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Laser-based Chaos Communication

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Abstract– The paper first reports statistical and nonlinear dynamical analysis of a sequence of 10 million paired outputs from two synchronized free-space lasers. The objective is to make an empirical study of such outputs prior to their use as message carrying waves in chaos communication applications. The results obtained from this investigation support the use of synchronized chaotic laser waves in a somewhat modified version of binary antipodal chaos shift-keying (CSK) communication. Instead of studying BER performance by mathematical simulation of chaotic sequences, the more realistic laser waves are used to obtain spreading sequences. CSK theory of BER involving Gaussian assumptions is adapted and comparisons made with empirical results which are not supported by Gaussian assumptions. Differences are not large, except at high spreading, but suggest future enhanced modelling.

1. Introduction

Chaos communication has been investigated over more than the past decade, see [1], [2], beginning with the use of mathematically generated chaos and continued by its electronic circuit generation. A key minimal requirement in most systems is that two waves of chaotic signals are synchronized, either exactly or approximately, and often this has been found difficult to achieve. The possibility of using pairs of lasers which can be synchronized, at least to a good approximation, to generate identical pairs of waves, has been demonstrated, [3], across an optical fiber network in Athens, using laser diodes. In this early experiment the encoding/decoding messages did not follow any of the later well known forms of chaos communication, but employed a simple form of additive masking. Thus when synchronized and digitized laser waves became available yielding 10^7 intensity pairs, of which 200 are illustrated in Figure 1, there was an opportunity to use them in an empirical investigation of a more sophisticated form of chaos communication – a modified form of antipodal chaos shift-keying (CSK) was chosen since it is possible to derive the associated theoretical bit error rate (BER) under ideal Gaussian assumptions and compare with empirical results.

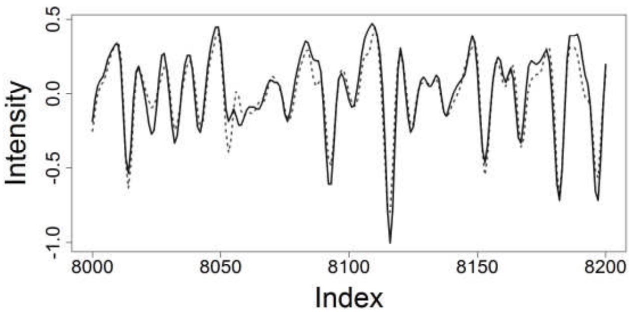

Figure 1. The solid line gives 200 of 10 million values from the laser at the transmitter and the dotted line is the corresponding synchronized wave generated by the laser at the receiver.

Initially, little was known about the statistical and dynamical properties of such synchronized laser waves. Thus, an initial task was an empirical study of the laser waves and this showed them to be well-behaved statistically and to be chaotic.

The data was then used empirically as input waves to the antipodal CSK system. Bit error rates have been calculated as a function of the amount of signal spreading employed and have given the useful results reported here. In contrast to conventional CSK systems, synchronization error is important, and its statistical properties are found to depart to some extent from a Gaussian assumption. Further, in the experimental circumstances considered, channel noise is absent, but theory is developed for its inclusion. For laser-based communication, lasers are imagined as being located at transmitter and receiver stations and connected over an optical fiber network. At the transmitter station the laser wave is broken into segments so as to spread each bit transmission and each is modulated in multiplicative binary way according its encoded bit message; this becomes the received segment after transmission. It is decoded at the receiver station by using the corresponding synchronized but unmodulated segment, in the manner specified by the communication system in use, a correlation decoder here. A simpler CSK system, known as on/off chaos shift-keying was experimentally investigated in [4] with laser generated chaos, and motivates the present investigation.

2. Statistical Analysis of Laser Waves

The stored 10 million pairs of data used in this investigation come from two synchronized distributed feedback (DFB) lasers under a free-space optical configuration. The two waves have been adjusted to have zero mean and equal overall amplitudes. From the illustration in Figure 1 it is seen that there is a high degree of synchronization, and that there are more outlying negative than positive values, indicating skewness in their distributions. This behaviour and constant level is evident across each entire series. The synchronization error wave is of interest, but not shown, and is fairly balanced in its positive and negative values. More information is revealed by statistical plots. Figure 2 gives the scatter plot of the two series showing the degree of synchronization, with correlation of 0.947, while the histogram and its smoothed version in Figure 3 show the skewed distribution of the laser waves at the transmitter. Several other statistical aspects were studied, such as auto- and cross- correlations indicating directionality, mutual information, power spectra, sequential variances giving further evidence for stationarity, and phase coherence with strong value of 0.892. Thus, in summary of the statistical properties, they were found to be stable and reasonably behaved, but certainly not Gaussian, and neither were the individual series linear in their dependency.

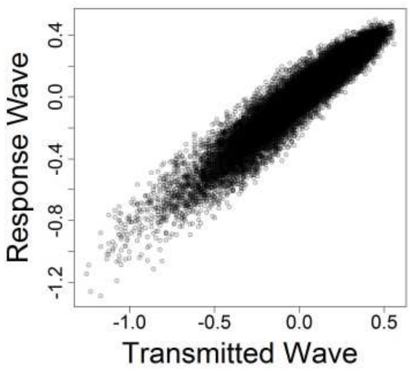

Figure 2. Scatter plot showing the degree of synchronization between the stored laser waves generated at the transmiiter and receiver.

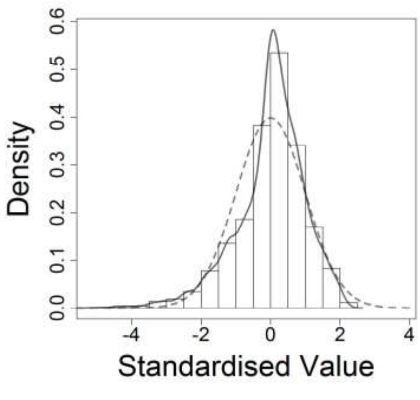

Figure 3. Histogram and smootherd histogram of the stored laser wave generated at the transmitter relative to a Gaussian distribution.

3. Chaotic Dynamics Analysis of laser Waves

The waves emitted by lasers are believed to be chaotic and thus useful in communications because of their unpredictability and noise like properties. This is the fundamental tenet of chaos-based communication systems, such as CSK. Actually, the bit error performance theory of such systems does not usually depend on such assumptions, but rather on the statistical characteristics of the chaotic waves. So both statistical and dynamical aspects are important. However, the theoretical assumptions that laser-generated waves realize chaotic characteristics should be empirically established, and there is scarce evidence in the literature. This is not surprising since the numerical investigation of nonlinear dynamics is delicate task, although with the advent of more software, it is becoming more practical, as will be demonstrated.

The chaotic and dynamical properties of interest are the time lag τ , the local dimension D_L, the Lyapunov exponents $\lambda_i, i = 1, 2, ..., D_L$, and the embedding or global dimension D_E. An observed laser time series is considered as a projection from a higher dimensional space. In order to ascertain the nature of its governing dynamical system, the space needs to be reconstructed. This can be done as a D_E -dimensional space using a time lag delay technique, see [6] for instance. There are several ways to ascertain the time lag, the two most popular are the first time that the autocorrelation function (ACF) of the signal crosses zero (Figure 4, perhaps also indicating some periodicity), and the first minimum of the average mutual information (AMI) of the signal (Figure 5). When the choice is appropriate, there is strong preference amongst practitioners for the mutual information technique because it takes into account linear and nonlinear dependency.

The local dimension D_L is equal to the number of non-trivial Lyapunov exponents calculable for the system, and is obtained by calculating the attractor dimension, and considering its ceiling. The maximum of the Hausdorff, information, and correlation dimensions were used to obtain the attractor dimension and the ceiling was found to be $D_L = 4$, [6].

Chaotic signals are characterized as emerging from systems which are deterministic in nature but are sensitive to their initial conditions, [7]. Lyapunov exponents are a way of quantifying this sensitivity to initial conditions because they signify the exponential rate of divergence between two nearby points in a dynamic system; a positive exponent implies that points are diverging away from each other at an exponential rate and this is used to signify a process that is unstable or chaotic.

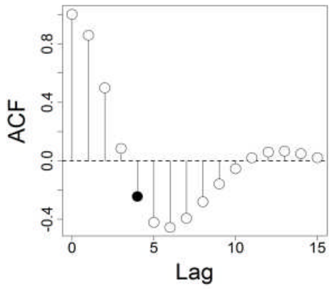

Figure 4. Autoorrelations of the stored laser wave generated at the transmitter, with first negative crossing appearing lag 4.

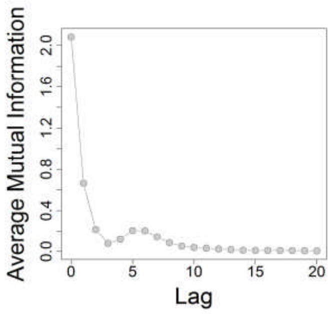

Figure 5. The average mutual infomation of the stored laser wave generated at the transmitter, with first minimum appearing at lag 3.

Figure 6 shows the output of the calculation for the Lyapunov exponents of the laser wave generated at the transmitter. Two positive and two negative Lyapunov exponents were calculated, with error bars, showing that they were not just chaotic but hyperchaotic, [6], since they possessed more than one positive exponent. Their sums were negative, confirming that the dynamical system is indeed stable.

Obtaining a reliable value for the minimal embedding dimension is difficult and many techniques have been advocated. The two most popular, false nearest strand and false nearest neighbour [12], [13], where found to be inconsistent and unreliable for this purpose. Therefore it was decided to use the fractal delay embedding prevalence theorem [12] and hence obtain $D_E \geq 2D_L + 1 = 9$.

The open source statistical package R [8] was used to carry out all the calculations and the various sub-packages used within R were: the fractal package [9] for calculating the Lyapunov exponents, the fdim package [10] for false nearest neighbour and false nearest strands for carrying out the attractor dimension calculations, and the tseries chaos package [11] for calculating the mutual information.

4. Chaos Shift-keying Using Laser Transmissions

There are several variants of chaos shift-keying systems for binary bit transmission, with the main distinction being between those which modulate and transmit a single chaotic wave (the antipodal version) and those which

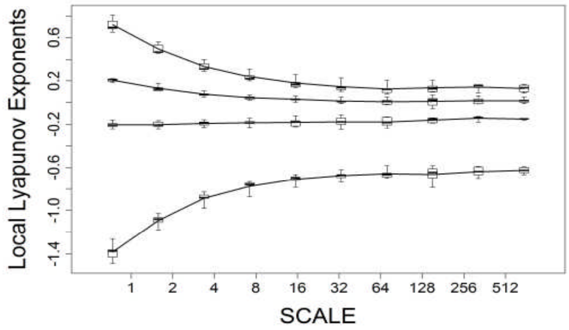

Figure 6. Lyapunov exponents of the stored laser wave generated at the transmitter assuming an embedding dimension of 9.

transmit one of two chaotic waves (the binary version) according to the bit value being transmitted. In the antipodal version, the one used in modified form here, both the transmitted modulated wave and the synchronized unmodulated wave are both available at the receiver. For ease of reference to the standard CSK versions, see [5].

For the present laser-based antipodal version of CSK, let $\{X_1, X_2, ..., X_N\}$ be a typical segment of N successive values of the chaotic laser wave generated at the transmitter, adjusted if necessary to have zero mean, and with variance σ_X^2; Let the binary bit $b = \pm 1$ be multiplicatively spread over the N successive values at the transmitter, the so-called spreading segment. In the experimental generation of the synchronized laser waves, as illustrated in Figures 1 and 2 there is no transmission noise. However, in communication use, additive transmission noise can be anticipated. Thus, the received message wave segment will be represented as

$$R_i = bX_i + \varepsilon_i,\ i = 1, 2, ..., N \qquad (1)$$

where $(\varepsilon_1, \varepsilon_2, ..., \varepsilon_N)$ is transmission noise, assumed to be Gaussian with variance σ_ε^2 . The synchronized wave $(S_1, S_2, ..., S_N)$ is generated at the receiver but this is subject to laser-induced synchronization error, as illustrated by the stored wave data in Figure 2; it is modelled as

$$S_i = \beta X_i + \eta_i,\ i = 1, 2, ..., N \qquad (2)$$

where β is a scaling constant which equalizes the amplitudes. It can be found by a preliminary regression analysis, using the stored waves. The $(\eta_1, \eta_2, ..., \eta_N)$ are the synchronization errors; thus $\sigma_\eta = 0$ corresponds to perfect synchronization. Decoding of the binary message b is an operation of statistical estimation and in CSK systems is routinely carried out by correlation decoding according to

$$\sum_{i=1}^{N} R_i S_i <,> 0 \Rightarrow \hat{b} = -1, +1 . \qquad (3)$$

It can be justified as maximum likelihood estimation of b when the synchronization errors are Gaussian and the transmission noise is zero.

The bit error of the correlation decoder with zero transmission noise, as with the present experimental data, is a small mathematical generalization of earlier CSK theory, such as in [5], and is

$$BER(N) = E\left\{\Phi\left(-\beta\left[\sqrt{N}\sigma_X \big/ \sigma_\eta\right]\sqrt{N^{-1}\sum_{i=1}^{N} X_i^2 \big/ \sigma_X^2}\right)\right\}. \quad (4)$$

All quantities in this expression except for N are set according to the laser hardware and thus *BER* is a function only of spreading length *N.* Unlike other CSK systems it does not involve a signal-to-noise ratio. Near exact calculation of (4) is undertaken by simulation of its expectation using the stored transmission laser wave divided into spreading segments of lengths $N = 2, 3, \ldots$. The actual *BER* is obtained as the frequency of errors when $b = 1$ is transmitted, also using spreading segments from the stored transmission wave, and is illustrated in Figure 7. There is encouraging agreement between the two curves, although less so at extensive spreading, the effect of the imperfect assumption of additive Gaussian synchronization error. There is an opportunity from (4) to investigate the effect of synchronization error by varying σ_η while keeping N fixed at selected values.

When there is transmission noise η as well as synchronization error, BER obviously increases, and its mathematical derivation becomes more complicated. Performance is now affected by the signal-to-noise ratio $SNR = 10\log\left(N\sigma_X^2 / \sigma_\varepsilon^2\right)$. The communication system is actually close to non-coherent CSK allowing the theory in [5] to be extended. An expression for the *BER* can be reported as

$$BER = E_X P\left\{\sum_{i=1}^{N}\left[U_i + \frac{1}{\sqrt{2}}\left(\sigma_\varepsilon^{-1} + \beta\sigma_\eta^{-1}\right)X_i\right]^2 \Bigg/ \sum_{i=1}^{N}\left[V_i + \frac{1}{\sqrt{2}}\left(\sigma_\varepsilon^{-1} - \beta\sigma_\eta^{-1}\right)X_i\right]^2 < 1\right\}, \quad (5)$$

where the U_i, V_i variables have independent and standardized Gaussian distributions. Thus, the stored transmission wave allows *BER* to be calculated empirically using (5) for any chosen *N*. The choice of σ_ε is based on any desired signal-to-noise ratio $SNR = 10\log\left(N\sigma_X^2 / \sigma_\varepsilon^2\right)$. The result (5) can be cast in terms of a doubly non-central F-distribution. Lower bounds in terms of *N* can be also be deduced. These associated results and enhanced synchronization modelling will be more fully reported elsewhere.

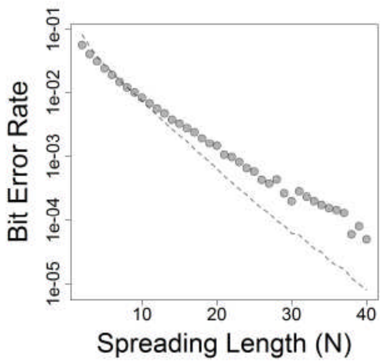

Figure 7. The BER of antipodal coherent CSK as a function of spreading using synchronized laser waves; dotted curves are empirical values, dashed curve correspond to theoretical values based on equation (4).

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