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Magnet Alignment Design of Pure-type Superconductor Undulator by Using Magnetization Simulation Based on T-method

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Abstract– As one of the important tasks in the next generation Free Electron Laser (FEL), the Pure-type High-Tc Superconductor (HTS) undulator has been developed. We have been developing a numerical simulation code based on the T-method for assisting design of the appropriate magnet alignment before the HTS magnetization. This paper presents the design for uniform distribution of sinusoidal vertical magnetic field component which is needed for stable FEL operation.

1. Introduction

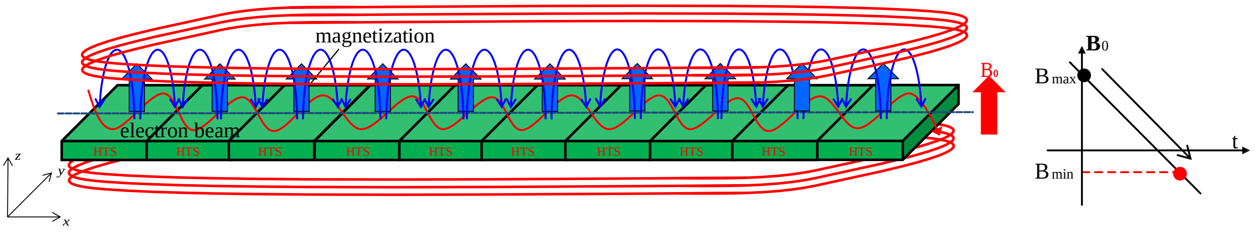
The next generation X-ray Free Electron Laser (X-FEL) is expected to contribute remarkable progress of various kinds of advanced technologies in biology and chemistry. In the development of the X-FEL, achievement of a small size and high intensity magnetic field undulator is one of the most important tasks, and it is considered to use High-Tc Superconductor (HTS) for undulator magnets. In fact, several types of HTS undulators were already proposed [1-2]. In general, it is known that very uniform sinusoidal magnetic field is required in the FEL undulator for stable operation. On the other hand, the adjustment of the individual magnets of the undulator is almost impossible after the HTS undulator is installed inside a cryostat and changed to a superconducting state. Accordingly, the appropriate magnet alignment has to be determined before the HTS magnetization. Then numerical simulation of the magnetization process of the HTS plays a very important role for determining the suitable magnet alignment in the device design stage. For this purpose, we have been working in a development of a numerical simulation code for the magnetization process of the HTS undulator based on the current vector potential method (T-method) combining with the critical state model for the shielding current in the HTS [3-4]. In particular, it was shown that the magnetic field distribution in the magnetization process of the Pure-type HTS undulator can be simulated appropriately to compare with magnetic field measurements [2] [4]. In this paper, the simulation code is applied to the design of the Pure-type HTS undulator magnets which leads to uniform sinusoidal distribution of the vertical component of the magnetic field along the electron trajectory. In addition, a single electron trajectory in the magnetic field produced by the Pure-type HTS undulator is calculated.

2. Pure-type Superconducting Undulator

An overview of the Pure-type HTS undulator constructed by ten HTSs is shown in Fig. 1(a). In the Pure-type HTS undulator, circulating horizontal shielding currents are induced in the individual HTS by applying the time dependent vertical magnetic field B_0 (see Fig. 1(b)). Then, alternative vertical magnetic field is formed along the traveling direction of the electron beam by the superposition of the magnetic field created by ten HTSs, and the electron performs undulator motion as in the typical FEL. In this paper, we show the design of the magnetic field created by ten HTSs by using the simulation of the HTS magnetization process.

3. Simulation of magnetization process of Pure-type HTS undulator based on T-method

We here use the current vector potential method (T-



(a) (b)

Fig. 1. Overview of Ten-magnet Pure-type HTS undulator

method) combing with the critical state model on shielding current for the simulation of the magnetization process [3-4]. The shielding current **J** induced in the HTS is expressed by using the current vector potential **T** defined by $\mathbf{J}=\nabla\times\mathbf{T}$ and the governing equation for **T** is the following integro-differential equation,

$$\nabla\times\frac{1}{\sigma}\nabla\times\mathbf{T}-\mu_0\frac{\partial\mathbf{T}}{\partial t}-\frac{\mu_0}{4\pi}\int_S\frac{\partial\mathbf{T}\cdot\mathbf{n}}{\partial t}\nabla'\left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right)dS'=\frac{\partial\mathbf{B}_0}{\partial t}, \qquad (1)$$

where, σ is conductivity, μ_0 is permeability, S is the surface of the HTS, **n** is a unit normal vector on S, and $\mathbf{B_0}$ is externally applied magnetic field. Then the current vector potential has to satisfy the following boundary and gauge conditions respectively,

$$\mathbf{T}\times\mathbf{n}=0, \quad \text{on S}, \qquad (2)$$

$$\nabla\cdot\mathbf{T}=0, \quad \text{in domain.} \qquad (3)$$

It is assumed that the shielding current is induced in *x*-*y* horizontal plane, then the HTS can be expressed by the thin-plate model, and the current vector potential has only *z*-component [4]. In addition, Ohm's law is modified to the following critical current model for describing the shielding current behavior in the HTS,

$$\begin{cases}\mathbf{J}=\mathrm{Jc}(|\mathbf{B}|)\dfrac{\mathbf{E}}{|\mathbf{E}|} & \text{if}\,|\mathbf{E}|\neq 0,\\[2ex] \dfrac{\partial\mathbf{J}}{\partial \mathrm{t}}=0 & \text{if}\,|\mathbf{E}|=0.\end{cases} \qquad (4)$$

where J_c is the critical current of the HTSs and **E** is the electric field. In particular, we here use the following Bean's model for the critical current,

$$\mathrm{J_C}(|\mathbf{B}|)=\mathrm{J_C}=\mathrm{const.} \qquad (5)$$

However, a straightforward implementation of the critical current model (4) in the T-method is not easy because the electric field **E** in the HTS can be calculated after the current **J** is obtained. Therefore, the following artificial conductivity scheme is used instead of (4),

$$\begin{aligned}\sigma_{\mathrm{new}}&=\sigma_{\mathrm{old}}\frac{\mathrm{J_C}}{\mathrm{J}} && \text{if } \mathrm{J}>\mathrm{J_C},\\ \sigma_{\mathrm{new}}&=\sigma_{\mathrm{old}} && \text{if } \mathrm{J}\leq\mathrm{J_C}.\end{aligned} \qquad (6)$$

In this work, we employ the Finite Difference Method (FDM) for a numerical scheme of (1), and the Runge-Kutta method for calculations of electron trajectories under the magnetic field in discretized space.

A flowchart of the simulations of the magnetization process of the Pure-type HTS undulator based on the T-method is indicated in Fig. 2. In particular, the magnetization simulation for all HTSs are iteratively executed in each time step for obtaining the converged solution to a consistent shielding current distribution to take into account an influence of the magnetic field created by all other HTSs.

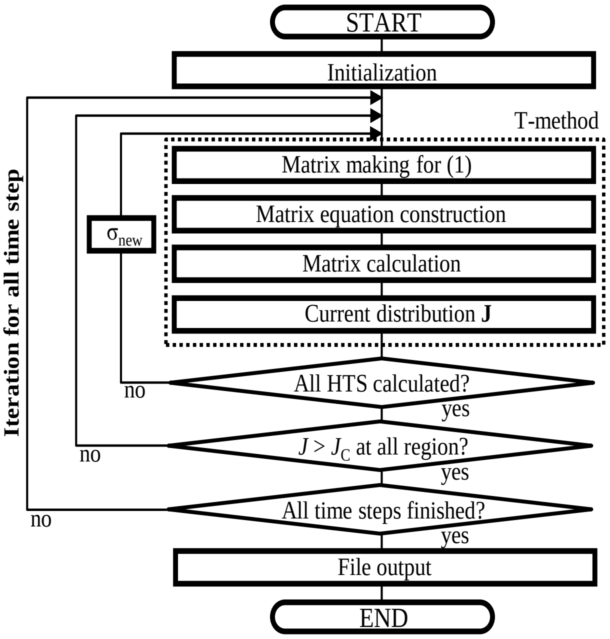


Fig. 2. Flowchart of simulation of magnetization process

4. Numerical example

Figs. 3 indicate the numerical results of the shielding current distribution on a horizontal plane (Fig. 3(a)), the magnetic field distribution on vertical cross section (Fig. 3(b)) and the profile of the vertical magnetic field component along the electron trajectory (Fig. 3(c)) for ten HTSs undulator of Fig. 1. The size of the individual HTS is taken to be 10mm × 15mm × 4mm, and J_c is 7×10^8 [A/m^2] for all HTSs. In general, a uniform sinusoidal distribution of the vertical magnetic field component along a longitudinal direction of the FEL undulator is required for stable laser operation. For obtaining such the uniform profile of Fig. 3(c), we here propose to use different values of J_c for the individual HTSs. Fig. 4(a) indicates a modified profile of the magnetic field component in which $J_c = 6.53\times10^8$ [A/m^2] for the left- and right-most HTSs, 6.87×10^8 [A/m^2] for the second left- and right-most HTSs, and 7×10^8 [A/m^2] for all others. It is shown that the uniform sinusoidal distribution of the vertical component of the magnetic field is obtained by appropriately choosing J_c values. The fluctuation of the amplitude of the vertical sinusoidal magnetic field component is suppressed within 1% in Fig. 4(a). Fig. 4(b) shows the estimated single electron trajectory with 2GeV energy for X-ray radiation, which is calculated by the Runge-Kutta method to use the calculated magnetic field distribution of Fig. 4(a). We can find that the center axis of the undulator motion of the electron takes a curved line, although the undulator magnetic field (Fig. 4(a)) is sufficiently uniform. Figs. 5 indicate the simulation of the magnetic field distribution (Fig. 5(a)) and the electron trajectory (Fig. 5(b)) in which an offset vertical magnetic field of -0.41T is additionally applied on that of Fig. 4(a). It is found that the further modification of the magnetic field of Fig. 5(a) gives the reasonable electron trajectory.

5. Conclusion

This paper has presented the design study of the HTS magnets alignment for the Pure-type HTS undulator by using the T-method with the critical current model. And predicted single electron trajectories in the calculated magnetic field have been also given. It is demonstrated that the sinusoidal vertical magnetic field component distribution and the electron trajectory can be improved to individually adjust the value of the critical current of the HTS at the design stage.

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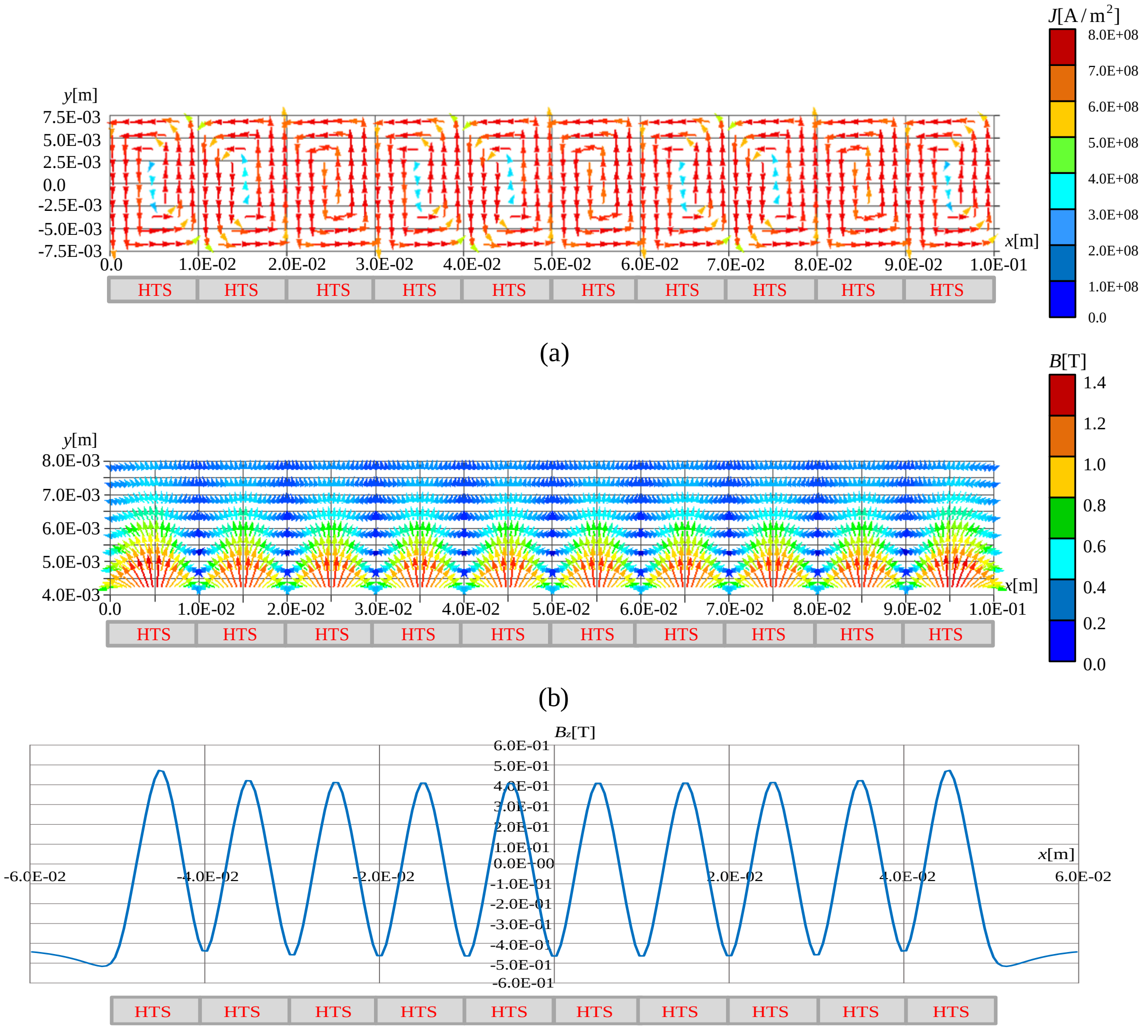
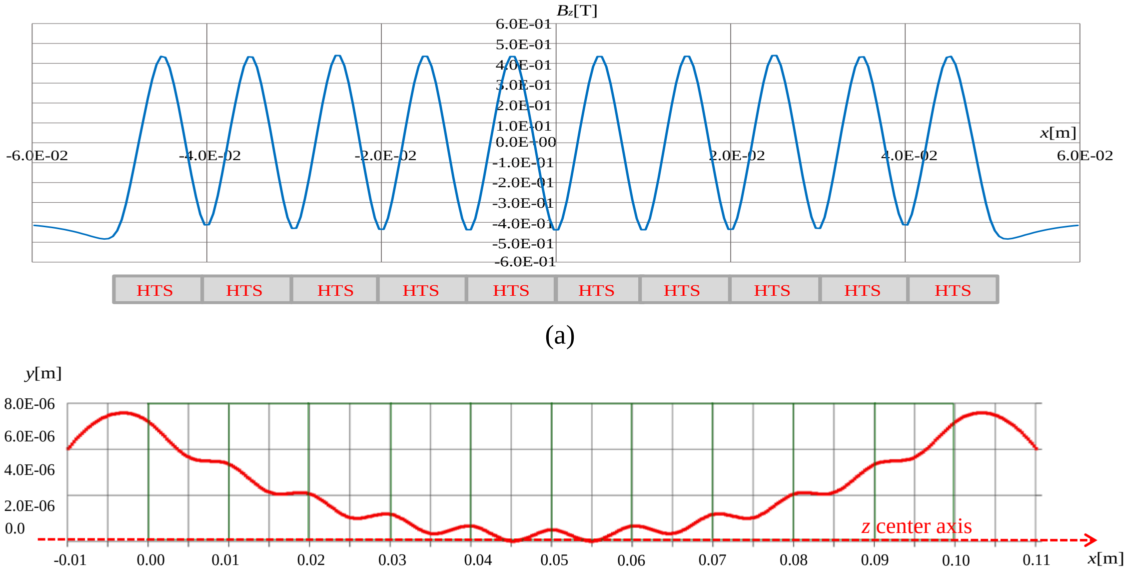


(c)

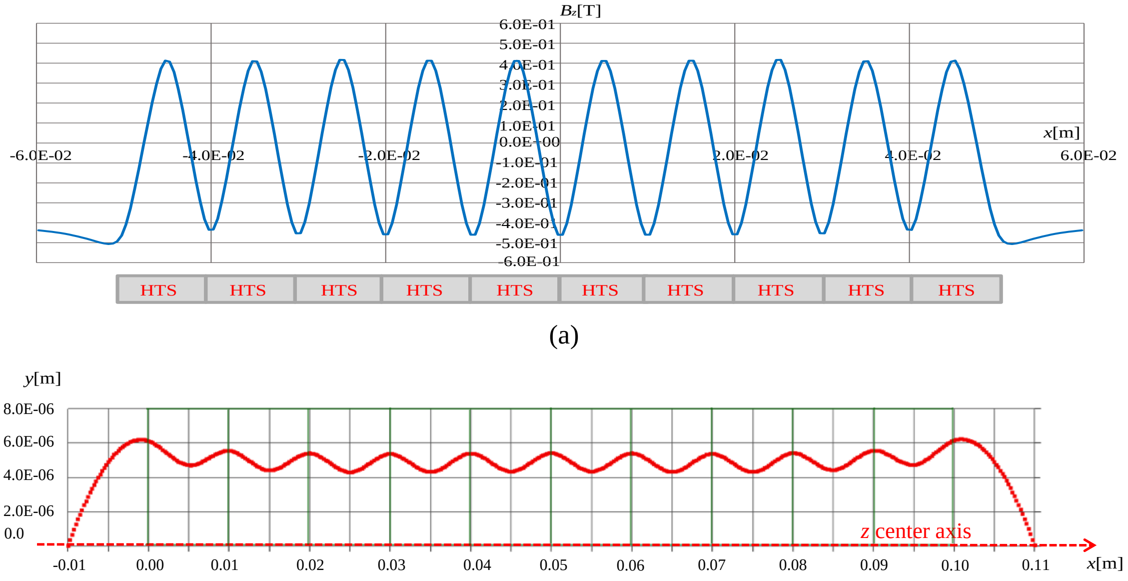
Fig. 3. Numerical results of HTS undulator magnetization process, (a) shielding current distribution on HTS, (b) magnetic field distribution in vertical lane on HTS and (c) profile of vertical magnetic



(a)

(b)

Fig. 4. Numerical results of HTS magnetization process for modified design, (a) modified magnetic field component distribution along electron trajectory and (b) estimated single electron trajectory with 2GeV energy (b)



(a)

(b)

Fig. 5. Numerical results of HTS magnetization process for further modification, (a) modified magnetic field component distribution for stable electron trajectory and (b) estimated stable electron trajectory with 2GeV energy