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Spike propagation of a Pseudofractal scale-free neural model

Yoshin Ito and Osamu Araki

Graduate School of Science, Tokyo University of Science
Kagurazaka 1-3, Shinjyuku-ku, Tokyo, 162-0601 Japan
Email: j1509603@ed.kagu.tus.ac.jp, brainics@rs.kagu.tus.ac.jp

Abstract– All-to-all or randomly selected synaptic connections have been assumed for the network structure of most neural network models. However, it is thought that biological neural networks in the real brain are more heterogeneous such as "small-world" rather than homogeneous. This research aims to clarify the characteristics of spike propagation in a Pseudofractal scale-free neural network model with the scale-free and the small-world properties. The main feature of this model is that the number of synaptic connections from a neuron differs depending on the generation of the neuron. The results of computer simulations show that the difference of latency of spike propagation between the generations depends on which generation of neurons is driven by external stimuli. This reflects heterogeneous routes of spike propagation in the network. The result suggests the possibility that different information processing is activated by different driven neurons.

1. Introduction

Recently, characteristics of complicated networks have been studied. Especially, the properties such as “Small Path Length”, “High Clustering”, and scale-free are paid attention to [1, 4]. Watts & Storogatz discovered that real world networks have common characteristics of “small world” such as the power grid network, nervous networks, and a collaboration network of film actors. The small world is featured as small path length and high clustering coefficient. This means that neighboring nodes are almost connected and some long paths are shortcut. The scale-free is a network whose degree distribution follows a power law. Thus, the fraction p of nodes having k connections to other nodes is as follows: $p(k) \propto k^{-\gamma}$ where γ is a constant value. This degree is the number of edges connected from a node. The examples of scale-free networks in the real world are shown in Table.1.

Table 1: Scale-free networks [1]

Network	γ
Inter net	2.1-2.5
Film actors	2.3-3.1
Protein interactions	2.4-2.5

On the other hand, the network of the brain is not clearly understood while the neural network in the brain has more complicated structure, although all-to-all or randomly selected synaptic connections have been assumed as neural network models. Recently, however physiological studies suggest that the neural networks have small-world and scale-free properties [2, 8].

Assuming that the neural network has the small world and free scale properties, how are the spikes propagated? This research aims to clarify the characteristics of spike propagation in a Pseudofractal scale-free neural network model, which has scale-free and the small-world properties (Fig.1) [4, 5].

2. Methods

2.1. Pseudofractal scale-free model

The growth starts from a single edge connecting two nodes at t=-1. At each time step, every edge generates an additional node, which is attached to both end nodes of the edge. Notice that the Pseudofractal scale-free model (PFS) at time step t=-1 can be made by connecting together the three t graphs (Nodes newly added are shown with open circle, and the other nodes are shown with filled circle.)(Fig.1). In PFS, three nodes at time t=0 are called the neuron of g (generation) =0. In the same way, nodes generated at the time of t= 1 are called g= 1 neuron, and nodes at t= 2 are called g= 2 neurons, and so on. The number of nodes at time t is as follows:

$$n = \frac{3}{2}\left(3^{t} + 1\right)$$

When nodes are generated, the index number is assigned counter-clockwise. For example, #0, #1, and #2 are counterclockwise assigned to the nodes at t=0. Similarly #3-#5 are assigned to the nodes generated at t=1, and #6-#14 are assigned to the nodes at t=2 (Fig.1 Table2).

The computer simulations in this research use PFS at t= 8 (9842 neurons, 19683 edges). We assume that the nodes of PFS are neurons and the edges are axons. Signals are supposed to be propagated from a neuron to other neurons. One way directivity (counterclockwise) is assumed in the edges (Fig.2). The g=0 neurons are specially called hub neurons because of the highest degree (Fig.2: A, B and C).

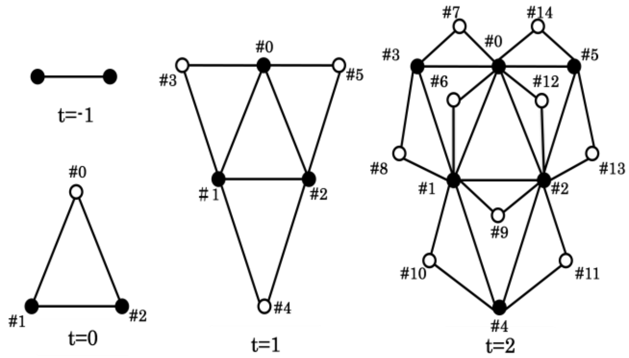

Figure 1: Pseudofractal scale-free model

Table 2: The numbers of nodes and index#, indices

t	0	1	2	3	4	5	6	7	8
All nodes	3	3	15	42	123	366	1095	3282	9843
Generated nodes	3	3	9	27	81	243	729	2187	6561
Index	0-2	3-5	6-14	15-41	42-122	123-365	366-1094	1095-3281	3282-9842
Input area index	2	5	12-14	33-41	96-122	285-365	825-1094	2553-3281	7656-9842
Output area index	0-1	3-4	6-11	15-32	42-95	123-284	366-824	1095-2552	3282-7655

2.2. Neuron model

For simplicity a network is assumed to be composed Integrate-and-fire neurons. The membrane potential V obeys the following differential equation:

$$\tau \frac{dV(t)}{dt} = -V(t) + I(t)$$

Where $I(t)$ shows the size of an external stimulus and τ shows the time constant that express attenuation. The spike occurs when membrane potential $V(t)$ exceeds a threshold V_1, and the membrane potential is reset to the resting potential V_0 immediately after the spike.

2.3. External stimulus

Since the spike interval observed within a brain has very strong irregularity, an external stimulus pattern is assumed to be a Poisson process [6, 7]. This means that the interval time of the spikes follows an exponential distribution.

The neurons which receive external stimuli are specific, and we analyze the signal propagation property. The Poisson trains for 30ms are repeatedly input as the external stimuli (Fig.3).

The neurons (input area) to which external stimuli are input are those between the hub neuron C and the hub neuron A. Precisely, neuron C is included in the input area but A is not. 1/3 of the generated neurons correspond to the input area (Table2).

The input area consists of neurons with various generations from g=0 to g=8.We assume that external stimuli in one trial are provided for the neurons with the same generation in the input area. In other words, we stimulate one generation in the input area and we compare the spike patterns in other areas.

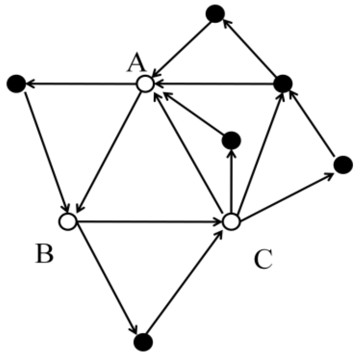

Figure 2: Hub neurons and directions of edges

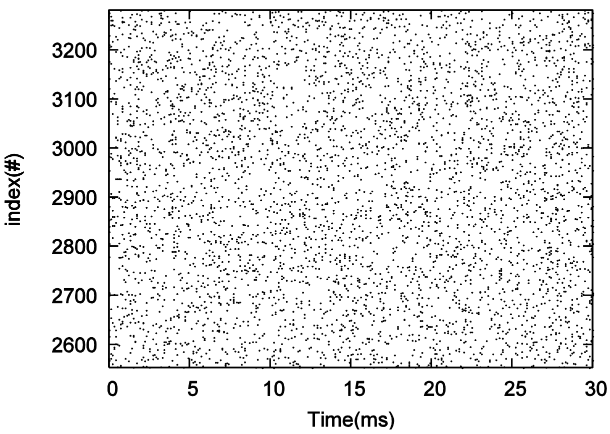

Figure 3: External stimuli to input area of g=7(index # 2553-3281)

3.Simulation Result

3.1. Raster Plot

The spikes of the hub neuron A are propagates to hub neuron C via many edges. We analyzed the spike pattern of the neurons through A to C (it is called output area) (Table- 2).

Figure 4 and 5 show raster plots of the g=7 neurons (index#1095-2553) in the output area when external stimulus is input into g=7 or g=8, respectively. These suggest that the spikes tend to periodically synchronize in the first hundreds milliseconds. In the long term, they become asynchronous. The same tendency is observed when the external stimulus is input to another generation. In order to quantify the periodic synchronization of raster plots, we calculated the autocorrelation of the spikes in the output area. The formula which calculates the cross-correlation of two spike patterns a (t) and b (t) is as follows:

$$R_{ab}(\tau) = \frac{1}{N-\tau} \sum_{t=1}^{N-\tau} a(t) b(t+\tau)$$

In the case of $a(t)=b(t)$, $R_{aa}(\tau)$ indicates the auto-correlation. Fig.6 slows a typical example of auto-correlation of output area neurons when g=7 neurons are input. The regular intervals of small peaks show that the average period is 10.7ms and the standard deviation is 0.01.

3.2. Cross-Correlation

Next in order to examine the difference which depends on the input generation, we calculated the Fig.7 is the cross-correlation of the raster plots for 100ms, which are partially shown by Fig.4 and 5. In all cases we simulated, we observed that $R_{ab}(\tau)$ has a high peak. The peak time of the cross-correlation seems to depend on which generation is stimulated despite the input patterns.

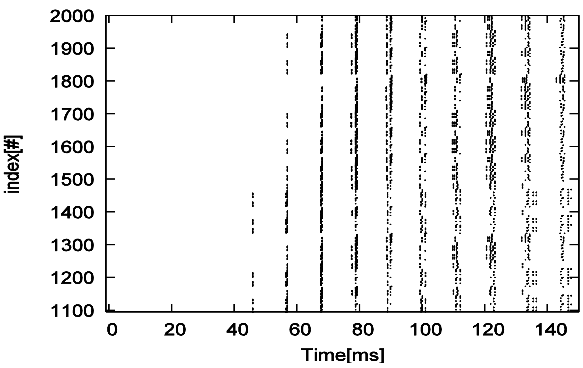

Figure 4: The raster plot of the output area g= 7 at the time of inputting into g= 7

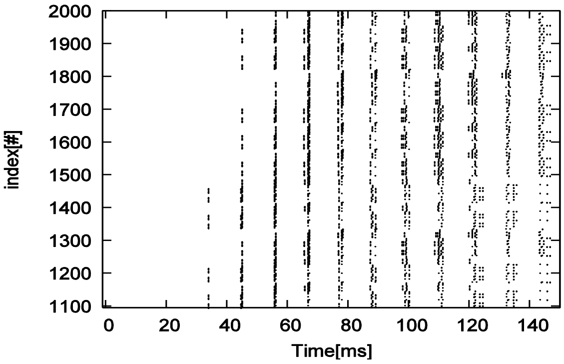

Figure 5: The raster plot of the output area g= 7 at the time of inputting into g= 8

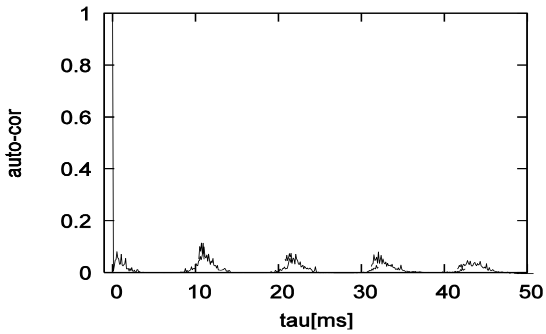

Figure 6: An example of auto-correlation

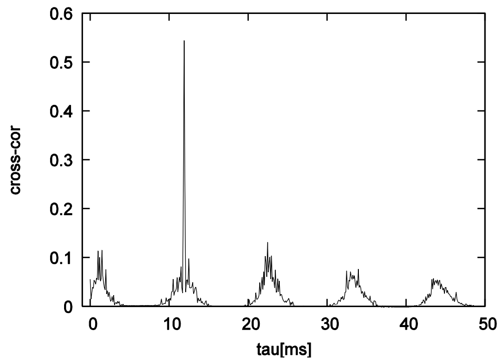

Figure 7: An example of cross-correlation

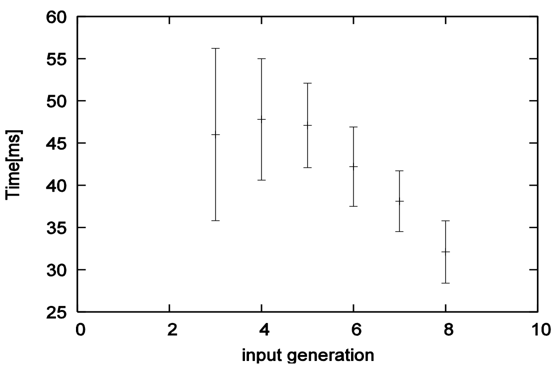

Figure 8: Input generation and the start time of spikes

Once external stimuli are input, the hub neuron A starts firing. Then, the spikes are propagated through the output area. After several analyses, we noticed that the delayed peak of cross-correlation is due to the latent periods of neurons which receive spikes from the hub neuron A. Figure 8 shows the time when the output area starts to fire in response to each generation input. Each plot indicates the average and the standard deviation of the time over 1000 trials. This means that the latency tends to be shorter as the input generation becomes younger (i.e. g becomes larger).

This is because higher generation neurons have more direct input connections to the hub neuron A (Fig.10). When g=8 neurons are stimulated, the distribution of the time of first spikes in the output area is shown in Fig.9. The average time is 32.1ms and the standard deviation is 3.7 over 1000 trials. This distribution seems to include Poisson and earlier distribution. Further analyses about this distribution are one of our future works.

4. Conclusion

In this research, we analyzed the spike propagation characteristics of PFS model. The raster plots in the output area showed the synchronization of spikes at the beginning and became asynchronous as the time proceeds. The spike latency becomes shorter as the input generation increases from g=5 because of large number of input connections. On the other hand, when g=2, 3, and 4, the latency is not clearly different. When input generation number is low, direct inputs to the hub from the input generation neuron are not enough to fire the hub neuron (Fig.10). Thus, the hub neuron in this case is driven by the neurons activated through more detours. The result suggests the possibility that different information processing is activated by the heterogeneous routes in the network.

5. References

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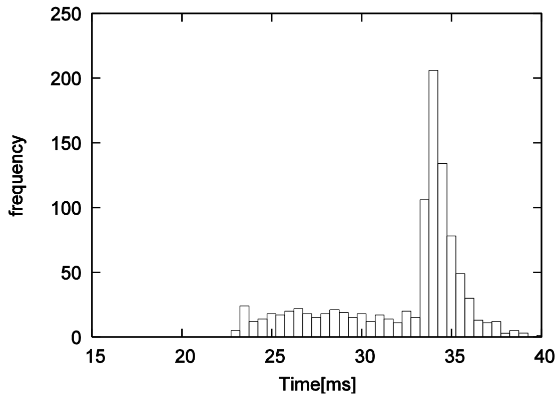

Figure 9: Histogram of start times in the output when g=8 input

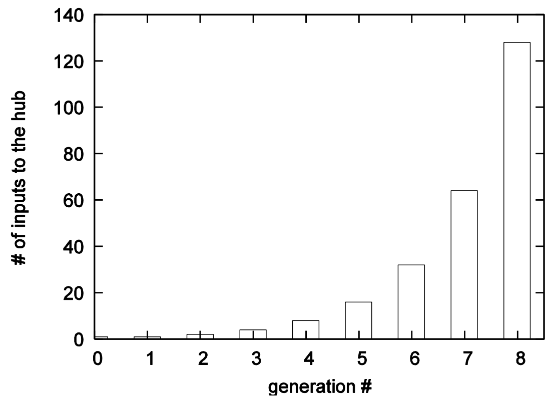

Figure 10: The number of input connections to the hub neuron