2011 International Symposium on Nonlinear Theory and its Applications
NOLTA2011, Kobe, Japan, September 4-7, 2011

Multiplexed Communications with Chaotic Optoelectronic Devices

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Abstract – In this paper, we describe a strategy to encrypt and decrypt multiple data streams using optical chaotic carriers. We modify a well-known electro-optic chaos generator by adding another delayed feedback loop. In each loop, a Mach-Zehnder modulator is used to produce a signal (later referred to as a *code*) to convey a particular message. A multiplexed transmission of two messages at multi-Gb/s is numerically demonstrated.

1. Introduction

Optical chaos-based communications have focused their attention on the transmission of a single message at fast bit rates [1,2] using various types of chaotic generators such as wavelength chaos generators (WCG) [3], electro-optic intensity chaos generators (ICG) [4], phase chaos generator (PCG) [5], edge-emitting laser with optical feedback [6], or optoelectronic feedback [7]. Recently, there has been a growing interest in increasing the spectral efficiency of such chaos-based cryptosystems by transmitting multiple messages on a single communication channel. Originally developed for chaotic maps [8] and continuous scalar time-delay systems [9,10], chaos-multiplexing concepts (with no messages transmitted) have rapidly found resonance with optoelectronic devices. Actual chaos multiplexing was achieved with multiple microchip lasers with detuned frequencies [11] or multimode lasers [12,13], extending the concept of wavelength-division multiplexing (WDM) on top of chaotic optical systems (such an approach is known as *chaotic WDM*).

In this paper, we aim at adapting the philosophy of a different multiplexing technique, known as code-division multiple access (CDMA), to the field of optical chaos-based communications. CDMA consists of the use of fixed orthogonal codes that spread out the spectrum of binary data before being summed and transmitted [14]. At the receiver, similar codes are used to recover each data stream through correlation-based detection [15]. Within this framework, a better spectral efficiency would be achieved for chaos-based communications since multiple messages could occupy the same frequency band. However, replacing these codes by chaotic signals, while maintaining the orthogonal properties, is challenging in terms of design, because they change for every bit of information transmitted. This requires a particular attention in the choice of state variables and nonlinear functions processing them, otherwise high levels of cross-correlation may exist [16] and could be detrimental to the various messages recoveries. In this paper, we propose to adapt the existing structure of ICG based on an electro-optic oscillator (EOO) but using two feedback loops comprising of a Mach-Zehnder modulator for the generation of chaotic signals to be later used to carry the various messages.

Our study is organized as it follows; we first describe our architecture and give its mathematical modeling, then we give insight on the properties of codes and their orthogonality. Finally, we demonstrate numerically the transmission of two binary messages at multi-Gb/s.

2. Theoretical framework and Modeling

2.1. Decryption of the Architecture

Our EOO-based multiplexing architecture for the emitter (**E**) is depicted in Fig. 1. The modified EOO has two cosine-square nonlinearities with different internal gains. It is made of a monochromatic CW semiconductor laser diode whose optical power P is divided in two separate arms and modulated by a Mach-Zehnder modulator (MZi) with respective constant-valued RF and DC half-wave voltages $V_{\pi_{RFi}}$ and $V_{\pi_{DCi}}$, biased by voltage V_{DCi}. Optical signals are linearly polarized and travel through different optical fibers DL_i inducing fixed time delays T_i. They are recombined and detected by a single photodetector PD (of efficiency S), and their polarization directions are rotated by π/2 relative to each other with a half-wave plate (λ/2) to prevent optical interferences. An electrical signal is generated by the PD before being amplified with gain G and filtered by a band-pass filter with low and high cut-off frequencies denoted f_L and f_H, respectively. Each loop has attenuation gain denoted $g_i < 1$, achieved through the use of a voltage divider D_i.

They induce different internal gains ω_i in the cosine-square nonlinearities; they reduce the electrical voltage $V(t)$ before driving their respective modulator MZi.

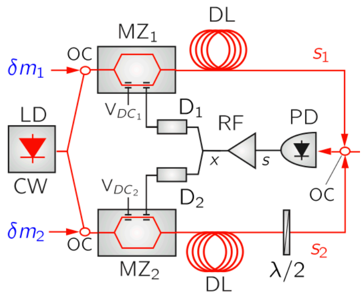

Fig. 1 – Description of the chaotic emitter E based on an EOO with two feedback loops with inclusion of messages. The acronyms are CW LD : continuous-wave laser diode, MZ$_i$: ith Mach-Zehnder modulator, DL$_i$: ith optical delay line, λ/2-P: half-wave plate, PD: photo-detector, RF: RF band-pass filter, D$_i$: ith voltage divider.

2.2. Modeling

Emitter **E** can be modeled using similar notations to [4]

$$\tau\frac{dx}{dt}+x+\frac{1}{\theta}\int_{t_0}^{t}x(u)du=s(t), \tag{1}$$

where $x(t)=\pi g_1 GV(t)/2V_{\pi_{RF1}}$ is the dimensionless variable of **E**, $\theta=(2\pi f_L)^{-1}$, $\tau=(2\pi f_H)^{-1}$, and s(t) is the multiplexed signal transmitted in the communication channel defined by $s(t)=\sum_{i=1}^{2}\beta_i\cos^2(\omega_i x_{T_i}+\phi_{0i})$, with $x_{T_i}=x(t-T_i)$, $\beta_i=\pi g_i GSP/2nV_{\pi_{RF1}}$ a nonlinear gain, $\phi_{0i}=\pi V_{DC_i}/2V_{\pi_{DC_i}}$ a phase, and $\omega_i=g_i V_{\pi_{RF1}}/g_1 V_{\pi_{RFi}}$ an internal gain modifying the pulsation of its cosine-square function.

3. Statistics of Chaos and Orthogonality

3.1. Statistical Properties and Generation of Codes

Investigating the statistics of the state variable $x(t)$ is important in helping the design of proper chaotic codes for multiplexed chaos-based transmissions. Scalar time-delay systems with a cosine nonlinear delayed feedback are known to exhibit hyper-chaotic regimes (existence of multiple positive Lyapunov exponents) with approximate Gaussian statistics [17]. With two feedback loops, our system exhibits strongly developed chaotic regimes [see Fig. 2(a)] with identical statistics to those of a single-feedback system [see Fig. 2(b)].

These Gaussian statistics are then processed by the two different nonlinearities associated with the Mach-Zehnder modulator in each feedback loop. This leads to the creation of two optical signals denoted by $s_i(t)=\beta_i\cos^2(\omega_i x_{T_i}+\phi_{0i})$, with i=1,2. These signals are natural candidates for an analog of CDMA codes in the context of optical chaos-based communications.

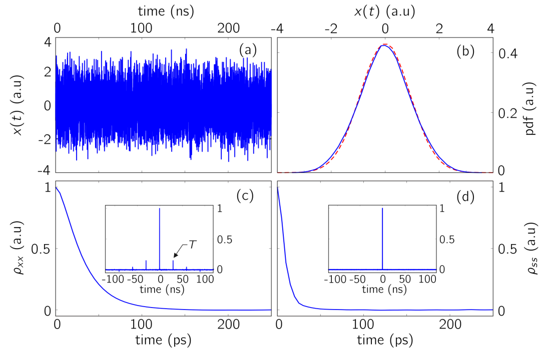

Fig. 2 – Time series of x(t) in (a), its probability density function (b) and its auto-covariance in (c). The auto-covariance of signal s(t) is given in (d). The parameters are τ = 25 ps, θ = 10 μs, $T_{i|i=1,2}$ = T = 30 ns, $\beta_{i|i=1,2}$ = 5, $\phi_{0i|i=1,2}$ = −π/4, $\omega_2 = 2\omega_1$ = 2, and time step Δt = 5 ps.

3.2 Orthogonality

In this subsection, we investigate the dependence of the orthogonality (or decorrelation) on parametric differences existing between the two codes, namely, the differences in nonlinear gain $\Delta\beta_{ij}=\beta_i-\beta_j$, internal gain $\Delta\omega_{ij}=\omega_i-\omega_j$, and phase $\Delta\phi_{0ij}=\phi_{0i}-\phi_{0j}$. Orthogonality is measured by the absolute value of the normalized cross-covariance coefficient defined as

$$\left|\rho_{s_i s_j}\right|=\left|\frac{\left\langle\left[s_i(t)-\langle s_i\rangle\right]\left[s_j(t)-\langle s_j\rangle\right]\right\rangle}{\left\langle s_i(t)-\langle s_i\rangle\right\rangle^{1/2}\left\langle s_j(t)-\langle s_j\rangle\right\rangle^{1/2}}\right|=\left|\frac{\Gamma_{s_i s_j}}{\Gamma_{s_i s_i}^{1/2}\Gamma_{s_j s_j}^{1/2}}\right|, \tag{2}$$

where $\langle\cdot\rangle$ is the time-average operator. This cross-covariance coefficient is calculated during the time interval T_b, corresponding to the duration of a bit transmitted. Orthogonality depends significantly on the choice of T_b, which controls the number of point used to perform the cross-covariance calculations. We have chosen $T_b=0.4\ ns$ in order to investigate orthogonality for multiplexed transmission at 2.5 Gb/s. We consider no difference in nonlinear gain ($\Delta\beta_{ij}=0$) to guarantee that each code has approximately the same variance; instead, the impact of the nonlinear gain $\beta_i=\beta$ will be studied. Figure 3 shows the evolution of $|\rho_{s_i s_j}|$ in the parametric plane $(\beta,\Delta\omega_{ij})$ with zero phase difference $\Delta\phi_{0ij}=0$ in (a) and $(\Delta\omega_{ij},\Delta\phi_{0ij})$ with $\beta=5$ in (b). The cross-covariance measurements are averaged over $5000T_b$. Figure 3(a) shows the existence of a large zone of quasi-perfect orthogonality for $\Delta\omega_{ij} > 1$ and $\beta > 2.5$. Figure 3(b) shows the existence of only four narrow zones of orthogonality that all merge as the difference in internal gain $\Delta\omega_{ij}$ increases. Further insight can be obtained by exploiting the approximate Gaussian statistics of $x(t)$, which remain valid for sufficiently large values of T_b. We demonstrate that the cross-covariance between two chaotic codes satisfy the following equivalence:

$$\Gamma_{s_i s_j} \approx \beta^2 \cos(2\Delta\phi_{0ij}) e^{-2\Delta\omega_{ij}^2 \sigma_x^2}, \tag{3}$$

with the variance of $x(t)$ depending on the nonlinear gain $\sigma_x^2 \propto \beta^2$. This highlights the key role of β and $\Delta\omega_{ij}$ in achieving high levels of orthogonality between the chaotic codes and their suitability for the transmission of data streams at high bit rates.

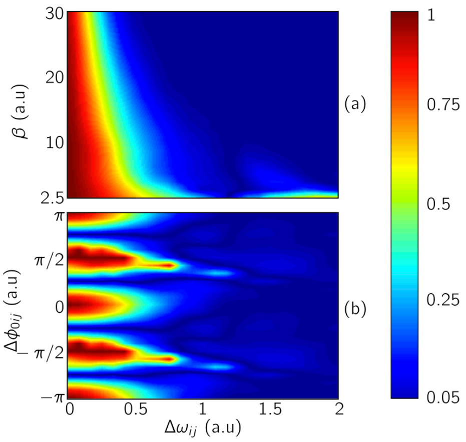

Fig. 3 – Evolution of the cross-covariance coefficient $|\rho_{si,sj}|$ in the parametric planes ($\Delta\omega_{ij}$,β) in (a) and ($\Delta\omega_{ij}$,$\Delta\phi_{0ij}$) in (b).

4. Multiplexed Chaos-Based Communications

In this section, we propose an architecture to encode and decode two messages, based on the emitter described in Sections 1-2.

We have included the messages $m_{i=1,2} = \{-1,1\}$ such that (i) they do not disturb s(t) too significantly (small modulation depth) and (ii) have their spectrum spread out. These two conditions are achieved by considering a new expression for the multiplexed signal

$$s(t) = \sum_{i=1}^{2} \left(1 + \delta m_i\right) \beta_i \cos^2(\omega_i x_{T_i} + \phi_{0i}), \tag{4}$$

with $|\delta| \ll 1$ the modulation depth. The multiplexing signal $s(t)$ drives both the emitter **E** and the receiver **R**, described by a dynamical equation similar to Eq. 1

$$\tau \frac{dy}{dt} + y + \frac{1}{\theta}\int_{t_0}^{t} y(u)\,du = s(t - T_c), \tag{5}$$

with $y(t)$ the dimensionless state variable of receiver **R** and T_c the propagation time in the communication channel (without loss of generality one may consider $T_c = 0\,\mathrm{s}$). With this particular coupling configuration, **E** and **R** exhibit complete chaotically synchronized evolutions.

The recovery of each message is based on a correlation-based technique similar to that used in CDMA. To perform the decryption, the codes $s_{i=1,2}(t)$ used at the emitter must also be available at the receiver. The solution is to exploit chaos synchronization to duplicate the codes and create *twin codes* $s_i'(t) = \beta_i \cos^2(\omega_i y_{T_i} + \varphi_{0i})$. Then, each user at the receiver calculates the cross-covariance $\Gamma_{ss_i'}$ between his twin code $s_i'(t)$ and $s(t)$. The expansion of $\Gamma_{ss_i'}$ leads to the following expression

$$\Gamma_{ss_i'} = (1 + \delta m_i)\Gamma_{s_i s_i'} + (1 + \delta m_j)\Gamma_{s_j s_i'}. \tag{6}$$

Assuming a high level of orthogonality between $s_i(t)$ and $s_j(t)$, it is possible to neglect the contribution of $\delta m_j \Gamma_{s_j s_i'}$ in Eq. 6. Furthermore, with the twin codes being identical to the original ones, we can finally derive the decoding equation

$$\delta m_{i=1,2|j\neq i} = \frac{1}{\Gamma_{s_i' s_i'}}\left(\Gamma_{ss_i'} - \Gamma_{s_i' s_i'} - \Gamma_{s_j s_i'}\right). \tag{7}$$

Equation (7) is similar to the decoding equation in [16], except covariance is used instead of correlation in our study. It guarantees the possibility to decrypt each message independently.

We have numerically simulated the full transmission chain and apply Eq. (7) to retrieve independently the two binary messages encoded at 2.5 Gb/s. Our findings are presented in Fig. 4. The parameters are identical to those used in Fig. 2 and a modulation depth $\delta = 1/64$ is considered. Dashed lines and solid lines represent the encrypted and retrieved messages, respectively. The simulations have been realized in optimal transmission conditions, assuming no noise, parameter mismatch, or distortions induced by the communication channel. Although no decryption errors are reported, small deviations are observed. They result from the neglecting of $\delta m_j \Gamma_{s_j s_i'}$ with imperfect orthogonality existing between the two codes. To quantify the robustness of our method we have applied realistic level of noise and parameter mismatch to the system (a few percent) and we have still observed a satisfactory quality of the decryption.

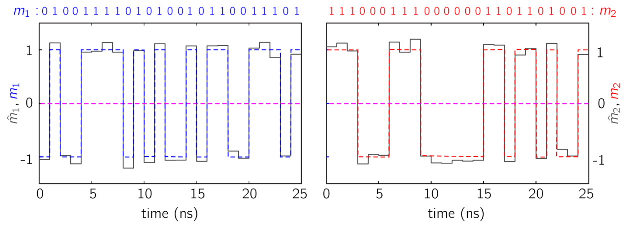

Fig. 4 – Decryption of two data stream at 2.5 Gb/s (bit rate of the OC-48 standard). The purple dashed line is used as a threshold detection to discriminate messages' values ("-1" and "1").

5. Conclusion

In this paper, we have proposed to modify an existing electro-optic chaos generator to multiplex two binary data streams. The modification consists of having two delayed feedback loops comprising a Mach-Zehnder modulator. The optical chaotic signals generated at their output are used as modulating carriers to transmit two binary messages, while exhibiting a high level of orthogonality, if the system's parameters are properly chosen. This has allowed multiplexing and demultiplexing of two binary messages at multi-Gb/s using a correlation-based detection, similar to that used by CDMA architectures. These results offer promising perspectives towards the realization of multi-user optical chaos-based communications.

Acknowledgments

The authors acknowledge the support of Fondation Supélec, of Conseil Régional de Lorraine, and of National Science Foundation (NSF) by grants ECCS 0925713 and ECCS 0523923.

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