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Particle swarm optimization using a chaotic restarting method

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Abstract—The particle swarm optimization (PSO) method is a population-based optimization technique which searches for solutions by updating simultaneously a number of candidate solutions called particles. Since in PSO the exploration ability is important to find a desirable solution, various kinds of methods have been investigated to improve it. In this paper, we propose a restarting PSO model, where all particles are basically updated by the same dynamical system in the original PSO, while particles trapped at undesirable local minima are restarted by initializing their velocities and positions, and updates by the chaotic dynamical system with sinusoidal perturbations during a certain period. The restarted particles can not only escape from the undesirable local minimum but also search for solutions extensively by the chaotic behavior, while particles execute the detail search around the global best solution. Therefore, this model can be expected to keep a balance of intensification and diversification of the search. Through computational experiments, we verify the performance of the proposed model by applying it to some global optimization problems.

1. Introduction

The particle swarm optimization method (PSO) is one of metaheuristic methods for global optimization inspired by swarms of birds or fish [5]. This method is a very simple algorithm with high performance. However, it is reported that for some cases almost all particles converge to an undesirable local minimum at early stages without an extensive search. Thus, in this method, the exploration ability is critical to find a desirable solution, and various kinds of improved methods have been investigated to improve the ability [2].

In this paper, we focus on improved PSOs which exploit the chaotic dynamics. In most of those method, some or all particles search for a solution broadly by using a chaotic sequence generated by the well-known function such as the logistic function [1, 3, 6]. However, since the used chaotic map is irrelevant to an optimization problem, the behavior of particles is not necessarily suitable to solve any problem.

On the other hand, we proposed a chaotic dynamical system which is derived by adding sinusoidal perturbation terms to the standard update rule of a particle and showed the sufficient conditions for parameters under which the proposed system is chaotic [10]. A particle with the system called a *chaotic particle* moves chaotically around the global and local best solutions, while it can be expected to search for a solution in the similar direction to the standard particle, that is, toward two best solutions. Since a chaotic particle often cannot execute the detail search, we proposed a PSO model which uses both of standard and chaotic particles, and verified the effectiveness of the proposed model than some improved PSO methods through numerical experiments. However, we observed that it is difficult to select the appropriate numbers of standard and chaotic particles for each problem, and that a considerable number of standard particles in the proposed model are trapped at a local minimum.

Therefore, in this paper, we propose a new PSO restarting inactive particles, which initializes particles whose velocity is sufficiently small by resetting its position and velocity by randomized numbers, and updates the particle by the chaotic dynamical system during a certain period. In addition, the detail search is required for the intensification, the model uses a small number of standard particles which are not restarted. Through some numerical experiments, we show the effectiveness of the proposed model.

2. Particle Swarm Optimization

In this paper, we focus on the following global optimization problems having many local minima and the rectangular constraint.

$$\min \quad f(x) \quad \text{s.t.} \quad x \in \prod_{i=1}^{n} [x_i^l, x_i^u].$$

In order to solve this problem, in the PSO system a number of candidate solutions called *particle* are simultaneously updated by exchanging the information each other. At each iteration, particles move toward a linear combination of two best solutions called *the local best* $l^i(t)$ and *the global best* $g(t)$, where the former is the best solution obtained by each particle i until iteration t and the latter is the best one obtained by all particles until iteration t. Then, the update formula of particle $i \in \{1, \ldots, L\}$ is given by

$$\text{(P1)} \quad \begin{aligned} v^i(t+1) &:= wv^i(t) + c_1 r^1 \otimes (g(t) - x^i(t)) \\ &\quad + c_2 r^2 \otimes (l^i(t) - x^i(t)), \\ x^i(t+1) &:= x^i(t) + v^i(t), \end{aligned}$$

where $x^i(t) \in \Re^n$ is a current point of particle i at iteration t, and w, c_1, $c_2 > 0$ are weights, while r_1, r_2 are randomized numbers uniformly selected from $(0, 1)^n$. The operation $\otimes : \Re^n \times \Re^n \to \Re^n$ is defined by $(s \otimes t)_i := s_i t_i,\ i = 1, \ldots, n$. We call a particle updated by (P1) the standard particle.

This simple approach has been applied to a great number of global optimization problems and is reported to have the ability to find high-quality solutions [5]. However, particles sometimes tend to converge to a local minimum at early stages for some problems, which is called the premature convergence. Hence, in order to improve the exploration ability, various kinds of improved methods have been investigated [2, 4]. In this paper, we focus on the PSO using a chaotic dynamical system.

3. Chaotic Particle Swarm Optimization

3.1. PSO using chaotic dynamical system

Mathematically, the chaos means an aperiodic deterministic behavior which is exceedingly sensitive to its initial conditions. Even though the model of the system is well defined and contains no random parameters, the behavior appears to be random. In the field of optimization, such a behavior is exploited in some metaheuristic methods [11, 13] for the global optimization problem. Those methods avoid being trapped at an undesirable local minimum by making use of the chaotic behavior, and aim to find a desirable solution within a practical time.

Recently, PSO making use of the chaotic dynamics have been investigated [1, 3, 6, 10]. Most of those methods use a chaotic sequence generated by a well-known function such as the logistic function. Thus, they can extensively search for solutions by the chaotic sequence. Alatas, Akin and Ozer compared twelve kinds of chaos-embedded PSO algorithms (CEPSOAs) which use one of eight kinds of chaotic maps for the benchmark global minimization problems [1]. They report that the following model (C) is superior to other models for these problems on average:

$$\text{(C)} \quad \begin{aligned} v^i(t+1) &:= wv^i(t) + c_1 r^1 s_1(t) \otimes (g(t) - x^i(t)) \\ &\quad + c_2 r^2 s_2(t) \otimes (l^i(t) - x^i(t)), \end{aligned}$$

where, $s_1(t)$ and $s_2(t)$ are chaotic sequences generated by Zaslavskii map defined by

$$\begin{aligned} u(t+1) &= (u(t) + 400 + 12v(t) + 1) \mod 1, \\ v(t+1) &= \cos(2\pi u(t)) + v(t)\exp(-3), \end{aligned}$$

where $\{u(t)\}$ is used as $\{s_1(t)\}$ or $\{s_2(t)\}$. This dynamical system can execute a diversified search because of the sensitiveness of an initial condition. It is reported that this method can be superior to some improved PSOs.

However, since the used chaotic sequence is irrelevant to the objective function, the performance of the search may significantly depend on each optimization problem. Therefore, we proposed a method of generating a chaotic sequence on the basis of the information of the local and global bests obtained by particles. Moreover, we showed the sufficient condition under which the proposed dynamics is chaotic in the sense of Li-Yorke [10]. Then, in the next section we introduce the chaotic dynamical system in brief.

3.2. Perturbation based chaotic PSO

In this section, let us consider the following dynamical system:

$$\text{(P2)} \quad \begin{aligned} v^i(t+1) &=: w^c v^i(t) + c_1^c r^1 \otimes (g(t) - x^i(t)) \\ &\quad + c_2^c r^2 \otimes (l^i(t) - x^i(t)) \\ &\quad - \beta\omega \begin{pmatrix} \sin(\omega x_1(t) - \omega \bar{q}_1^i(t)) \\ \vdots \\ \sin(\omega x_n(t) - \omega \bar{q}_n^i(t)) \end{pmatrix}, \end{aligned}$$

where positive constants β and ω are the amplitude and the frequency of the perturbation, respectively. $\bar{q}^i(t)$ is a divided point of global and local bests defined by

$$\bar{q}^i(t) =: \frac{c_1^c l^i(t) + c_2^c g(t)}{c_1^c + c_2^c}.$$

This model is derived from an interpretation that a particle with (P1) can be regarded as a stochastic steepest descent method with an inertial term and a step-size 1 for a problem minimizing $x^\top \left(\frac{1}{2}x - \bar{q}^i(t)\right)$. Thus, $x^i(t)$ quickly converges to $\bar{q}^i(t)$ if $w = 0$, $r_j^1 = r_j^2 = 1/2$, $j = 1, \ldots, n$, c_1 and c_2 are sufficiently small and $l^i(t)$ and $g(t)$ are not updated. Then, by adding perturbation terms to (P1), the obtained system (P2) is chaotic around the $\bar{q}^i(t)$. In fact, we can show the sufficient conditions of parameter values in which (P2) is chaotic.

Theorem 1 *Suppose that $w = 0$ and $r_j^1 = r_j^2 = 1/2$, $j = 1, \ldots, n$ and that $l^i(t)$ and $g(t)$ are not updated for some particle i. In addition, if c_1^c, c_2^c, ω and β satisfy the inequalities,*

$$\frac{c_1^c + c_2\ c}{2} - \frac{2}{3\pi}\beta\omega^2 < 1 \quad \text{and} \quad \frac{c_1^c + c_2^c}{2} + \frac{2}{\pi}\beta\omega^2 > 4, \quad (1)$$

then system (P2) *is chaotic in the sense of Li-Yorke.*

We can prove this theorem by showing that $\bar{q}^i(t)$ is a snap-back repeller of (P2). It is well-known that if a system has a snap-back repeller which is a kind of fixed point, then the system is chaotic in the sense of Li-Yorke [7], and that there exists various kinds of chaotic sequences generated by the system which approach the snap-back repeller, and at the same time, which are repelled from the point. Thus, (P2) can be used in order to search for a solution extensively without risk of being trapped at local minima.

Moreover, if we select c_1^c, c_2^c, β and ω such that

$$0 < c_1^c + c_2^c \leq 4, \tag{2}$$

$$\beta\omega^2 > 2\pi, \tag{3}$$

then, (1) is satisfied. Here, c_1^c and c_2^c are usually selected to satisfy (2) because it guarantees that the next point $x^i(t+1)$ of particle i is closer to $\bar{q}^i(t)$ than a current point $x^i(t)$. Hence, it is easy to select parameters, for example, a sufficiently large ω and a small β, though (P2) has more parameters than (P1). This results is simpler than conditions shown in [10].

Now, in the next section, we propose a new PSO model which exploits the system (P2).

4. Restarting chaotic particle swarm optimization

As mentioned above, particles with the chaotic system (P2) can search for solutions extensively, while it might not be able to execute the detail search around the global best. Therefore, in order to keep a balance of diversification and intensification of the search, it is suitable to use both of standard and chaotic particles. In [10, 12] we proposed some models which use both type particles simultaneously, and verified the good performance of the proposed models for some benchmark problems. However, we also observed that an appropriate selection of the numbers of standard and chaotic particles is difficult for each problem, and moreover, that standard particles are often trapped at around the global best, which is called an *inactive* particle, until the solution is drastically changed.

Therefore, in this paper we propose a PSO which restarts inactive particles by using the chaotic system (P2), where all particles basically search for solutions based on the standard dynamics (P1). If a particle is trapped and, in addition, if the following equality is satisfied:

$$\sum_{j=1}^{n} \left(\frac{v_j^i(t)}{x_j^u - x_j^l} \right)^2 < n\, v_{\mathrm{th}}^2, \tag{4}$$

where v_{th} is a small positive constant, the particle i is restarted by initializing its position and its velocity by randomized numbers, and set $l_i(t) =: x^i(t)$. In addition, during a certain period, the particle is updated by the chaotic dynamical system (P2). Since the proposed particles having standard and chaotic modes, we call it a *dual-mode particle*. Moreover, if so many particles are restarted simultaneously, the intensification ability might be weakened. Thus, the proposed model uses a small number of standard particles which are not restarted. Therefore, this method can keep the diversity of the search by the chaotic dynamical system, and execute the intensification of the search by the standard system. We call this model the PSO using the chaotic restarting system (PSO-CR).

Next, we compare the proposed model with the existing PSOs which restarts inactive particles [8, 9] Those methods restart a particle by initializing its velocity or an element of the velocity by randomized numbers. On the other hand, the proposed method initializes not only its velocity but also its position and its local best, which might seem to discard the important information of promising regions. However, the best solution obtained is always kept as the global best. In addition, we observed that when inactive particles are trapped at an undesirable solution, several active particles move around there in some experiments. Therefore, the proposed method can be expected to offer lower risk of losing the important information. Moreover, the method prevents restarted particles from being trapped again at the same local minimum.

5. Numerical experiments

In this section, we show the results of numerical experiments, where we applied the proposed model PSO-CR, original PSO and CEPSOA to six benchmark problems, Rastrigin, Rosenbrock, Griewank, 2n-minima, Schwefel and Levy No.5 functions. The maximal number of iteration was 5000, and the number of particles was 20 in all models. We executed a preparatory experiment to select parameter values for CEPSOA and PSO-CR, and to select the appropriate numbers of standard and dual-mode particles for each problem. Then, for PSO-CR we used fifteen standard and five dual-mode particles which was effective for all problems, and selected suitable two kinds of sets of parameter values, called parameter sets A and B. As parameter set A, we used $(w, c_1, c_2) = (0.0, 1.0, 2.5)$ in (P1), $(w^c, c_1^c, c_2^c) = (0.0, 0.3, 0.3)$ in (P2) and $v_{\mathrm{th}} = 0.0001$ for Rosenbrock, Levy No.5 and Rastrigin functions, while as parameter set B, $(w, c1, c2) = (0.0, 0.2, 2.5)$ in (P1), $(w^c, c_1^c, c_2^c) = (0.0, 0.8, 0.8)$ in (P2) and $v_{\mathrm{th}} = 0.001$ for other functions, and moreover, we set $(\beta, \omega) = (0.0002, 500)$ in both sets. The average function values of the global bests obtained by three methods are shown in Table 1, where the bold and underlines figures indicate the first and second best results among three methods, respectively. The

Table 1: Comparison of PSO, CEPSOA and PSO-CR for six benchmark problems (Average objective function values)

Function	Dim.	PSO	CEPSOA Best param. set	PSO-CR Two param. set	PSO-CR Best param. set
Griewank	40	0.59826	0.019738	0.015283	**0.007162**
Rastrigin	40	170.92029	69.708011	23.970452	**3.163157**
Rosenbrock	40	36.585367	41.662483	35.634834	**28.072611**
Levy No.5	40	1.157437	0.097857	0.000001	**0.000000**
2n-minima	40	0.133943	0.066422	0.057356	**0.034573**
Schwefel	40	0.277059	0.146946	0.150968	**0.137543**

table shows that PSO-CR using the best parameter set obtained the lowest average function values for all problems, and that PSO-CR using parameter sets A and B obtained the second best results for five problems except Schwefel function. These results demonstrate that PSO-CR can find high-quality solutions by restarting method and dual-mode particles, and that it is not so difficult to select appropriate parameter sets for many kinds of problems, though PSO-CR has more parameters than PSO-IWA and CEPSOA.

6. Conclusion

In this paper, we have proposed a PSO which restarts inactive particles to avoid stagnation of the search. The proposed model uses two kinds of particles, a large number of dual-mode particles and a small number of standard particles. The former particle is basically updated by the standard dynamical system, while it is restarted and updated by the chaotic dynamical system during a certain period if its velocity is sufficiently small. The latter particle is updated by the standard dynamical system and never restarted in order to execute the detail search around the global best. Thus, the proposed model can be expected to search for solutions without the stagnation and keep a balance of intensification and diversification in the search. Through some numerical experiments, we have observed that the proposed model execute more effective search than some exiting improved PSOs, and that it is not difficult to select its parameter values.

As future research, we should investigate a condition of restarting particles which is crucial for the proposed PSO model to search for solutions efficiently.

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