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Interactions Between Propagating Wave Phenomena in a Large Number of Coupled Bistable Oscillators

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Abstract—Different types of propagating waves can emerge in a simple model of inductor-coupled bistable oscillators. In this study, we numerically investigate interactions between the propagating wave phenomena in the coupled bistable oscillators. The propagating waves are reflected when one of the coupling parameter values in the oscillator array is changed. Various interactions between the two propagating waves in the 16 coupled oscillators are shown.

1. Introduction

Propagating wave phenomena emerge in wide variety of dynamical systems, and attract considerable attentions. Examples include the propagating waves in FitzHugh-Nagumo dynamics [1], in reaction-diffusion system [2], and in micro-electro mechanical systems [3, 4]. Because the dynamical systems correspond to a class of nonlinear coupled oscillators, a study of propagating wave phenomena in a nonlinear coupled system is significant.

In our previous study [5], we numerically studied a ring of six-coupled bistable oscillator system, and confirmed that various propagating waves occur though a global bifurcation of map based on heteroclinic tangle. Kamiyama *et al.* succeeded in distinguishing these propagating waves as well as attractive quasi-periodic oscillations [6]. In addition, these propagating waves in six-coupled bistable oscillators can be observed in an actual circuit experiment [7].

The interaction of traveling excitations is a subject of great interest [8]. In this study, we show that the traveling direction of the propagating waves in larger number of oscillators can be changed by varying the coupling strength at a particular site of the oscillator array. The results implies that it is natural to assume that multiple propagating waves emerges in the coupled system. We numerically investigate the interaction between the two propagating waves moving the opposite direction with each other. In particular, we use a coupling factor as a control parameter and pay attention to interaction for two distinctive propagating waves previously reported in the literatures [7].

2. Inductor-coupled bistable oscillators

Figure 1 shows one-dimensional array of inductor-coupled bistable oscillators. An individual bistable oscillator ($O_k, k = 1, 2, \ldots, N$) comprises an inductor (L), a capacitor (C), and a nonlinear conductance (NC). These oscillators are connected by inductors (L_0). When the voltage–current characteristics of NC are assumed to be written by the fifth-order polynomial: $i^k_{NC} = g_1 v_k - g_3 v_k^3 + g_5 v_k^5, g_1, g_3, g_5 > 0$, the individual oscillator has two steady-states: a stable focus and a limit cycle oscillation [7].

From Kirchhoff's law, the circuit equation of Fig. 1 is written as

$$\frac{d^2 v_k}{dt^2} + \frac{g_1}{C}\left(1 - \frac{3g_3}{g_1}v_k^2 + \frac{5g_5}{g_1}v_k^4\right)\frac{dv_k}{dt} + \left(\frac{1}{LC} + \frac{1}{L_0C}\right)v_k - \frac{1}{L_0C}(v_{k+1} - v_k + v_{k-1}) = 0,$$
$$k = 1, 2, \ldots, N \quad (v_0 = v_N, v_{N+1} = v_1). \tag{1}$$

Substituting

$$t = \sqrt{LC}\tau, \quad v_k = \sqrt[4]{g_1/5g_5}\; x_k,$$
$$\varepsilon \equiv g_1\sqrt{L/C}, \quad \alpha \equiv L/L_0, \quad \beta \equiv 3g_3/\sqrt{5g_1g_5}, \tag{2}$$

into Eq. (1) yields the following normalized equation:

$$\dot{x}_k = y_k,$$
$$\dot{y}_k = -\varepsilon(1 - \beta x_k^2 + x_k^4)y_k - x_k + \alpha(x_{k-1} - 2x_k + x_{k+1}),$$
$$(\cdot = d/d\tau). \tag{3}$$

The parameter ε indicates the degree of nonlinearity, whereas α is the coupling factor. The parameter β determines the amplitude of oscillation.

In our previous work [5, 7], we use the different normalized parameters: $\varepsilon' = g_1/\sqrt{(C/L) + (C/L_0)}, \alpha' = L/(L + L_0)$. In the circuit experiments, we recognize a difficulty that a key parameter for the observed phenomena becomes confusing by using the parameters. In particular, with the parameters, the value of coupling inductor L_0 changes both values of parameters. Therefore, in this study, we use the normalized parameters in Eq. (2).

In the following results, we use the parameter values $\varepsilon = 0.447$ and $\beta = 3.2$ to compare the results in Ref. [7], and integrating Eq. (3) by using a fourth-order Runge-Kutta method with step size 0.01.

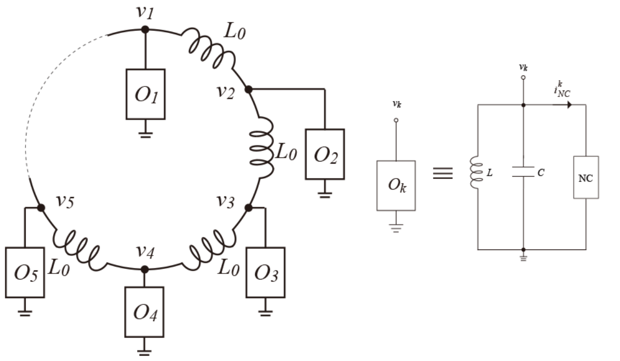

Figure 1: Circuit diagrams of a large number of coupled bistable oscillators.

3. Interactions between propagating waves

Several types of propagating waves, where a spatiotemporally localized excitation propagates in one direction with constant speed, exist in the inductor-coupled bistable oscillators [7]. When we set the coupling strength at a particular site of the oscillator array much larger or much smaller than the all other α, the propagating waves are correctly reflected at the site. Figure 2 shows the reflection of two different types of propagating waves when we set the coupling strength between x_9 and x_{10} and between x_9 and x_8 is a hundred times larger than that for all other oscillators ($\alpha = 0.2$). Comparing the trajectories on the phase plane with those in Ref. [7], the two propagating waves in Figs. 2 (a) and (b) are identical to those reported in the literature denoted by PW_2 and PW_3, respectively. In the following, for simplicity, we use the same notation to identify the propagating waves. PW_2 is generated through a global bifurcation of maps based on heteroclinic tangle, and the bifurcation point is near a pitch-fork bifurcation point of a standing wave [5]. As far as our numerical calculations are concerned, PW_2 and PW_3 appear around the different pitch-fork bifurcation points of the distinctive standing waves, namely $\alpha_c = 0.1172$ and $\alpha_c = 0.1272$, respectively. Moreover, the reflection of PW_2 is shown in Fig. 3, when we use the coupling strength between x_9–$x_{10,8}$ is a hundred times smaller than $\alpha = 0.2$ for all other oscillators. It is seen from the figure that the amplitude of x_9 is almost zero, whereas those in Fig. 2 (a) undergoes a large amplitude of oscillation.

The above results imply that the traveling direction of the propagating waves can be easily changed by varying the coupling strength at a particular site. Therefore, for a large number of coupled oscillators, interaction between the localized propagating excitation becomes a subject of interest. We now pay attention to the interaction between the propagating waves in the 16 coupled bistable oscillators.

Figures 4(a)–(d) show the 3D plots of the interactions between the two PW_2 moving the opposite direction for $\alpha = 0.2$, 0.18, 0.17, and 0.15, respectively. For $\alpha = 0.2$, PW_2 disappears right after the first collision, whereas for $\alpha = 0.15$, the propagating waves survives and are reflected successively at particular sites (around x_4 and x_{12}). Moreover, for the intermediate α, the spatiotemporally localized excitations are observed as shown in Fig. 4 (b) and (c). For larger α than 0.2, PW_2 disappears without collision.

Next, we investigate the interaction between the two PW_3. In contrast with PW_2, PW_3 appears for wide range of α greater than α_c. In addition, almost same interaction phenomenon for α is observed for PW_3. Figures 5 show the 3D plot and the corresponding time series of x_k for $\alpha = 0.2$, respectively. It is seen that the successive reflection of the two propagating waves occur. Comparing with the results in Fig 4 (d), x_k at the points of collision oscillate with large amplitude of oscillation.

4. Concluding remarks

We numerically investigated propagating waves and their interaction in sixteen inductor-coupled bistable oscillators. It was shown that the traveling direction of the propagating waves is changed by varying the coupling strength at a particular site of the oscillator array. Furthermore, we observed the interaction for the two distinctive propagating waves by changing the coupling strength of the oscillator array.

Acknowledgments

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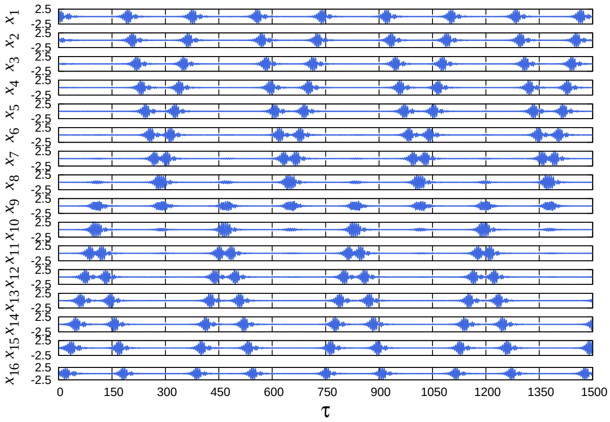
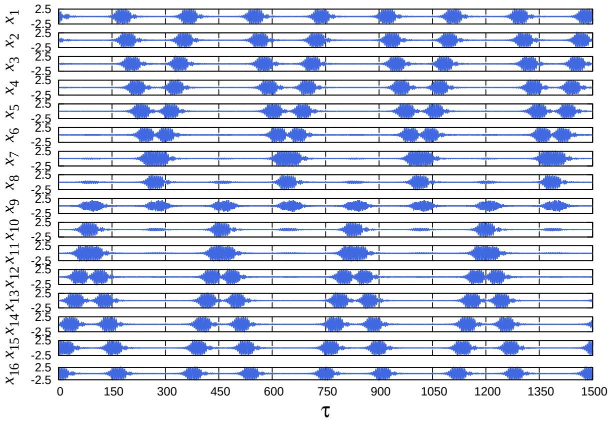
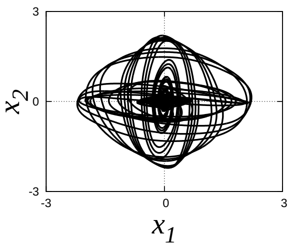


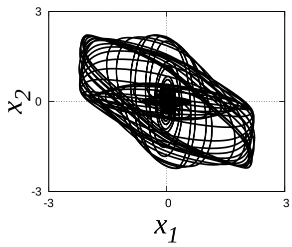
(a) PW_2.



(b) PW_3.



(c) Trajectories on the phase plane of (a).



(d) Trajectories on the phase plane of (b).

Figure 2: Reflection of propagating waves. the coupling strength between x_9 and x_{10} and between x_9 and x_8 is a hundred times larger than that for all other oscillators ($\alpha = 0.2$).

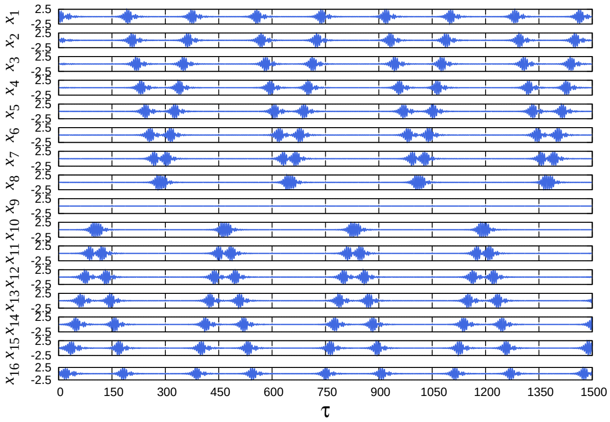
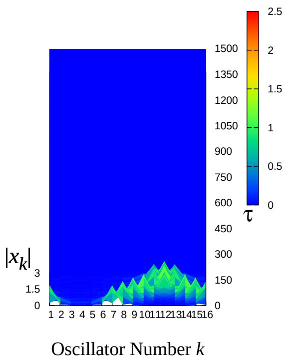
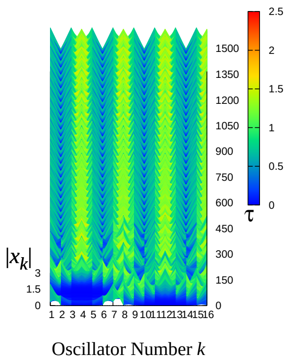


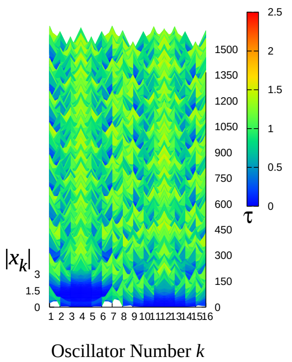
Figure 3: Reflection of propagating wave PW_2, when the coupling strength between x_9 and x_{10} and between x_9 and x_8 is a hundred times smaller than that for all other oscillators ($\alpha = 0.2$).



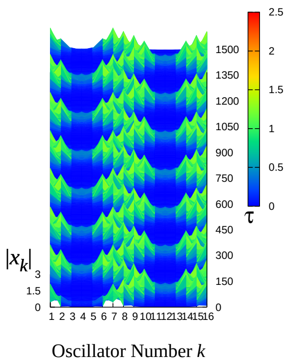
(a) $\alpha = 0.20$.



(b) $\alpha = 0.18$.

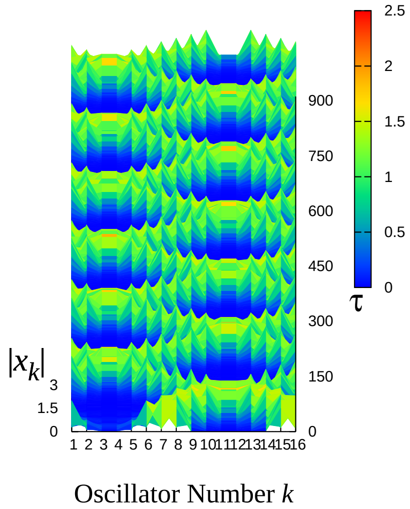


(c) $\alpha = 0.17$.

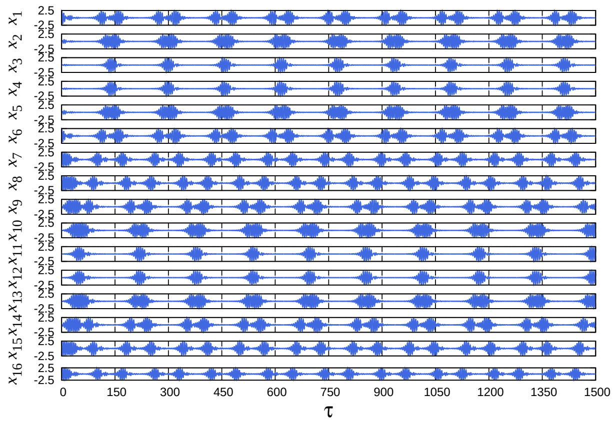


(d) $\alpha = 0.15$.

Figure 4: 3D plot of interactions between two PW_2 moving the opposite direction with each other for four values of α.



(a) 3D plot.



(b) Timeseries.

Figure 5: Interaction between two PW_3 moving the opposite direction with each other for $\alpha = 0.2$.