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Correlation between Lyapunov exponents and pitches of chaotic sounds

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Abstract—We report chaotic properties of several sounds including human voices and saxophone sounds. We found that the male voice has a larger maximal Lyapunov exponent than the female voice. In addition, by comparing the soprano, the alto, and the tenor saxophones, we found that the instruments with lower pitches tend to have larger maximal Lyapunov exponents.

1. Introduction

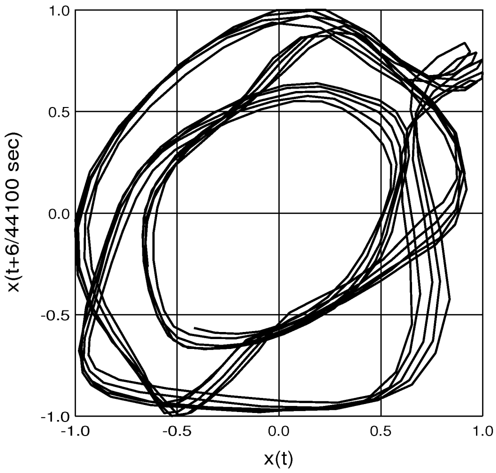
Chaotic properties have been pointed out in sounds of reed type wind instruments, especially human voices, a clarinet and a saxophone. For example, Tokuda [1, 2] showed that the sounds of Japanese vowel /a/ are chaotic and have low-dimensional attractors. The correlation dimensions are estimated by Keefe and Laden [3, 4] for several types of the sounds of saxophones. The results show that they would be strange attractors.

However, few studies are found about differences of Lyapunov exponents depends on kinds of musical instruments. In this study, the two human voices which are by the tenor singer (male) and by the female vocalist, and sounds of three types of saxophones (soprano, alto and tenor) will be analyzed.

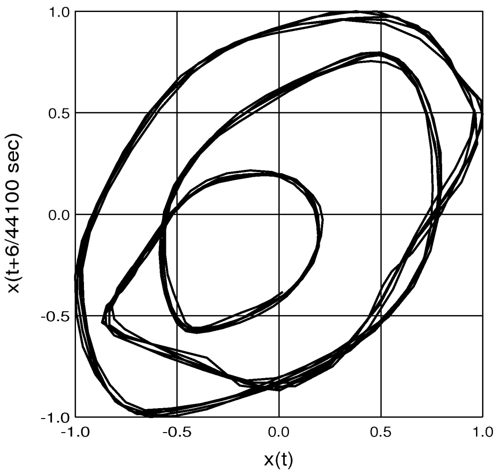
2. Data preparation

In this study, we selected datasets that are available for research without copyright problems. Real World Computing (RWC) database [5, 6] was developed by the Real World Computing Partnership (RWCP) in Japan. The database was recorded carefully under the controlled environment by the acoustics experts, and can be used freely for only research purposes. These data sets are provided in Windows wav file (44.1 kHz, 16bit monoral). Musicians' properties and details of musical instruments are concealed. We used the same A_4 pitch sounds (440 Hz) which is used of the tuning in an orchestra. Each size of the data is about one second (44100 points).

We plots used 500 points in the delay coordinates of the sounds are shown in Fig. 1. To enhance the visibility, we selected the delay dimension $d = 6$ which is not necessarily the best delay. We can find that each instrument has specific patterns in the delay coordinates.



Human voice (male)



Human voice (female)

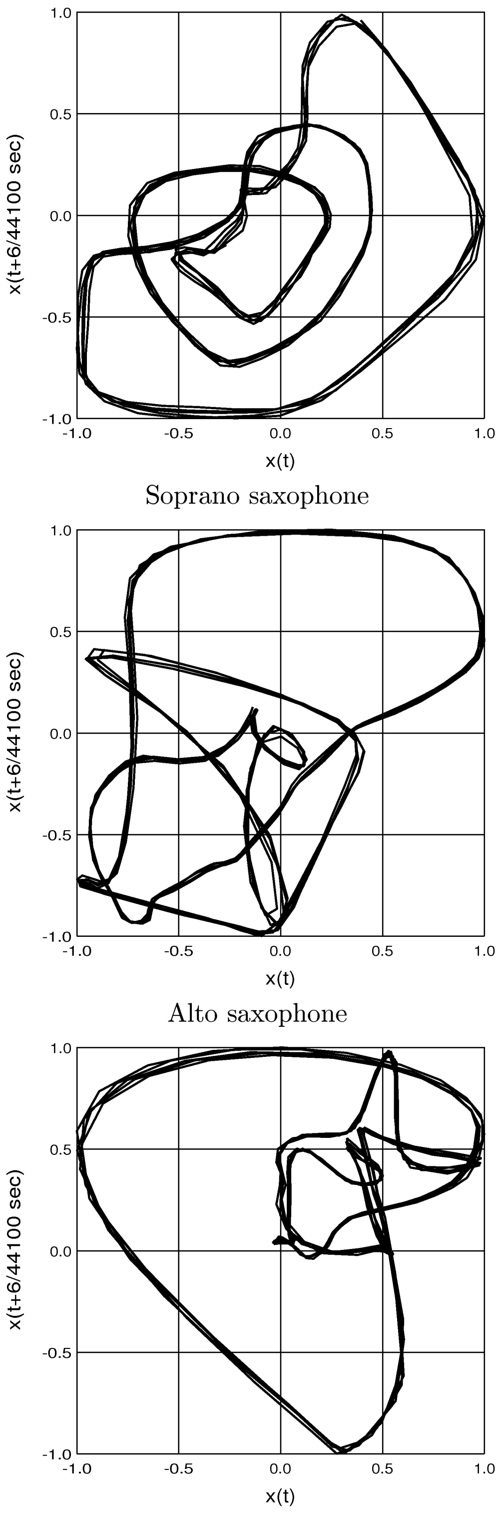


Fig. 1: Plots the sounds in the delay coordinates. The wide varieties of attractors are found.

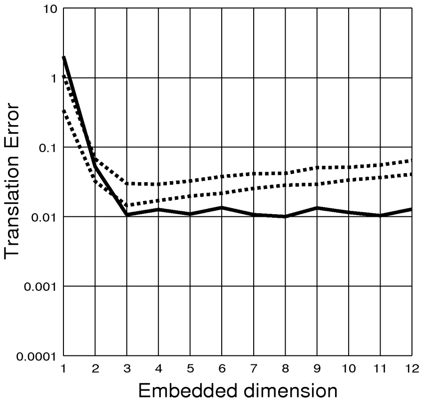
It is important that stationarity of the time series. Hence, we verified the stationarity as following steps. (1) We searched a proper delay which is related to the first minimum of mutual information [7]. We regarded the delay d as the best delay. (2) We subsampled the given time series every d points. (3) We estimated the embedded dimension by the false nearest neighbors (FNN) method [8]. The concept of the FNN method is as follows. If we assume a wrong embedded dimension, neighboring points on that dimension will be not neighboring points in the higher dimensions. So, checking these false nearest neighbors will give the best embedding dimension n. For the test, we used TISEAN [9] programs by Rainer Hegger *et al.* and we used 5% for the rate of errors. (4) We tested the stationarity by Kennel's method [10]. We regarded less than 2.326 of the test value as a stationarity dataset. The results are presented in Table 1.

Name	d	n	Length
Soprano saxophone	7	10	1285
Alto saxophone	9	10	1555
Tenor saxophone	8	10	1375
Human voice (male)	7	10	1428
Human voice (female)	8	12	2250

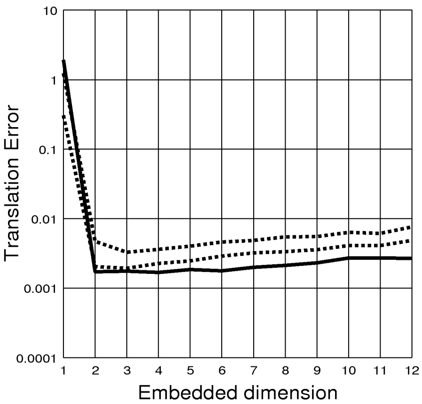
Table 1: Estimated delay and embedded dimensions of sounds of musical instruments by using mutual information and FNN method.

A surrogate test is one of the statistical methods widely used in nonlinear time series analysis. When we want to evaluate a property of a time series we are interested in, we make many data sets called "surrogate data sets" by preserving the property of null-hypothesis and randomizing the given time series. Then, we compare the original time series with surrogate data sets by a statistical test. If we find to a difference as a result, it means the null-hypothesis is rejected. A difficult point for making surrogate data sets is how to randomize the original data by preserving the property of null-hypothesis.

Now, we used three different surrogate tests with the Wayland statistic [11] as a test statistic. These tests are phase-randomized Fourier-transform surrogate[12], iterative amplitude adjusted Fourier-Transform surrogate [13], and pseudo periodic surrogate (Small's test) [14]. The results show that these sounds are nonlinear time series and have determinism beyond pseudo-periodicity.The results of pseudo-periodic surrogate tests for the human voice are shown in Fig. 2 as examples. The solid lines in Fig. 2 indicate the test statistic for the original dataset. These lines, for most embedding dimensions, are out of the intervals indicated by the two dashed-dotted lines, which indicate the maximum and minimum values obtained using the 39 surrogate datasets for the test statistic.

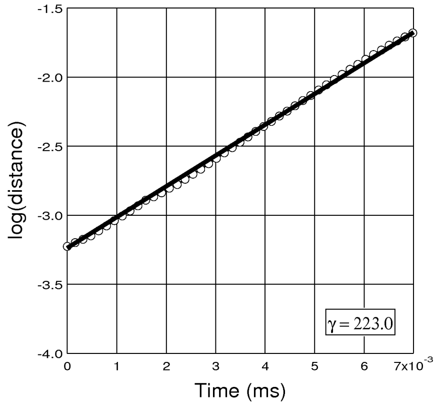


Human voice (male)

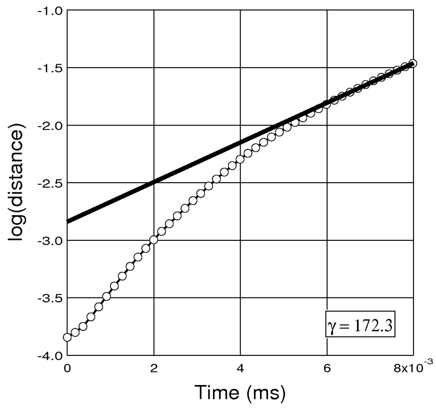


Human voice (female)

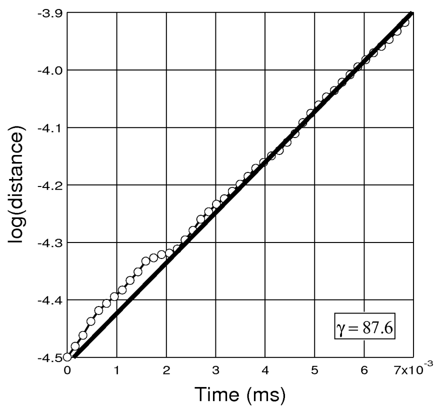
Fig. 2: Results of pseudo-periodic surrogate tests for human voice. The solid lines in Fig. 2 indicate the test statistic for the original dataset. These lines, for most embedding dimensions, are out of the intervals indicated by the two dashed-dotted lines, which indicate the maximum and minimum values obtained using the 39 surrogate datasets for the test statistic.



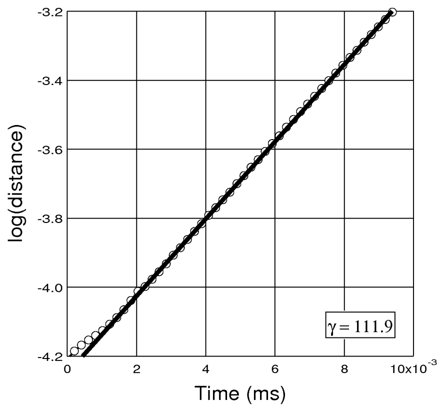
Human voice (male)



Human voice (female)



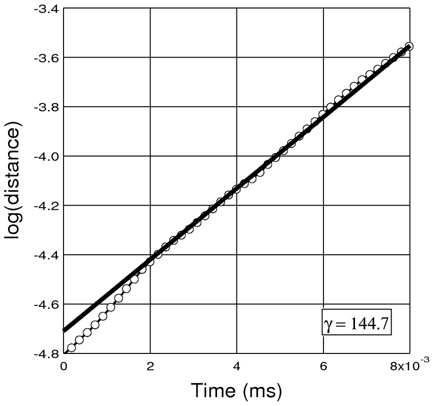
Soprano saxophone



Alto saxophone

3. Estimation of maximal Lyapunov exponent

We calculate the maximum Lyapunov exponents of the sounds by using the Kantz's method [15]. In Fig. 3, we show the average distance between a selected point and neighboring points with time in delay coordinates. In the linear part that is about to saturate in a logarithmic plot, the slope of linear part gives the maximal Lyapunov exponent γ.



Tenor saxophone

Fig. 3: Estimation of the Lyapunov exponent by using Kant'z method. The number of neighboring points is 8 fixed. The slope of linear part in these logarithmic plots gives the maximal Lyapunov exponent.

We confirmed the positive maximal Lyapunov exponents in the human voices and saxophone sounds. The Lyapunov exponents are shown in Table 2. These results show the chaotic properties in these sounds.

Musical instrument	γ
Human voice (male)	223.0
Human voice (female)	172.3
Soprano saxophone	87.6
Alto saxophone	111.9
Tenor saxophone	144.7

Table 2: Estimated Lyapunov exponents γ

4. Disucussion

We found that the male voice has a larger maximal Lyapunov exponent than the female voice, although sample sounds have the same A_4 pitch. By comparing the soprano, the alto, and the tenor saxophones, we found that the instruments with lower pitches tend to have larger maximal Lyapunov exponents.

We think that a reason for this tendency is instability of sounds by deviation from the most appropriate pitch of the musical instrument. Hence, this tendency may be independent of the type of the instrument. Conversely, Lyapunov exponents of sounds would give the most appropriate pitch of the musical instrument.

5. Conclusion

We confirmed chaotic properties of several sounds including human voices and saxophone sounds by Kantz's method. We found that the male voice has a larger maximal Lyapunov exponent than the female voice. In addition, by comparing the soprano, the alto, and the tenor saxophones, we found that the instruments with lower pitches tend to have larger maximal Lyapunov exponents.

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