Performance analysis of Bidirectional Associative Memories by using the Inverse Function Delayless model

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Abstract– There are some problems in the conventional Hopfield model such as spurious memory and the presence of a non-optimal solution due to local minimum state. The Inverse function Delayed (ID) model has been proposed by Nakajima[5] as a neuron model. The ID model can solve such problems at 100% rate by its negative resistance effect in its dynamics, which can destabilize undesirable states selectively by this effect. However, in the neural network made by ID neurons, we can not solve the problems that have big size in real-time because of the high cost of the calculation time of the ID model. In the Inverse Function Delayless model, we proved that such problem can be avoided by realizing the destabilized region due to a discrete operation. In this report, I attempt to introduce the effect of IDL to the associative memory system of hetero associative type. However, it was associated systems BAM type in order to fulfill the effects of the IDL. In the of the proposed system, we can expect the expansion of the Basin size and speed of convergence time to the memory state .

1. Introduction

Recent years, the speed of information processing device is expected to increase with the increasing amount of information. According to its high-speed parallel processing capacity, Neural network has been researched as an information processing system of the next generation.

However, in macro models of conventional such as the Hopfield model[1,2,3], the problems such as spurious memory caused by the solution of the network not to escape from a local minimum has been known.

ID (Inverse function Delayed) model [4,5], it has been extended to the physiological model from the macro model, delays have been set to the output and the membrane potential. Such delays output was assumed sufficiently small. Because of such delays, in case of using the function of N-shape, the negative resistance is occured to the output function of the system. We found that it was possible to avoid the spurious memory problem when the ID model was used in the neural network. However, we found it was difficult to solve the problems in a practical time when the size of that is big.

On the other hand, the Inverse Function Delayless model[6], we proved that such spurious memory problem can be avoided by realizing the destabilized region. Moreover, by allowing a discrete operation, the convergence speed of the network is sufficiently fast. In this study, We attempt to introduce the effect of IDL to the associative memory system of hetero associative type and to analyses the performance of the neural network. Particularly it is associative system BAM type in order to fulfill the effects of IDL model.

2. Inverse function Delayed model

First, we will explain the ID neural network model[4, 5,]. The ID model is described as following differential equations.

$$\tau \frac{du_i}{dt} = \sum_j w_{ij} x_i - u_i, \tag{1}$$

$$\tau_x \frac{du_i}{dt} = u_i - g(x_i), \tag{2}$$

where w_{ij}, τ and τ_x are a connection weight, the time constant of the internal state and the output, respectively. The conversion time τ_x should be taken into consideration in geneal cases, under the condition $\tau_x << \tau$. The inverse function $g(x)$ is used instead of the conventional activation $f(u)$.

Eq. (1) is differentiated with time, and substitutions of a single unit and Eq. (1) lead to the following differential equations.

$$\tau_x \frac{d^2 x}{dt^2} + \eta \frac{dx}{dt} = -\frac{\partial U}{\partial x}, \tag{3}$$

$$\eta = \frac{dg(x)}{dx} + \frac{\tau_x}{\tau}, \tag{4}$$

where U is the potential of the ID model. The first and the second term on the left hand side of Eq. (1) express the inertia and the friction term, respectively. When the $g(x)$ is defined as a N-shape function, the negative resistance is occured to the network and it is possible to avoid the spurious memory.

3. Inverse function Delayless model

In the ID model, delay is caused by a inertial term τ_x. The IDL model increases the speed of calculation by removing this term.

In the Eq. (3), by removing the inertial term($\tau_x = 0$), we can get Eq. (5).

$$\frac{dx_i}{dt} = \frac{1}{g'(x_i)}\frac{1}{\tau}\left(\sum_j w_{ij}x_j + h_i - g(x_i)\right) \tag{5}$$

In IDL model[6], the system's dynamics can be described as Eq. (5). To increase the speed of the calculation, Here we defined $\frac{1}{g'(x_i)}$ as following equation, which is referred to velocity amplitude function.

$$A(x) = B(\frac{1}{1+\exp\{\gamma(-L-x)\}} - \frac{1}{1+\exp\{\gamma(+L-x)\}}) + \theta \tag{6}$$

where B, γ, L, θ are the maximum speed, the gain of the sigmoid function , the accelerated range and the minimum speed.

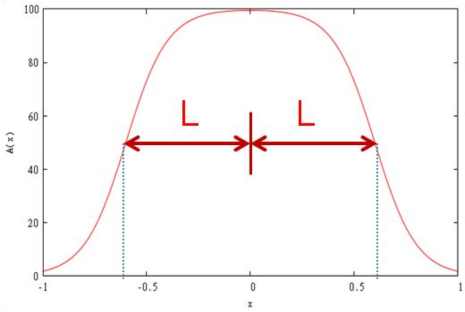

Figure 1: Velocity amplitude function

Moreover, we can discretize Eq. (5) into the following vector-valued difference equation.

$$x_i(t+\Delta t) - x_i(t) = A(x_i(t))\Delta t \frac{1}{\tau}\left(\sum_j w_{ij}x_j + h_i - g(x_i)\right) \tag{7}$$

In this study, we use Eqs. (6) and (7) to build the associative system.

4. Bidirectional Associative Memories

The Bidirectional Associative Memories(BAM)[7] is a variation of associative neural networks which has two layers. The principal function of BAM is to store and retrieve multiple patterns. Let N_1 and N_2 be the numbers of neurons these layers have. The state of unit is represented by a binary value ± 1, and the state of BAM is given by the pair $\{s_1, s_2\}$, the state of the two layers is defined as $s_1 \in \{1,-1\}, s_2 \in \{1,-1\}$.

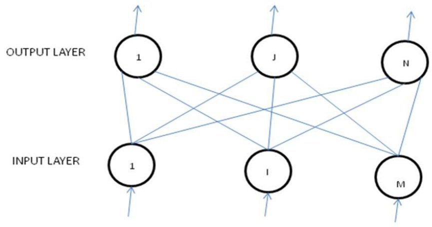

Figure 2: Structure of BAM

The synaptic weight w_{ij} is given by

$$w_{ij} = \frac{1}{N}\sum_{u=1}^{p} P_i^u P_j'^u \tag{8}$$

The input h_i between the two layers is defined as the following equation.

$$h_i = \sum_{j=1}^{N_{1or2}} w_{ij} s_j \tag{9}$$

In order to fulfill the effects of the IDL model to the hetero associative systems BAM, we can expect the expansion of the Basin size and speed of convergence time to the memory state .

5. Simulations and Results

5.1. Storage Capacity

Let $P_1, P_2, \ldots\ldots, P_n$ be the binary patterns which are stored. Here the memory capacity is defined as $\alpha_c = \frac{P_{\max}}{N}$, where $P_{\max}$ is the maximum number of patterns that can be stored perfectly.

We simulated with BAM model by using the Hopfield neurons and IDL neurons for the gain parameter $\beta = 4$. IDL model with the negative resistance has $w_{ii} = 1$, and Hopfield model both with the self-connections $w_{ii} = 1$ and without the self-connections $w_{ii} = 0$.

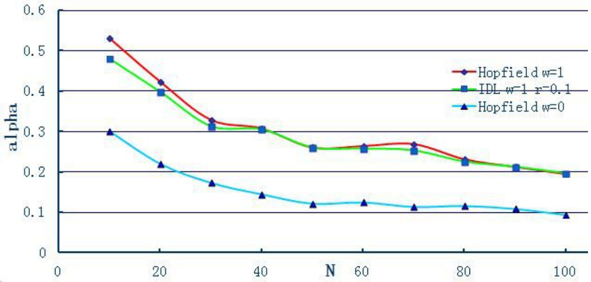

Figure 3 Memory capacity of the BAM model

Figure 3 shows the memory capacities of the IDL model and the Hopfield model as a function of the amount of neuron units N. The results demonstrate that both α_c of IDL model and the Hopfield model are the same values on the condition having the same potential energy .

The reason for the capacity of Hopfield model with self-connections is that the feedback of the neurons itself can increase the memory capacity of the neural network.

5.2. Retrieval Dynamics

In order to investigate the dynamical behavior of recalling process, we use the overlap

$$m^{\mu}(t) = \frac{1}{N}\sum_{i=1}^{N} P_i^{\mu} x_i(t) \qquad (9)$$

The overlap means the similarity between a state of current network $x(t)$ and a embedded pattern P^{μ}. Figures 4,5, and 6 show the time evolutions of overlap of the BAM model by using IDL neurons with negative resistance and of Hopfield neurons.

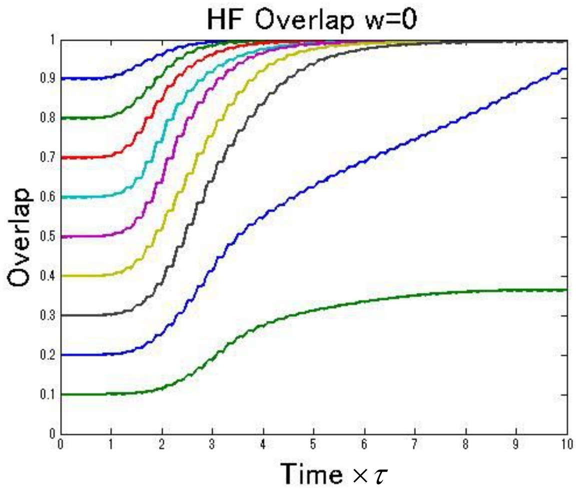

Figure 4 Time evolution of overlap for the BAM model with Hopfield neurons

$(N = 100, P = 10, \beta = 4.0, w_{ii} = 0, \quad \tau = 1, \quad \Delta t = 0.01)$

When the self-connection is small, the overlap of the state decrease. We assume that the reason is due to many spurious state when the self-connection is set for the Hopfield model. Here, we define m_c as the critical overlap, which is the smallest overlap of initial state still to be able to flow to the embedded memory state. Both m_c of Hopfield neurons without self-connections and IDL neurons are about 0.3.

However, Hopfield model without self-connections has low memory capacity.

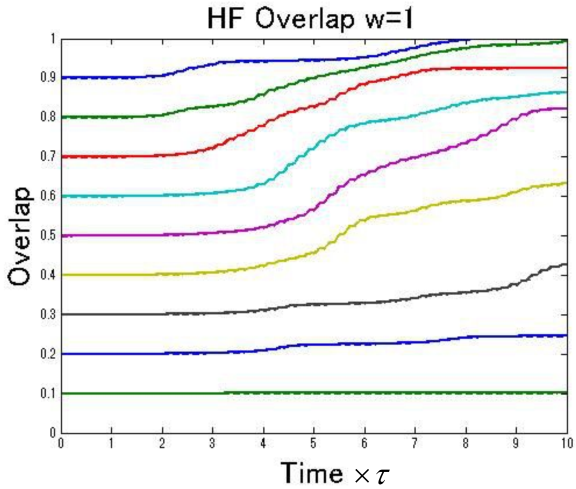

Figure 5 Time evolution of overlap for the BAM model with Hopfield neurons

$N = 100, P = 10, \beta = 4.0, w_{ii} = 1, \quad \tau = 1, \quad \Delta t = 0.01$

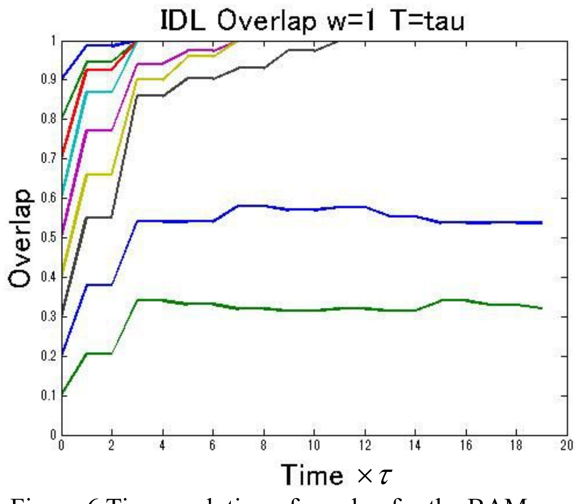

Figure 6 Time evolution of overlap for the BAM model with IDL neurons

$\left(\begin{array}{c} N = 100, P = 10, \beta = 4.0, w_{ii} = 1 \\ B = 100, \gamma = 10, L = 0.6, \tau = \Delta t = 1 \end{array} \right)$

The memory capacity of Hopfield model with self-connections is bigger than the one with self-connections. it can be seen that the speed of calculation in the network is significantly slow, at the same time the basin size is decreased.

In addition, the parameter $\Delta t/\tau$ shows that how much times the system calculated when it succeed to flow the embedded patterns. We can find that $\Delta t/\tau$ of IDL neurons is about 11, while the $\Delta t/\tau$ of Hopfield neurons is 800. As IDL model has been discretized, the results show that the convergence time of IDL neurons with negative resistance is much faster than that of the Hopfield neurons even when the self-connection is zero.

5.3. The basin of attraction

The basin size is an important criterion for the associative system, which shows the basin of attraction in the associative memory system.

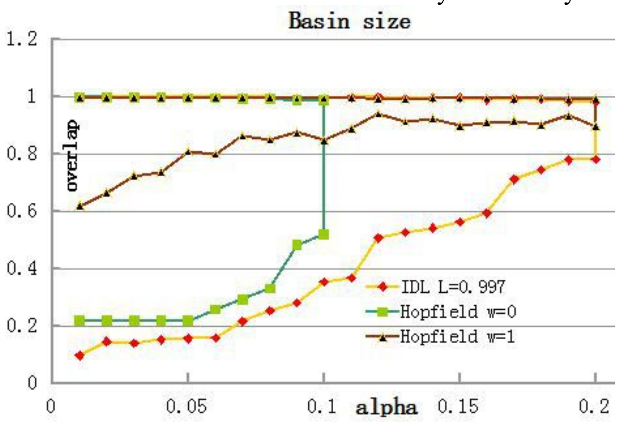

Figure 7 The basin size of the BAM model with Hopfield neurons with self-connections, without self-connections and IDL neurons

$$\begin{pmatrix} N=100, \alpha=0.1, \beta=4.0, \\ \text{Hopfield with} \quad w_{ii}=1, \quad \tau=1, \quad \Delta t=0.01 \\ \text{Hopfield with} \quad w_{ii}=0, \quad \tau=1, \quad \Delta t=0.01 \\ \text{IDL}: \ w_{ii}=1, \quad B=100, \gamma=10, L=0.6, \tau=1 \end{pmatrix}$$

Figure 7 shows the numerical results of the basin of attraction of the BAM model by using IDL neurons, Hopfield model with the self-connections and without the self-connections.

From the Figure 7, we can find that the basin size of Hopfield model with self-connections is smaller than the model without self-connections though the memory capacity of the network with self-connections is larger than the network without self-connections.

In addition the basin size of the BAM model with IDL neurons is larger than that with Hopfield neurons. Moreover the basin size of Hopfield neurons which have the 100% convergence rate only for the $\alpha = 0.1$, when the α of IDL neurons can be 0.2.

Finally, we can see that the effect of IDL model can keep the basin size of BAM system, even the self-connections has been set.

6. Conclusion

In conclusion, we fulfilled the effects of IDL model to the BAM system. We demonstrated that the storage capacity of IDL neurons did not change, but the basin size was larger, and calculation speed was getting faster than Hopfield model. Finally, as the future work, we plan to implement the BAM system by using IDL neurons with a programmable FPGA .

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