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Evolutionary Game Model of Water Resource Development Problem

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Abstract—There are many kinds of conflict in the world, and they are very complex and controversial. To analyze conflicts, the methodology of games can be employed. In this paper, we model a negotiation process between prefectures over water resource development by the evolutionary game theory. We consider the upstream prefecture consists of two sections. We propose a model that they play games with the downstream prefecture and change their strategies depending not only on their own payoffs but also on total payoffs of the upstream prefecture.

1. Introduction

Because of great import of water for sustaining modern day civilizations, water resource development problems tend to be very complex and controversial. The water resource development problem is a matter to adjust the supply-demand gap between the prefecture which can obtain plentiful water resources easily and the prefecture not so [1]. We call the former the supply prefecture and the latter the demand prefecture. When we try to adjust the supply-demand gap of water resources between these prefectures, the conflict which is caused by differences of their desires arises necessarily. The water resource development problem in the real world has various characteristics in each area.

To model a water resource development problem, the technique of the game theory can be employed. For example, the Lake Biwa conflict in Japan is analyzed by the hypergame [1]. The main purpose of this paper is to abstract common conflict patterns found in the water resource development problem and model the problem by the evolutionary game. In the evolutionary game, a player selects a strategy by trial and error, and dynamic characteristics of the selection process are described by replicator dynamics [2, 3]. By using the evolutionary game, we can discuss how players' decisions change. The water resource development problem is modeled as a game between an upstream prefecture and a downstream prefecture. We consider that the upstream prefecture consists of two sections that make decisions on regional developments and water allocations to the downstream prefecture, respectively. We propose a model that each section plays a game with the downstream prefecture and changes its strategy depending not only on its own payoffs but also on total payoffs of the upstream prefecture. We show a simplified simulation of the proposed model.

2. Preliminaries

An evolutionary game in an n-population model is defined as an interaction of n populations. We suppose that there exist n large populations $P_1, P_2, \ldots, P_n$. Repeatedly, players are randomly drawn from these populations to play game — one player from each population [2]. This section gives brief introductions to n-population evolutionary games [2, 3].

In this paper, we use the following notations:

- $N = \{1, 2, \ldots, n\}$: an index set of populations;
- $\Phi_i = \{1, 2, \ldots, m_i\}$: a set of pure strategies of P_i ($i \in N$);
- Δ_i : a set of population states of P_i,
- $s_i = (s_i^1, \ldots, s_i^{m_i})^T \in \Delta_i$: a population state of P_i, where s_i^k is the proportion of players with a pure strategy $k \in \Phi_i$.
- $\Delta = \times_{i \in N} \Delta_i$: sets of population state combinations;
- $R_i : \Delta \to \mathbb{R}$: player i's payoff function; and
- e_l^k : the l-dimensional unit vector such that the kth element equals 1.

As a special case, we consider a two-population game, and write players' payoff function as payoff matrices. Let $A = (a_{ij})$ and $B = (b_{ij})$ be P_1's and P_2's payoff matrix, respectively. For $k \in \Phi_1$ and $h \in \Phi_2$, suppose that $a_{kh} = R_1(e_{m_1}^k, e_{m_2}^h)$ and $b_{hk} = R_2(e_{m_2}^h, e_{m_1}^k)$. Hence, if player of P_1 adopts $s_1 \in \Delta_1$ and player of P_2 adopts $s_2 \in \Delta_2$, then the payoff which the former (*resp.* the latter) earns is $s_1^T A s_2$ (*resp.* $s_2^T B s_1$).

Replicator dynamics is one of key concepts of evolutionary game theory. It describes the evolution of distribution of strategies in populations. In each population, suppose that the rate of increase of players with a strategy k is expressed as the difference between the payoffs which a player with a strategy k earns and the average payoff the population earns. Hence, replicator dynamics of the n-population model is formulated as follows:

$$\dot{s}_i^k = s_i^k [R_i(e_{m_i}, s_{-i}) - R_i(s)], \tag{1}$$

where $s_{-i} = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n)$ is the population state combination results from s by removing the population state s_i. If a player with a pure strategy k earns more (*resp.* less) payoff than the average payoff, the proportion of players with a strategy k will increase (*resp.* decrease).

3. Model

We consider a water development problem for two prefectures such as the Lake Biwa conflict in Japan [1]. In the same river basin, the upstream prefecture is the supplier and the downstream prefecture is demander. The upstream prefecture allocates water to the downstream prefecture to replenish the shortfall of the downstream prefecture's water resources. In this situation, desires of prefectures conflict with each other. So, it is necessary to adjust the overall interests to resolve that problem.

We have the following assumptions:

1. The upstream prefecture:
   - is less developed than the downstream prefecture and is eager to implement regional development,
   - is unwilling to allocate water to the downstream prefecture because drawdown of water caused by the allocation may damage such as facilities,
   - can allocate water if the allocation helps with implementation of regional development.
2. The downstream prefecture:
   - desires to use sufficient water with no responsibility,
   - considers that shortage of water resources interferes with daily life,
   - has the fund for purchase of water from the upstream prefecture,
   - saves water in order to reduce financial burdens to get water and shows its attitude to work toward saving water by investment in facilities.

The upstream prefecture makes a two decisions, "whether implement regional development" and "whether allocate water to the downstream." They are decisions on financial resources and water resources, respectively. So, we consider the upstream prefecture consists of two sections that make decisions on regional developments and water allocations to the downstream prefecture, respectively. We call them "Water allocation Section (WS)" and "Development Section (DS)," respectively. We call the downstream prefecture "D." Thus, this problem is modeled as a three-population game among WS, DS, and D as shown in Fig. 1.

WS and DS play the game with D separately. However, these two sections are governed by "Governor (Gov)" which considers total payoffs of the upstream prefecture. Gov attempts to govern these sections by giving payoffs. That is, WS and DS earn payoffs by Gov while playing the game with D.

3.1. Players and Their Strategies

In this three-population game among WS, DS, and D, WS and DS have two strategies. WS's strategies are "Don't allocate water to D (Don't A.W.)" and "Allocate water to D (A.W.)." DS's strategies are "Don't implement regional development (Don't Dev.)" and "Implement regional development (Dev.)." On the other hand, D has two funds: that for water saving equipment and that for purchase of water from the upstream prefecture. The total amount of the funds is limited. So, D has three strategies related to how its budget is used for the funds: "Do-nothing," "Purchase of water (P.W.)," and "Save water (S.W.)."

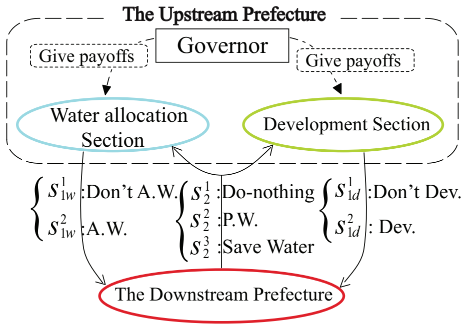

Figure 1: Relationships among populations.

In this game, population states are described as follows:

1. The upstream prefecture:
   $s_{1w} = [s^1_{1w}, s^2_{1w}]^T$: WS's population state,
   $s_{1d} = [s^1_{1d}, s^2_{1d}]^T$: DS's population state.
2. The downstream prefecture:
   $s_2 = [s^1_2, s^2_2, s^3_2]^T$: D's population state.

Then, when WS and DS play the game with D separately, payoff functions of the populations are given as follows:

$$\begin{cases} R_{1w}(s_{1w}, s_{1d}, s_2) = s_{1w}^T A_w s_2 & : \text{WS's payoff,} \\ R_{1d}(s_{1w}, s_{1d}, s_2) = s_{1d}^T A_d s_2 & : \text{DS's payoff,} \\ R_2(s_{1w}, s_{1d}, s_2) = s_2^T B_w s_{1w} + s_2^T B_d s_{1d} & : \text{D's payoff.} \end{cases} \quad (2)$$

A_w and A_d are WS's and DS's payoff matrices in the game with D, respectively. B_w and B_d are D's payoff matrices in the game with WS and DS, respectively.

3.2. The Payoff Given by Governor

When WS and DS play the game with D separately, these sections earn payoffs defined by Eq. (2). In this game, the purposes of WS and DS are maximization of their own payoffs. However, they may not equal the total payoff of the upstream prefecture. WS can allocate water to D if D uses a large amount of its budget as the fund for purchase of water and DS can use it for the implementation of regional development of the upstream prefecture, that is, if DS can earn more payoffs than losses of WS's payoffs which made by the allocation. Then, we consider that Gov which has a certain policy governs WS and DS by giving payoffs to each section. We define the given payoffs as follows:

$$\begin{cases} f_w(s_{1w}, s_{1d}, s_2) : \text{the given payoff to WS,} \\ f_d(s_{1w}, s_{1d}, s_2) : \text{the given payoff to DS.} \end{cases}$$

When Gov gives these payoffs, each section plays a game with D and changes its strategy depending not only on its own payoffs but also on a policy of Gov as *total payoffs of the upstream prefecture*. Introducing the payoffs given by

Table 1: Payoff matrix C.

WS\DS	Don't Dev.	Dev.
Don't A.W.	c_{11}	c_{12}
A.W.	c_{21}	c_{22}

Gov into the payoff functions of populations, we modify Eq. (2) as follows:

$$\begin{cases} R_{1w}(s_{1w}, s_{1d}, s_2) = s_{1w}^T A_w s_2 + f_w(s_{1w}, s_{1d}, s_2), \\ R_{1d}(s_{1w}, s_{1d}, s_2) = s_{1d}^T A_d s_2 + f_d(s_{1w}, s_{1d}, s_2), \\ R_2(s_{1w}, s_{1d}, s_2) = s_2^T B_w s_{1w} + s_2^T B_d s_{1d}. \end{cases} \quad (3)$$

In this paper, we consider that Gov has a certain policy to combination of strategies of WS and DS, and give the payoff which depends on the policy to each section. We define a payoff matrix C which characterize the Gov's policy as show in Table 1. An order of elements of matrix C depends on the Gov's policy. When Gov has a policy which gives priority to the implementation of regional development over the water allocation, the order is $c_{22} > c_{12} > c_{21} > c_{11}$. When Gov has a policy which allocates water to D in order to gain fund from D and use the fund to implement regional development, the order is $c_{22} > c_{21} > c_{12} > c_{11}$. We set the given payoffs f_w and f_d determined by the matrix C as follows:

$$\begin{cases} f_w(s_{1w}, s_{1d}, s_2) = s_{1w}^T C s_{d1}, \\ f_d(s_{1w}, s_{1d}, s_2) = s_{1d}^T C^T s_{w1}. \end{cases} \quad (4)$$

Note that $\sum s_{1w}^i = \sum s_{1d}^i = \sum s_2^i = 1$. From Eq.(1), replicator dynamics of this model is formulated as follows:

$$\begin{cases} \dot{s}_{1w}^2 = s_{1w}^2\{(e_2^{2T} - s_{1w}^T)(A_w s_2 + C s_{1d})\}, \\ \dot{s}_{1d}^2 = s_{1d}^2\{(e_2^{2T} - s_{1d}^T)(A_d s_2 + C^T s_{1w})\}, \\ \dot{s}_2^2 = s_2^2\{(e_3^{2T} - s_2^T)(B_w s_{1w} + B_d s_{1d})\}, \\ \dot{s}_2^3 = s_2^3\{(e_3^{3T} - s_2^T)(B_w s_{1w} + B_d s_{1d})\}. \end{cases} \quad (5)$$

4. Simulation

In this section, we fix payoff matrices A_w, A_d, B_w, and B_d as follows:

$$A_w = \begin{bmatrix} 0 & 5 & 2 \\ -5 & 2 & -1 \end{bmatrix}, \quad A_d = \begin{bmatrix} -1 & 2 & 1 \\ 3 & 6 & 4 \end{bmatrix}, \quad (6)$$

$$B_w^T = \begin{bmatrix} -4 & -5 & -2 \\ 5 & 7 & 4 \end{bmatrix}, \quad B_d^T = \begin{bmatrix} 3 & -1 & 2 \\ -1 & 2 & 0 \end{bmatrix}. \quad (7)$$

These matrices are set based on the following assumptions [1, 4, 5]:

- WS does not want to allocate much water to D.
- Since DS's mission is the implementation of regional development, DS earns more payoffs if D's fund for purchase of water increases.
- If WS allocates much water, D uses a large amount of its budget as the fund for purchase of water, otherwise it does as the fund for water saving equipment.
- It is undesirable for D that DS pays much attention to implementation of regional development. However, D expects that DS uses D's fund for purchase of water to implement DS's regional development.

We consider the following four cases that Gov has different policies and the matrix C is set for the each policy.

(Case 0) Don't intervene ($C = O$).

(Case 1) Give priority to the implementation of development over the water allocation to D. The order of elements of the matrix C is $c_{22} > c_{21} > c_{12} > c_{11}$.

(Case 2) Give the same priority to the implementation of development and the water allocation to D. The order of elements of the matrix C is $c_{22} > c_{12} = c_{21} > c_{11}$.

(Case 3) Give priority to the water allocation to D over the implementation of development. The order of elements of the matrix C is $c_{22} > c_{12} > c_{21} > c_{11}$.

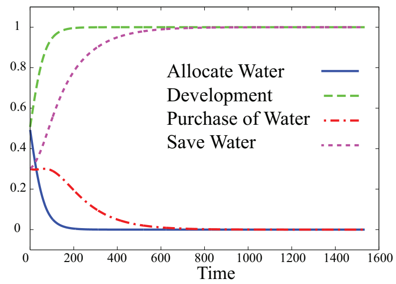

Figure 2: Example of transition behavior (Case 0).

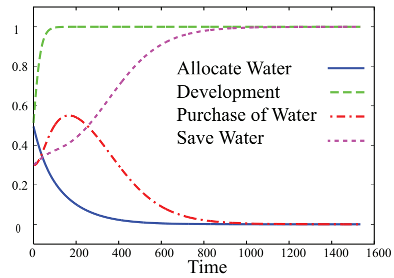

Figure 3: Example of transition behavior (Case 1).

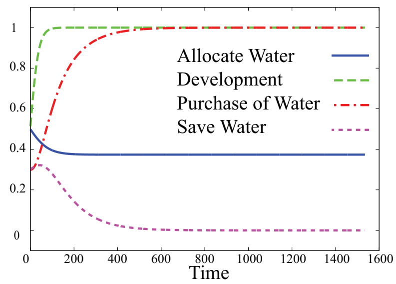

Figure 4: Example of transition behavior (Case 2).

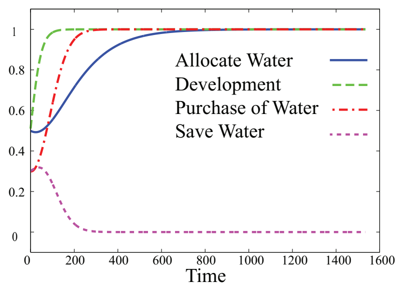

Figure 5: Example of transition behavior (Case 3).

In this section, we show that how the stability of equilibrium points changes according to Gov's policies. Figures 2–5 show examples of a transition behavior of these four cases, where the initial state is $s_{1w}^2 = 0.5, s_{1d}^2 = 0.5, s_2^2 = 0.3, s_2^3 = 0.3$. As shown in Figs. 2–5, the proportion of players with a strategy "Dev." s_{1d}^2 converges to $s_{1d}^2 = 1$ in all cases. Then, Figs. 6–9 show phase portraits except s_{1d}^2. In Cases 0 and 1, there exists a unique asymptotically stable equilibrium point $p_{0,1}$. Thus, all orbits starting from interior converge to population state combination $p_{0,1}$: $s_{1w}^2 = 0, s_2^2 = 0, s_2^3 = 1$, which is non-cooperative equilibrium. In Case 3, on the other hand, there exists a unique asymptotically stable equilibrium point p_2. Thus, all orbits starting from interior converge to population state combination p_2: $s_{1w}^2 = 1, s_2^2 = 1, s_2^3 = 0$, which is cooperative equilibrium. In Case 2, there exist two non-isolated equilibrium points $l_{P.W.}$ and $l_{S.W.}$. All orbits starting from interior converge to either population state combination $l_{P.W.}$ or $l_{S.W.}$ depending on the initial state. In Fig. 8, when the proportion of players with a strategy "W.A." s_{1w}^2 converges to more (*resp.* less) than 1.77, orbits converge to $l_{P.W.}$ (*resp.* $l_{S.W.}$).

As above, the Gov's policy affects strategies of each population, i.e., population states converge to a cooperative state when Gov has a cooperative policy for D, and population states converge to a non-cooperative state when Gov has a non-cooperative policy.

5. Conclusions

In this paper, we modeled a negotiation process between prefectures over water resource development by the evolutionary game theory. We considered the upstream prefecture consists of two sections. We proposed a model that they play games with the downstream prefecture and change their strategies depending not only on their own payoffs but also on total payoffs of the upstream prefecture.

Acknowledgments

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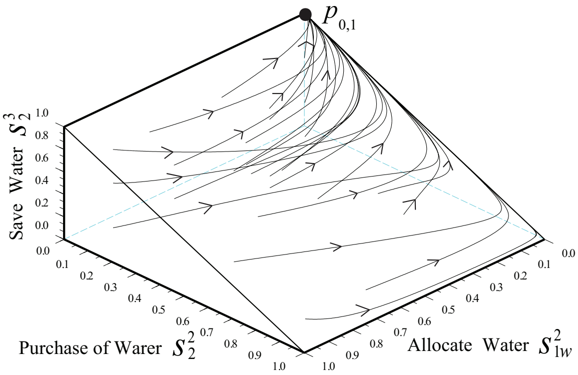

Figure 6: Phase portrait of Case 0.

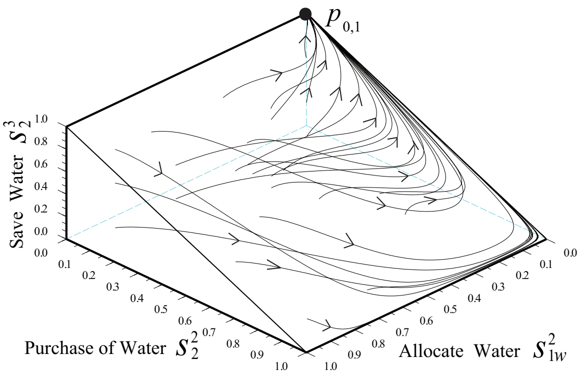

Figure 7: Phase portrait of Case 1.

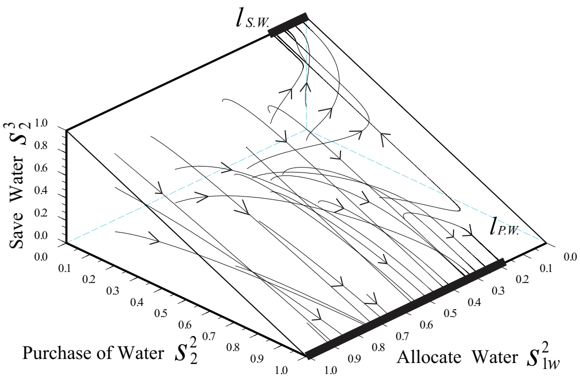

Figure 8: Phase portrait of Case 2.

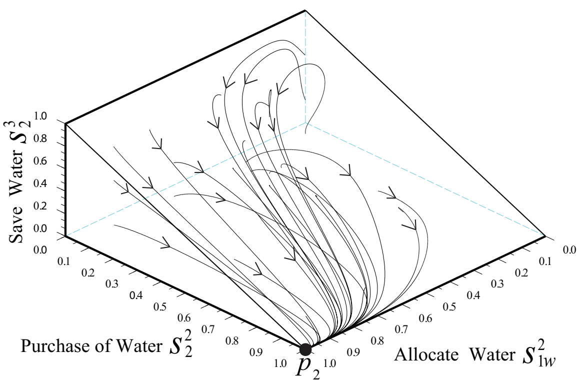

Figure 9: Phase portrait of Case 3.