The effect of integrator leakages on idle tone of double loop delta sigma modulator

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Abstract—The delta sigma modulator shows idle tone by zero or DC signal input due to limit cycles. In this study, we attend integrator leakages of double loop delta sigma modulator to suppress the idle tone. From bifurcation and FFT analysis, we observe parameter sets of integrator leakages which can suppress idle tone in a signal band.

1. Introduction

The delta sigma modulator is widely used as analog-to-digital and digital-to-analog converters. This modulator can realize the high resolution by using over sampling technique for low bit quantization. Therefore, the delta sigma modulation is applied to audio or biosignal processing. However, when zero or DC signal is injected, the periodic output (called "*limit cycle*") occurs. The *limit cycle* causes idle tone in the signal band, and the resolution of modulator deteriorates due to the idle tone.

To overcome this problem, there are two techniques; dither and chaotic modulation. The dither is a general way to suppressed the idle tone. However, the noise or harmonic wave are caused by intermodulation between the dither and signal. On the other hand, the chaotic modulation is proposed by Schreier in[1]. He showed that the idle tone is reduced by his chaotic modulation, and realized chaotic modulation with change of the integrator leakage. Feely and coworker represented that the integrator leak affect the chaotic output of modulation[2, 3]. Hence, the appearance of the idle tone in the signal band is suppressed by changing the value of the integrator leakage.

In this study, we consider the double loop delta sigma modulator including integrator leakages. We investigate the effect of integrator leakages on the idle tone and normal performance of delta sigma modulator by using bifurcation analysis. The idle tone is related with periodic output of quantizer. The state of output from quantizer is depend on that of integrator output. Therefore, to classify states (generating idle tone, normal performance and divergence) of output of modulator, we analyze periodic solution of integrator output, and we represent application of obtained bifurcation map for finding the idle tone.

As a result, from bifurcation map, we can obtain parameter region classified by three state; period point, chaos and divergence. We present that normal performance of delta sigma modulator is confirmed in parameter sets in the chaos region from FFT analysis.

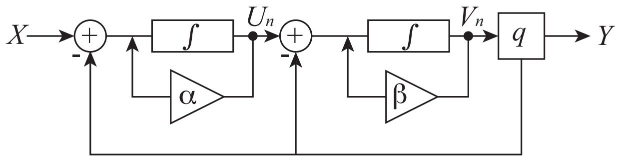


Figure 1: Schematic diagram of double loop delta sigma modulator.

2. Model of double loop delta sigma modulator

Figure1 shows a schematic diagram of double loop delta-sigma modulator. X and Y are input and output signals of delta-sigma modulator, respectively. U_n and V_n are outputs of integrators, and are shown as following:

$$U_{n+1} = X + \alpha U_n - q(V_n) \tag{1}$$
$$V_{n+1} = X + \alpha U_n + \beta V_n - 2q(V_n) \tag{2}$$

where, q is a function of behavior of a quantizer described as following:

$$q(x) = \begin{cases} 1, & \text{for} \quad x \geq 0 \\ -1, & \text{for} \quad x < 0. \end{cases} \tag{3}$$

In Eqs. (1) and (2), α and β correspond to prior and later integrator leakages, respectively. The integrator leakage of the ideal delta-sigma modulator is assumed as 1. When the integrator leakage is smaller than 1, the periodic output of integrator is observed at 0 input ($X = 0$). This periodic output evokes the idle tone in the signal band.

On the other hand, for integrator leakages > 1, the output of later integrator becomes chaos. Then, periodicity of output is lost by this chaos. Hence, the idle tone is suppressed even though input magnitude is 0.

However, the modulator becomes unstable since the output of integrator diverges easily. In this study we change the value of α and β between 0 and 2.0.

3. Result

Figure 2(a) shows the one-parameter bifurcation map for $X = 0$. The horizontal and vertical axes are integrator leakages (α,β) and the output of integrator V_n, respectively. When α and β are smaller than 0.65, the integrator output V_n becomes 2-periodic point. Hence, the output of delta sigma shows the periodic output (-1,1,-1,1..., or 1,-1,1,-1...). Then α and β are included between 0.65 and 1.0, V_n becomes 4-period points. When integrator leakages are larger than 1.12, the later integrator output is divergence. In this case, system of the delta sigma modulator is unstable.

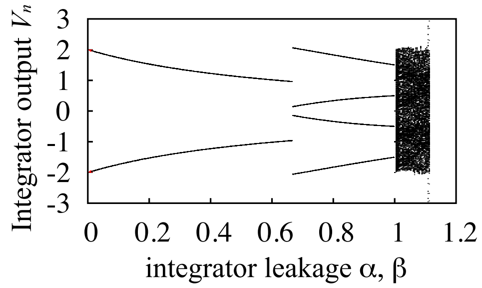
Figure 2(b) is enlarged diagram around $\alpha = \beta = 1.0$ of Fig.2(a). This figure shows that 4-period point exists at $\alpha = \beta = 1.0$. Hence, the output of the ideal delta sigma modulator has strong periodicity when $X = 0$. We can observe that the 5-period point appear over the bifurcation point of $\alpha = \beta = 1.0$. This period point becomes one chaos as integrator leakages increase.

Figure 3(a) and 3(b) show the power spectrum of output of delta sigma modulator Y. The horizontal and vertical axes are logarithmic normalized frequency and power spectrum, respectively. From Fig. 3(a) we can observe that the strong idle tone occurs in the band at a point of $\alpha = \beta = 0.4$. The power spectrum of idle tone appears at a half of sampling frequency. On the other hand, the idle tone is suppressed in the band, in the case of $\alpha = \beta = 1.1$ from Fig. 3(b) This parameter corresponds to one which we can observe chaos in Fig. 2(a). Moreover, we obtain the shaping characteristic of 40[dB/dec] in double loop delta sigma modulator.

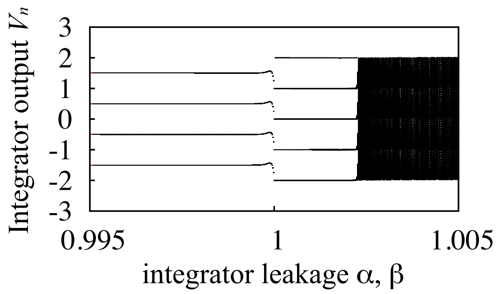
Figure 4 indicates a two-parameter bifurcation diagram obtained by using brute force method. The horizontal and vertical axes are integrator leakages α and β, respectively. The period point, chaos and divergence regions are classified by colored regions (See in Fig.4), *i.e.* in red region, we can find 2-period point state. The region in which we can observe chaos is colored by dark blue. The divergence of output corresponds to the colored region by black. The sub-captions indicate magnitude of the input DC signal.

From Fig. 4(a), for $X = 0$, 2-period and 4-period points are found in a parameter region of $\alpha \leq 1.0$ and $\beta \leq 1.0$. In the case of $\alpha > 1$ and $\beta > 1$, a chaos region is observed. The divergence region becomes wide as integrator leakages increase. From this figure, the delta sigma modulator has a wide parameter region in which the chaotic solution is observed as β becomes larger than 1.0.

Figures 4(b)-4(f) show that the region of 2-period point becomes narrow as magnitude of the DC input increases. On the other hand, various regions of period points appear; 3-, 5-, 7- and more period points. From Figs. 4(c)-4(f), the region of 3-period point becomes wider than that of 2-period point as the DC signal increases. However, system of delta sigma modulator has wider parameter region in which we can observe the divergence of integrator output as integrator leakages are lager than 1.0. Hence for leaky system, the output of delta sigma modulator is very likely to diverge by inputing the large DC signal.

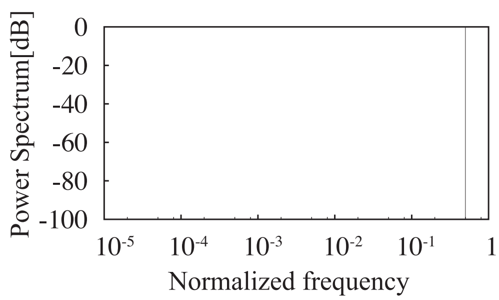


(a) One-parameter bifurcation diagram of V_n

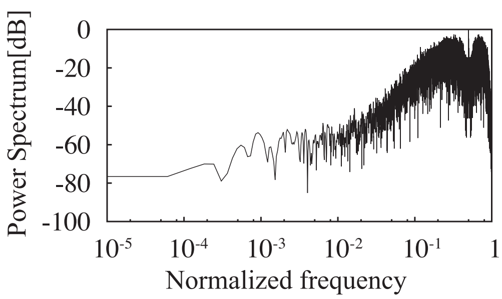


(b) Enlarged figure around $\alpha = \beta = 1.0$ of Fig. 2(a)

Figure 2: One-parameter diagram of integrator output V_n.



(a) $\alpha = \beta = 0.4$



(b) $\alpha = \beta = 1.1$

Figure 3: Power spectrums of output.

Figure 5 illustrate one-parameter bifurcation diagram for $X = 0.2$ and 0.8. We also find various regions of period points when α and β are smaller than 1. From this figure,

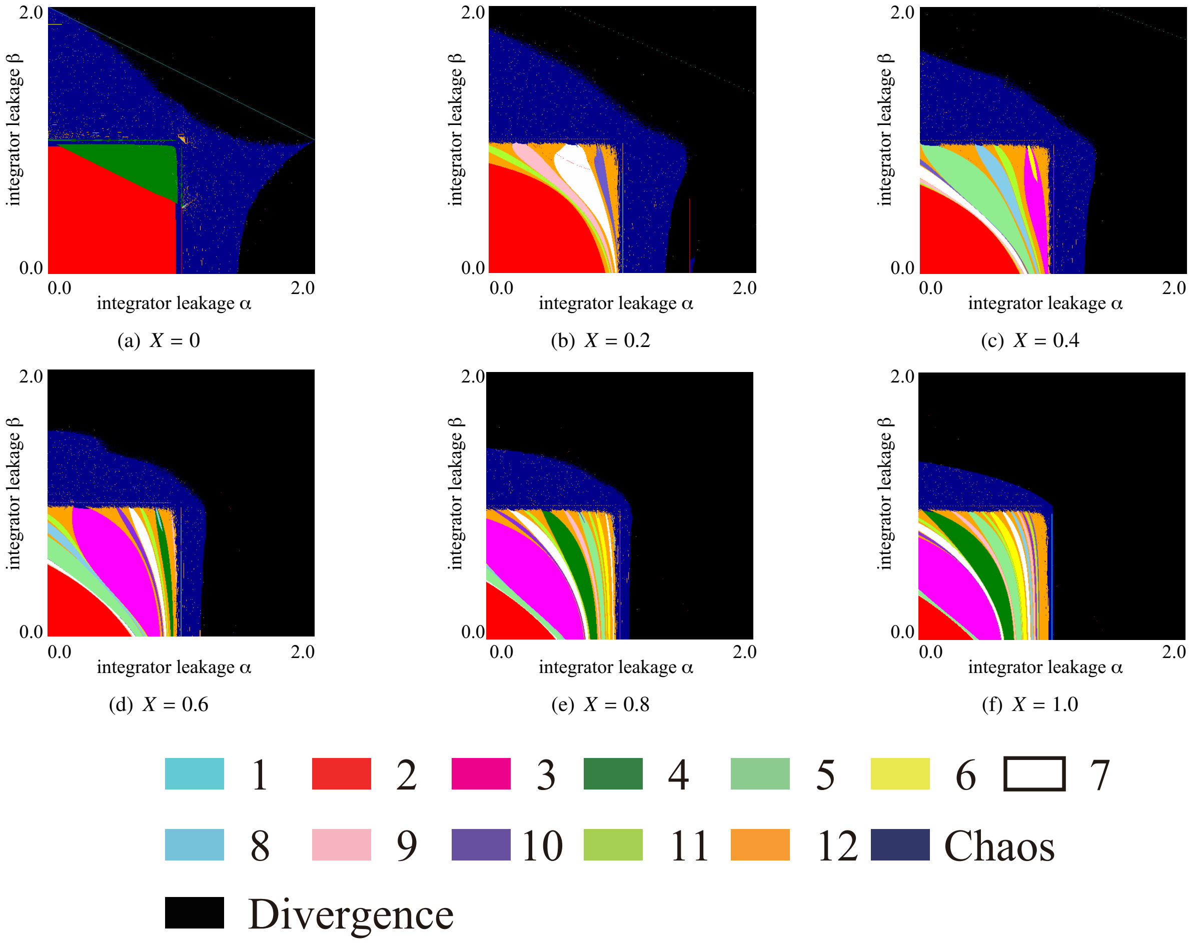


Figure 4: Two-parameter bifurcation diagram.

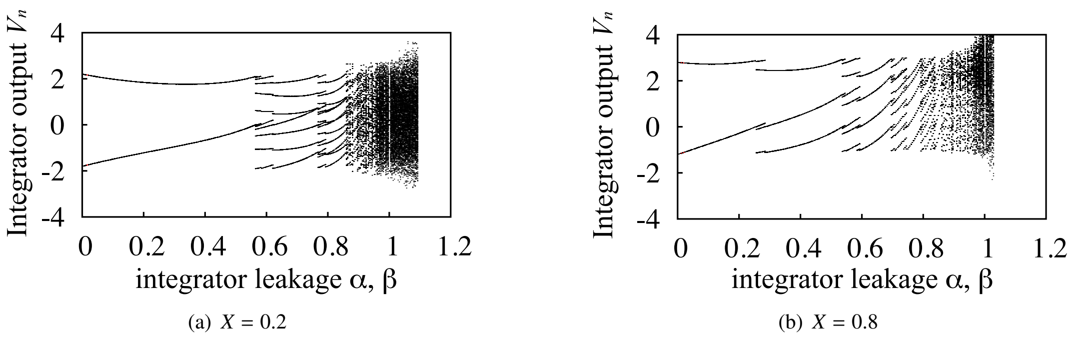
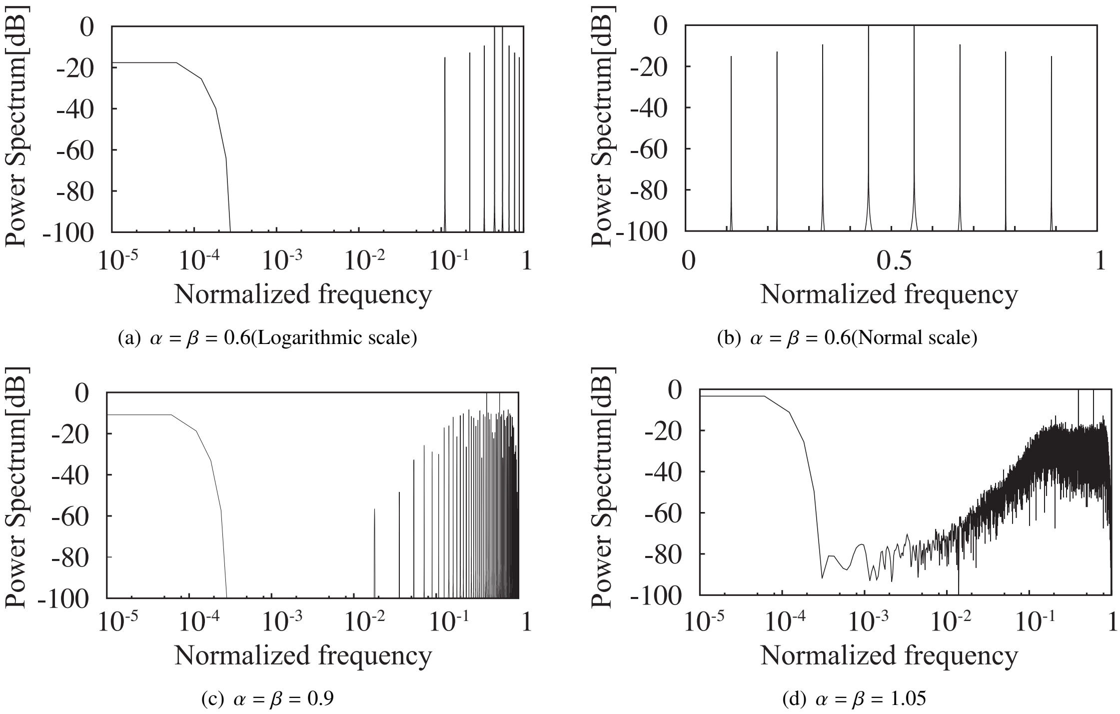


Figure 5: One-parameter bifurcation diagram in Fig. 4.



(a) $\alpha = \beta = 0.6$(Logarithmic scale)

(b) $\alpha = \beta = 0.6$(Normal scale)

(c) $\alpha = \beta = 0.9$

(d) $\alpha = \beta = 1.05$

Figure 6: Power Spectrum of output for $X = 0.2$.

the parameter region of 2-period point decreases as the appearance of parameter region of other period points. This figure shows that the divergence region is observed at small integrator leakages as magnitude of the DC input increases. Moreover the offset of V_n becomes high by increasing magnitude of the DC input.

Figure 6 shows power spectrum of output Y for $X = 0.2$. From Figs. 6(a) and 6(c), we can observe strong idle tones in signal band. In the case of $\alpha = \beta = 0.6$, eight strong tones appear in the band. Figure 6(b) shows that these tones divid the band among nine. The number of these intervals corresponds to that of period points at $\alpha = \beta = 0.6$ (See Figs.4(b) and 5(a)). Hence, we can guess the frequency which holds strong power (tone) by bifurcation diagram.

Figure 6(d) indicates the power spectrum at the points in chaotic region of Fig. 4(b). We can observe the noise shaping curve of 40 [dB/dec], thus the delta sigma modulation behaves normally. However, we find weak tone at near by a half of sampling frequency from Fig. 6(d). This weak tone does not affect the output signal since it can be eliminated by using low-order filter. Therefor, in this case the delta sigma modulator can operate when input is the DC signal.

4. Conclusion

We investigate the effect of the change of integrator leakages on the output signal of the double loop delta sigma modulator by using bifurcation and FFT analyses.

From bifurcation diagram, we observe various period points, chaos and divergence regions by changing integrator leakages. At the parameter including the chaos region, the strong idle tone in signal band is suppressed whereas the magnitude of input is 0. The divergence region becomes wide as $\alpha > 1.0$ for large magnitude of the DC input. The region of period point is complicated as magnitude of the DC input increases. We find that these period points are related to the strong idle tone in signal band by FFT. Hence, we guess that a frequency of the strong tone is predicted by the period point observed in bifurcation diagram.

Our open problems are clarifying a relationship between the weak idle tone and bifurcation ?and finding the values of integrator leakages that can suppress the weak idle tone in large magnitude of DC inputs.

Acknowledgments

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References

[1] R. Schreier, IEEE Trans. on CASS-I, vol.41(8), pp.539–547, 1994.

[2] O. Feely and L. O. Chua, IEEE Trans. on CASS-I, vol.38(11), pp.1293–1305, 1991.

[3] R. Farrell and O. Feely, IEEE Trans. on CASS-II, vol.45(6), pp.691–710, 1998.