing sophisticated and excellent information processing occurring in brain[4, 5, 6].

Inspired by Nara's idea, in realizing memory search functions based on chaotic wandering, Kuroiwa and his colleagues have studied sensitive response of chaotic wandering to memory pattern fragments in CNN [7, 8]. As the complexity of chaotic wandering increases, the delocalizing effects in visiting attractor basins expands from an intra-cycle of memory patterns to an inter-cycle, suggesting that it is possible to construct a hierarchical structure in the memory pattern space by controlling the complexity. In addition, the chaotic wandering responds to memory pattern fragment sensitively and robustly, that is, once a memory pattern fragment is applied to CNN, its chaotic orbit quickly moves to the vicinity of the corresponding memory pattern within several iteration steps.

The results suggest us that it is possible to realize a hierarchical memory search with chaotic wandering by embedding memory patterns with a hierarchical structure. However, for the memory patterns with a certain structure, their basin volumes and furthermore visiting measures of the basins become quite smaller.

In realizing the hierarchical memory search based on chaotic wandering in CNN it is important to ensure that basin volumes of the memory patterns and visiting measures of the basins are sufficiently large. Therefore, the purpose of this paper is to investigate how to construct the memory patterns which gives sufficiently large basin volumes of theirs in RNN, focusing on the randomness in the memory patterns.

2. RNN with Multi Cycle Memory Pattern

2.1. Associative Memory-type Recurrent Neural Network Model

Let us explain RNN briefly. Let us denote an internal state of the ith neuron at a time step t to be $u_i(t)$. The updating rule of RNN is represented as follows:

$$u_i(t+1) = \sum_{j=1}^{N} w_{ij} z_j(t), \tag{1}$$

where N denotes the total number of neurons and $\{w_{ij}\}$ represent synaptic connections from the jth neuron and the ith one.

In this paper, the output of the ith neuron is given by the following output function with continuous value of $[-1, 1]$,

$$z_i(t+1) = \tanh\left(\beta u_i\,(t+1)\right), \tag{2}$$

where β corresponds to the steepness of the output function.

2.2. Orthogonal Learning Method

In this paper, we employ an orthogonal learning method to determine synaptic connections. In general, basin volumes of the memory patterns becomes larger with the orthogonal learning method. In addition, it is difficult to embed multi cycle memory pattern because of the overlap between memory patterns. The orthogonal learning method

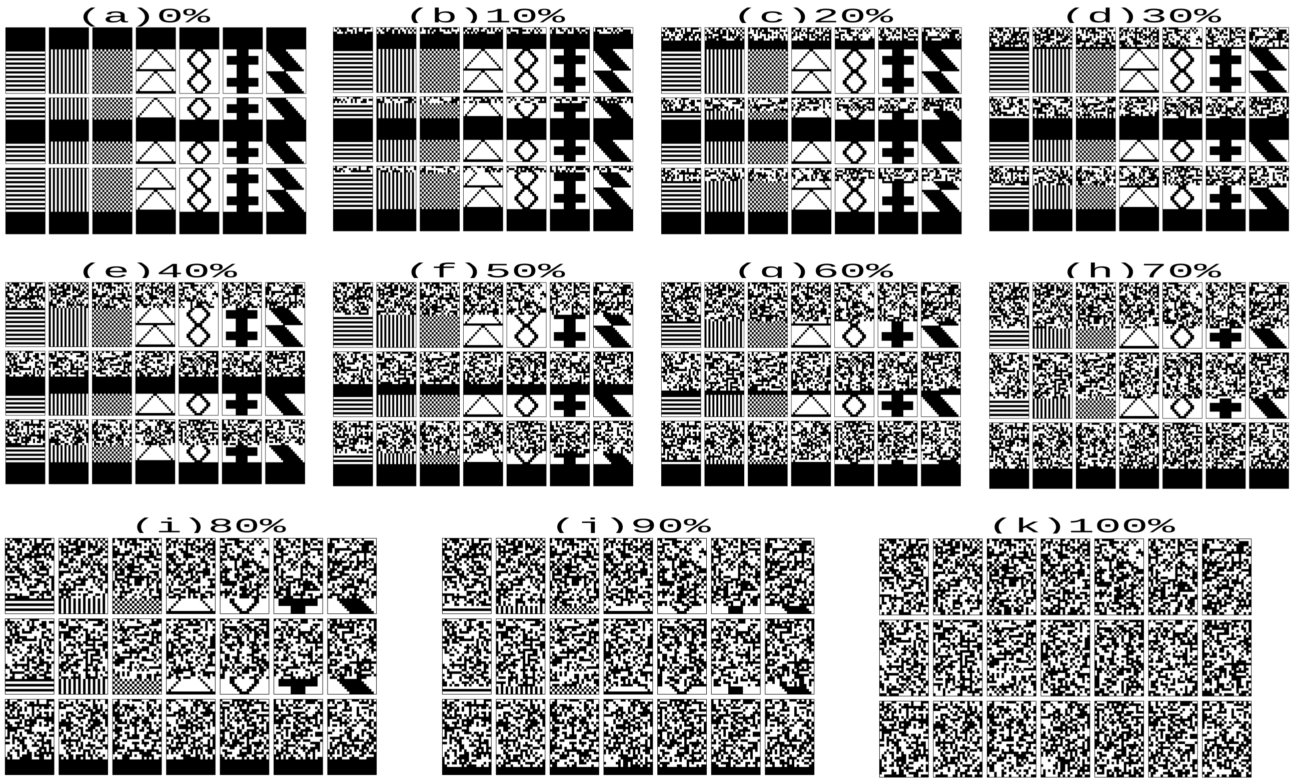

Fig. 1: Eleven kinds of memory patterns with changing the ratio of the randomness. (a) Original memory patterns without randomness. (b) Memory patterns including 10% randomness. (c) 20%. (d) 30%. (e) 40%. (f) 50%. (g) 60%. (h) 70%. (i) 80%. (j) 90%. (k) 100% random patterns.

is written as follows:

$$w_{ij} = \sum_{a=1}^{P} \sum_{\mu=1}^{L} v_i^{(a)(\mu+1)} (v_j^{(a)(\mu)})^{\dagger} \tag{3}$$

where $\boldsymbol{v}^{(a)(\mu)}$ denotes μth memory pattern vector among ath cycle, $(\boldsymbol{v}^{(a)(\mu)})^{\dagger}$ is a conjugate vector of $\boldsymbol{v}^{(a)(\mu)}$, P represents the total number of cycle and L represents the total number of patterns for each cycle.

The conjugate vector is defined as follows:

$$(v^{(a)(\mu)})^{\dagger} = \sum_{b=1}^{P} \sum_{\nu=1}^{L} (O^{-1})^{(a)(\mu)(b)(\nu)} v^{(b)(\nu)}, \tag{4}$$

where O^{-1} is an inverse matrix of the overlap matrix calculated by,

$$O^{(a)(\mu)(b)(\nu)} = \sum_{k=1}^{N} v_k^{(a)(\mu)} v_k^{(b)(\nu)}. \tag{5}$$

Note that applying the equation (3), P limit cycle memory patterns with a period of L are embedded in RNN.

3. Computer Experiments

3.1. Purpose and Method

The purpose of this paper is to investigate effectivity of the randomness in memory patterns. In this paper, the effectivity means that basin volumes of the memory patterns becomes sufficiently large. Therefore, we apply 11 kinds of the memory patterns with changing the ratio of the randomness as shown in Fig. 1. Memory patterns are composed of $P = 3$ cycles, each cycle contains $L = 7$ patterns, and each pattern consists of 20×30 pixels of ± 1, implying $N = 600$. In our memory patterns, the pattern overlap among an intracycle is larger than that among an inter-cycle.

The basin of the attractor is a set of configurations which converges into its attractor. Therefore, the basin volumes are calculated as follows. We evaluate basin volumes of each memory pattern in RNN starting from 20,000 different random initial patterns. At each time step of the updating of RNN, we check whether an output of RNN corresponds to any one of memory patterns or not. If the output coincides with any one of memory patterns, we regrade that the system is in the cycle where the memory pattern belongs. Note that we regard the inverse pattern as corresponding memory pattern. On the other hand, when the system does not coincide until 40 time steps, we stop the calculation and we regard that either the system could not converge or the system is among a pseudo cycle pattern. In this paper, we denote the above state as being among pseudo cycle. The summarized evaluation procedure of basin volumes is as follows:

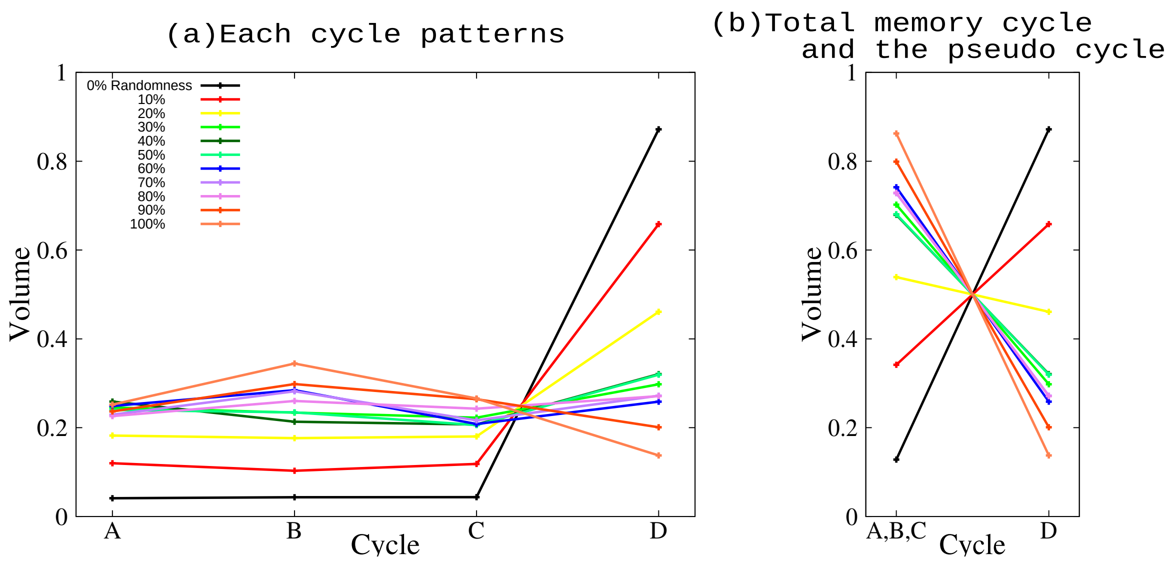

Fig. 2: Basin volumes. (a) For each cycle pattern. (b) For total memory cycle and the pseudo cycle. 'A', 'B' and 'C' indicate memory cycles, and 'D' represents the pseudo cycle.

1. Choose one initial configuration from 20,000 different random patterns.
2. Update RNN according to equation (1).
3. At each time set, check whether the output of RNN corresponds to any one of memory patterns.
4. When the time step arrive at 40 steps, stop the calculation and we regard that the system is among the pseudo cycle.
5. Until all the 20,000 random patterns are applied, continue the procedure.

Through the computer experiments, we employ the same parameter value of $\beta = 100$.

3.2. Basin Volumes

Results of the basin volumes are given in Fig. 2. In Fig. 2(a), the basin volumes for each cycle patterns including the pseudo cycle are depicted. In Fig. 2(b), the basin volumes for total memory cycle and the pseudo cycle are given. The indexes 'A', 'B' and 'C' indicate memory cycles, and 'D' represents the pseudo cycle.

The basin volumes of the memory cycle patterns without randomness is quite smaller than that of the pseudo cycle. Indeed, the basin volume of the total memory cycle pattern is 0.18, and that of the pseudo cycle is 0.82. On the other hand, as the randomness increases more than 30%, the dependence of the basin volumes on the randomness almost disappears and the basin volumes of the memory patterns becomes much larger than that of the pseudo cycle. For 30% randomness, the basin volume of the total memory pattern is 0.70, and that of the pseudo cycle is 0.30. For 100% randomness, the basin volume of the total memory pattern is 0.86, and that of the pseudo cycle is 0.14. The basin volume of the total memory pattern monotonically increases as the ratio increases. Therefore, the randomness in the memory patterns is practical in ensuring that their basin volumes are sufficiently large.

4. Discussions

Now, we consider reasons why the basin volume of memory cycle patterns increases as the ratio of the randomness increases. We expect that the distance between memory patterns and a random pattern affects the fact. Therefore, we evaluate the distance between the dirty A1 pattern and each memory pattern for each ratio of the randomness. The dirty A1 pattern is constructed by inverting pixels which are selected in order from top left to bottom right with the given number of pixels. The distance is calculated by inner product between the dirty A1 pattern vector and the conjugate vector of each memory pattern defined by equation (4).

The results are given in Fig. 3. The horizontal axis represents the number of inverted pixels. The vertical axis denotes the averaged distance. The distance between the original A1 monotonically decreases to zero as the inverted pixels increases regardless of the ratio of the randomness. On the other hand, for the ratio of 0% and 10%, the distance between the others is meaningful larger than zero. The fact would encompasses that the basin volume of the pseudo cycle increases as the radio of the randomness decreases.

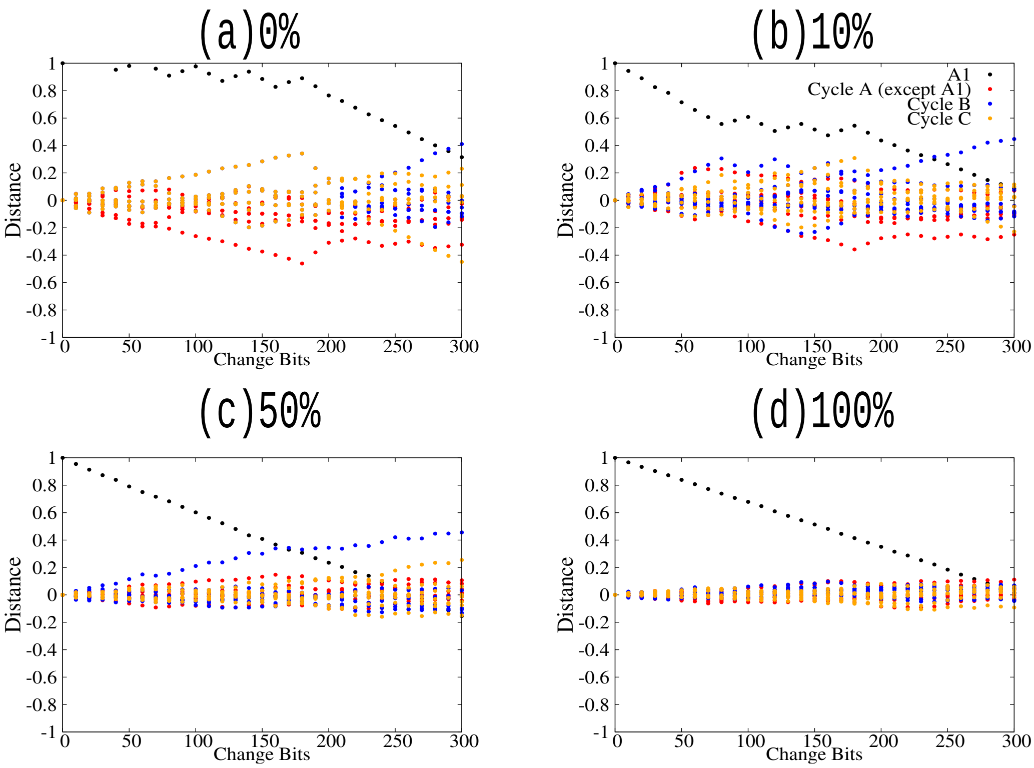

Fig. 3: Distance between the dirty A1 pattern vector and the conjugate vector of each memory pattern. The horizontal axis represents the number of inverted pixels. The vertical axis denotes the distance.

5. Conclusions

In this paper, we investigate effectivity of the randomness in memory patterns of RNN. We evaluate basin volumes of the memory patterns with changing the ratio of the randomness. Results are as follows:

- The basin volumes of the memory cycle patterns without randomness is quite smaller than that of the pseudo cycle.
- As the randomness increases more than 30%, the dependence of the basin volumes on the randomness almost disappears and the basin volumes of the memory patterns becomes much larger than that of the pseudo cycle.

Therefore, the randomness in the memory patterns is practical in ensuring that their basin volumes are sufficiently large. In near future, we evaluate visiting measures of the memory patterns in CNN with changing the ratio of the randomness.

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