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Destabilization of intrinsic localized modes in magnetically coupled cantilever array by parametric excitation

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Abstract—Energy localized vibrations known as intrinsic localized modes or discrete breathers were observed in a magnetically coupled cantilever array. It has been experimentally confirmed that the observed ILMs are stable against small perturbation. In this paper, it is shown that such stable ILMs can be destabilized by periodically varying on-site potentials. The destabilized ILM becomes a traveling ILM under the fixed boundary condition.

1. Introduction

Energy localization is known as intrinsic localized mode (ILM) or discrete breather (DB) in nonlinear coupled oscillators. ILM was discovered in an anharmonic crystal as a spatially localized and temporally periodic solution [1]. Many theoretical/numerical researches have been reported for diverse physical systems [2]. Existence of ILM in real physical systems is now recognized by researchers owing to experimental studies such as an observation of ILM in antiferromagnet [3]. In addition to the observation of ILM in natural structures, ILM was also identified in artificial structures, for instance, Josephson junction ladders [4], optic wave guides [5], electronic circuits [6, 7], and micro-cantilever arrays [8].

Intrinsic localized modes in micro-cantilever arrays can be manipulated by a locally induced impurity [9] and moved by tuning the frequency of an external excitor [10]. For manipulation method, we proposed "capture and release manipulation" in which the stability change of ILM is utilized [11]. In this method, an initially excited ILM begins to wander in the array when the nonlinear coupling coefficient is varied in a step-like manner. On the other hand, a stable ILM can also be destabilized by sinusoidally changing the nonlinear coupling coefficient, namely, parametric excitation [12]. Although the capture and release manipulation requires that the micro-cantilever array has to be designed to be close to the bifurcation point at which the stability of ILM is changed, the parametric excitation can be applied to any micro-cantilever arrays. Therefore, the parametric excitation is a useful way to create a traveling ILM from a static ILM in practical engineering.

In our previous work, a macro-mechanical cantilever array with magnetic forces were proposed [13]. The cantilever array can be classified into nonlinear Klein-Gordon (NKG) lattice because a nonlinearity caused by the magnetic force only exists in on-site potentials. The coupling is linear. In this array, ILM was successfully excited and manipulated by locally adding an impurity. Recently, a cantilever array having nonlinearity in coupling forces has been proposed [14]. It is already confirmed that an unstable ILM can travel the array with keeping its energy concentration if a small perturbation is applied. This implies that the magnetically coupled cantilever array permits ILM to move in the whole array.

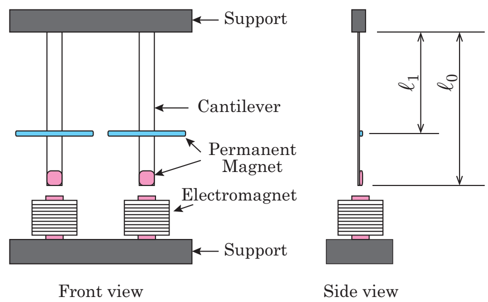

Figure 1: Configuration of magneto-mechanical lattice

In this paper, a possibility of creating a traveling ILM via parametric resonance is discussed. In next section, a model describing motions of cantilevers are briefly introduced. Then, a stable ILM, its Floquet multipliers, and eigenvectors are shown. Finally, unstable regions where the stable ILM loses stability and begins to move are computed. It is discussed whether the destabilized ILM begins to travel the array or not via parametric resonance.

2. Magnetically coupled cantilever array

Figure 1 shows a schematic image of a magnetically coupled cantilever array [14]. Each cantilever has a permanent magnet at the lower end and an electromagnet beneath the tip which makes the restoring force nonlinear. The other permanent magnet is attached below the middle of each beam. The magnetic force between the permanent magnets causes a nonlinear coupling force between adjacent cantilevers. By using the magnetic charge model and nondimensionalization [14, 15], the following equation of mo-

Table 1: Nondimensionalized parameters of Eq.(1). Unit length and unit time are chosen at 1 mm and 28.57 ms, respectively.

x_n	Displacement of nth cantilever
$\chi'_O = 133.1$[a]	On-site magnetic force coefficient
$\chi'_I = 243.1$[a]	Inter-site magnetic force coefficient
$d'_1 = 3$	Gap between permanent magnet and electromagnet
$d'_2 = 2.8$[b]	Gap between coupling permanent magnets

[a] This value is estimated at $I_{EM} = 30$ mA.
[b] Coupling magnets are attached at $\ell_1 = 50$ mm. The length of cantilever is $\ell_0 = 70$ mm.

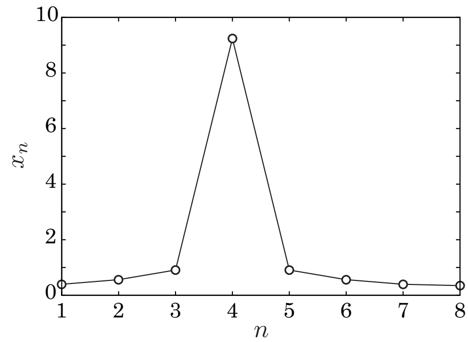

Figure 2: Wave form of an intrinsic localized mode centered at $n = 4$.

tion is obtained:

$$\ddot{x}_n = -(2\pi)^2 x_n - \chi'_O \frac{x_n}{\left(x_n^2 + {d'_1}^2\right)^{3/2}} - \chi'_I \frac{x_n - x_{n+1}}{\left\{(x_n - x_{n+1})^2 + {d'_2}^2\right\}^{3/2}} - \chi'_I \frac{x_n - x_{n-1}}{\left\{(x_n - x_{n-1})^2 + {d'_2}^2\right\}^{3/2}}. \quad (1)$$

Parameters in the equation is listed in Table 1. In numerical simulations, the number of cantilevers is set at 8 and the boundary condition is assumed to be periodic, namely, $x_0 = x_8$, $x_9 = x_1$.

3. Floquet multipliers and eigenvectors

Several types of ILM coexist in the magnetically coupled cantilever array [14]. In this paper, a site-centered ILM shown in Fig.2 is focused on. The period of the ILM is $T_{ILM} = 0.98$. The current flowing in electromagnets is fixed at $I_{EM} = 30$ mA.

The stability of ILM is determined by computing Floquet multipliers [16]. In Fig. 3, all the Floquet multipliers are on the unit circle in the complex plane. Therefore, the ILM is not linearly unstable. In this paper, such ILM is simply called stable ILM.

Floquet multipliers of ILM can be classified into two groups by referring the shape of eigenvectors. One group has spatially localized eigenvectors whereas the other has spatially extended eigenvectors [16]. In Fig.3, eigenvalues having spatially localized eigenvectors are labeled as

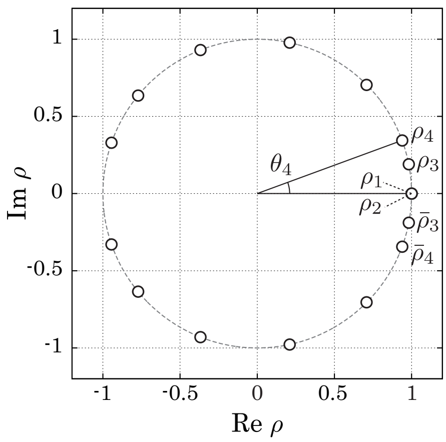

Figure 3: Wave form of an intrinsic localized mode centered at $n = 4$.

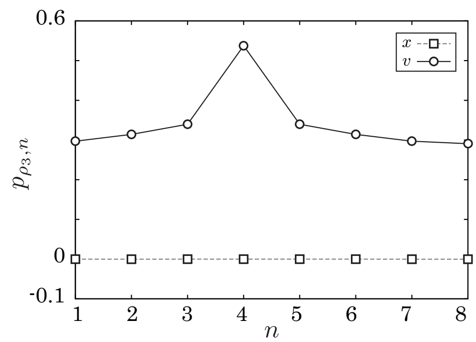

Figure 4: Real part of eigenvector of ρ_3.

ρ_k ($k = 1, 2, 3, 4$), where $\bar{\rho}_k$ denotes the complex conjugate of ρ_k. ρ_1 and ρ_2 are called the growth mode and the phase mode [16]. They are always located at +1 because the ILM is periodic solution and the system is Hamiltonian system.

Real part of eigenvector of ρ_3 and ρ_4 is shown in Fig.4 and Fig.5, respectively. In this case, x-components of the eigenvectors are very small comparing with the v-components. Hereafter, the eigenvector of ρ_k is denoted by p_{ρ_k}. As shown in Fig. 4, the v-components has a peak at $n = 4$. The symmetry of the eigenvector p_{ρ_3} is the same as the ILM. Therefore, the symmetry of ILM will not be changed when the ILM is perturbed along p_{ρ_3}. On the other hand, the v-components of p_{ρ_4} is antisymmetric with respect to $n = 4$ as shown in Fig.5. If the ILM is perturbed along p_{ρ_4}, the amplitude of the central cantilever of ILM is not changed because the v-component of p_{ρ_4} is zero. For the other cantilevers, the phases are changed in antisymmetric to $n = 4$. This deformation will force the ILM to move along the array.

4. Parametric destabilization

The frequency of the fluctuation caused by a perturbation along p_{ρ_4} is given by $\Omega_4 = \theta_4 / T_{ILM}$. Therefore, to amplify

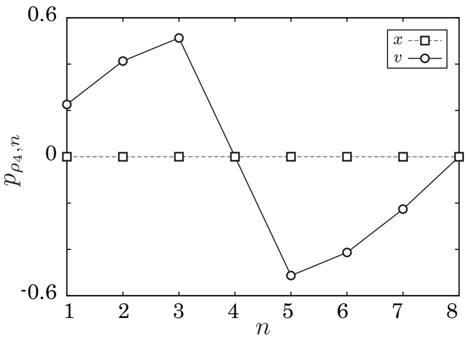

Figure 5: Real part of eigenvector of ρ_4.

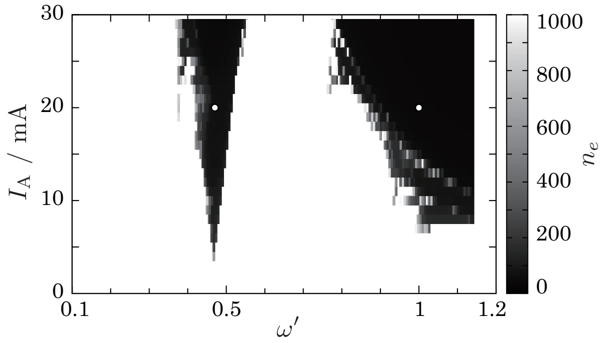

Figure 6: Unstable regions resulting from the parametric resonance. The displacement of ILM exceeds 10^{-6} at $t = n_e T_{\mathrm{ILM}}$.

the fluctuation, a parameter in Eq.(1) should be varied with the frequencies $2\Omega_4$, Ω_4, $2\Omega_4/3$, $\Omega_4/2$, and so on according to the previous work in a micro-cantilever array [12]. As shown in Fig.1, the cantilever array has electromagnets. Thus, the on-site potentials can be adjusted in time. In this section, the current flowing the electromagnets are varied sinusoidally $I_{\mathrm{EM}} = I_{\mathrm{A}} \sin \nu t + I_0$, where I_0 is fixed at 30 mA.

In Fig.6, unstable regions where the ILM becomes unstable and fluctuated along the array are shown. The fluctuation is detected based on the energy distribution (see Ref. [12] for details). The color n_e is the elapsed period when the displacement of ILM's position exceeds 10^{-6}. The horizontal axis $\omega' = \Omega_4/\nu$ corresponds to ω in the Mathieu equation $\ddot{x} + \omega^2(1 + \epsilon \sin t)x = 0$.

Two regions are located at $\omega' \simeq 0.5$ and 1. The region at $\omega' \simeq 0.5$ has a similar shape to those in the Mathieu equation. However, the lower peak of the region does not rigorously coincide with $\omega' = 0.5$. The reason is still unclear.

The other region around $\omega' \simeq 1$ is somewhat different in shape to the case of the Mathieu equation. For the region, the period of the parametric excitation is about several ten times of T_{ILM}. The ILM is thus subjected to the state for relatively long time in which χ'_{O} is too small to exist ILM. Therefore, the ILM is possibly destabilized by decreasing χ'_{O} below a critical value at which a bifurcation occurs. The region at $\omega' \simeq 1$ is emerged due to both the bifurcation and

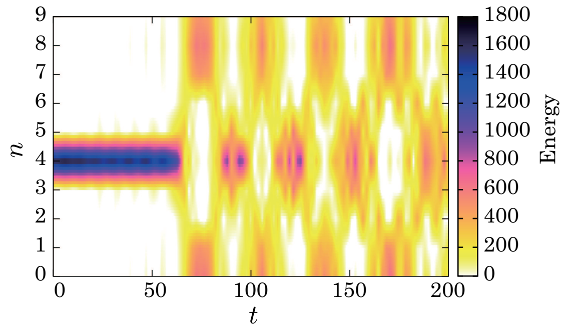

Figure 7: Behavior of a destabilized ILM when $\omega' = 0.47$ and $I_{\mathrm{A}} = 20$ mA.

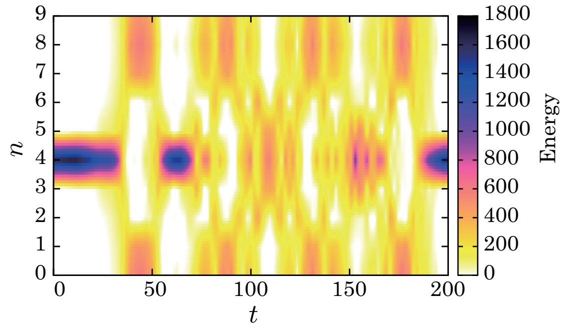

Figure 8: Behavior of a destabilized ILM when $\omega' = 1$ and $I_{\mathrm{A}} = 20$ mA.

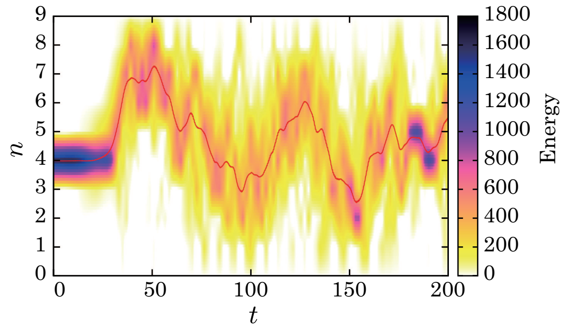

Figure 9: Traveling ILM resulting of parametric resonance in the array with fixed boundaries. The solid curve indicates the center of ILM. The parameter setting is the same as in Fig.8 except the boundary condition.

the parametric resonance.

To show how the ILM behaves when it is destabilized by the parametric excitation, time development of the energy distribution is plotted in Fig.7 and Fig.8. The open circles in Fig.6 indicate where the parameter is chosen. For both figures, the energy is initially concentrated at $n = 4$, but it is split into two parts. The two separated energy concentrations collide at the opposite side of the array. After the first collision, they are reflected and collided at the original position $n = 4$. This implies that the ILM is destabilized not only along p_{ρ_4} but also p_{ρ_3} or other eigenvectors. On the other hand, if the array has fixed boundaries as well as the real experimental setup, the ILM begins to travel the array as shown in Fig.9. Breaking the translational symmetry of the ringed array may prevent ILM from splitting.

5. Conclusion

In this paper, the possibility of creating a traveling ILM via parametric resonance was investigated. As a result, it was clarified based on the shape of unstable regions in a parameter space that a stable ILM can be destabilized via parametric resonance as well as in the micro-cantilever array. However, destabilized ILMs break into two parts instead of traveling in the array under the periodic boundary condition whereas there exists an eigenvector having antisymmetric shape. On the other hand, a traveling ILM was successfully created in the array having fixed boundaries. This implies that boundary conditions plays crucial roles for creating traveling ILMs especially for a small degree of freedom system. In the future work, the destabilization of ILM by the parametric excitation will be confirmed in our experimental system in which the cantilever array has fixed boundaries. It will be investigated numerically and experimentally whether the effect of the boundaries can be utilized to create traveling ILMs.

Acknowledgments

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