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Effects of Asymmetric and Symmetric STDP on the Topology of a Recurrent Neural Network Model

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Abstract—A simulation experiment showed that temporally asymmetric STDP leads to self-organization of feedforward networks starting from a pacemaker neuron, by contrast, symmetric STDP does not[Masuda et al. 2007]. However, how the topology of a network without the pacemaker neuron is changed by the STDP is not well understood. The purpose of this study is to clarify how the topology of the network without a pacemaker neuron is changed by asymmetric or symmetric STDP. We executed numerical experiments and analyzed the topology of synaptic connections using some measures. The results suggest that asymmetric STDP turns a recurrent network into a feedforward network while symmetric STDP keeps the recurrent structure.

1. Introduction

Synaptic connections are modified depending on the temporal difference between pre- and postsynaptic action potentials [1]. This phenomenon is termed ”Spike-Timing-Dependent synaptic Plasticity(STDP)”. Several kinds of STDP were found in various regions of the brain. Additionally, different STDP were observed even in the same region [2]. Asymmetric or symmetric STDP is found in cultured neurons or in a slice preparation respectively and independently [3, 4]. The functional difference of each STDP has been studied recently.

A simulation experiment showed that temporally asymmetric STDP leads to self-organization of feedforward networks starting from a pacemaker neuron, because asymmetric STDP gets rid of feedback paths [5]. By contrast, symmetric STDP leaves feedback paths. The pacemaker neuron, which fires with the constant frequency, is found in several brain regions. This neuron is rarely affected by other neurons [6]. Without the pacemaker neuron, how does the topology of the network change depending on the difference of STDP? The purpose of this study is to clarify the difference of the topological change of a recurrent neural network by asymmetric or symmetric STDP.

2. Methods

2.1. Neural Network Model

We adopt a spiking neuron model, which is based on the chaotic neuron model but the output function is the Heaviside function [7]. After it fires, the neuron becomes more refractory to further firing for a time. Assuming that x_i represents an internal activity of each neuron and y_i represents an output value, the dynamics of this model are represented by the following equations:

$$
\begin{aligned}
x_i(t+1) = &\sum_{j=0}^{N}\sum_{d=0}^{t}\exp(-\frac{d}{\tau})s_j w_{ji} y_j(t-d) \\
&+\sum_{d=0}^{t}\exp(-\frac{d}{\tau})A_i(t-d) \\
&-\alpha\sum_{d=0}^{t}\exp(-\frac{d}{\tau_{ref}})y_i(t-d)-\theta,
\end{aligned}
\tag{1}
$$

$$
y_i(t) = \begin{cases} 1 & if \quad x_i > 0 \\ 0 & if \quad otherwise, \end{cases} \tag{2}
$$

where N is the total number of the neurons, s_j is the discrimination parameter as to #j neuron is excitatory or inhibitory ($s_j = 1.1$ or $s_j = -4.0$, respectively), w_{ji} is the synaptic weight from #j neuron to #i neuron, $A_i(t)$ is the external input to #i neuron, α is the scaling parameter of the refractory effect, τ is the decay time constant of the membrane potential, τ_{ref} is the decay time constant of the refractory effect, θ is a threshold for firing. For simplicity, time is assumed to be discrete.

We investigated a recurrent neural network composed of 200 neurons. 80% of all neurons are excitatory, and 20% are inhibitory. The rate of connections is about 20%, that is, each neuron receives synaptic inputs from about 20% of all neurons except itself ($w_{ii} = 0$). The initial synaptic weights are generally asymmetrical, that is, $w_{ij} \neq w_{ji}$, and the random values are in a uniformly distributed in [0, 1].

2.2. Asymmetric and Symmetric STDP Learning Rules

According to the experimental data [3, 4], we assume that the window function of asymmetric STDP (Fig.1(a)) is represented by the following equation:

$$\Delta w = \begin{cases} A_p \exp(-\frac{\Delta t}{\tau_p}) & if \quad \Delta t > 0 \\ -A_d \exp(\frac{\Delta t}{\tau_d}) & if \quad \Delta t < 0 \\ 0 & if \quad \Delta t = 0, \end{cases} \quad (3)$$

where Δt is the temporal difference between presynaptic and postsynaptic action potentials ($t_{post} - t_{pre}$), A_p and A_d are scaling parameters of the changes in synaptic efficacy, and τ_p and τ_d are the rates of exponential decays. Additionally, we assume that the window function of symmetric STDP (Fig.1(b)) is represented by the following equation:

$$\Delta w = a(1 - b\Delta t^2)\exp(-c\Delta t^2), \quad (4)$$

where a, b, and c are the parameters which characterize the shape of the window function. The modification of the synaptic weight obeys the following multiplicative equation:

$$w_{ij}(t+1) = w_{ij}(t) \times (1 + \Delta w), \quad (5)$$

The amount of potentiation is equal between asymmetric and symmetric STDP, and the amount of depression is also under the same condition. Ratio between amount of potentiation and depression is set 0.9 : 1.0 to avoid bursting.

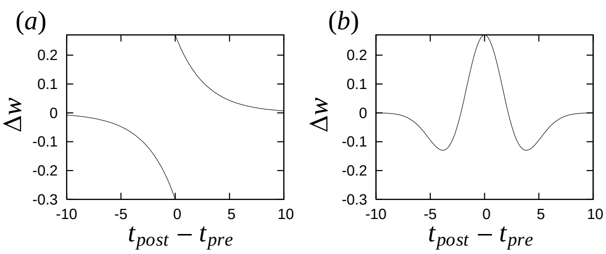

Figure 1: Window functions. (a)Asymmetric STDP (b)Symmetric STDP

2.3. External Inputs

Independent Poisson-process spike trains are assumed to be the external input spatiotemporal patterns for the neurons. The amplitude of the external input is set uniform. To examine the effects of the changes in the synaptic weights by spatiotemporal patterns, the external input with the period of 100 ms is applied repeatedly.

2.4. Topological Measures

We measured characteristic path length to evaluate the depth of the network, and circular path length to detect the feedback paths. In this study, we assume that a path exists if a synaptic weight of the connection is beyond the mean synaptic weights (path threshold) of all connections. Therefore, the path threshold is changed temporally, and the number of paths is almost the same between asymmetric and symmetric STDP. We also take into account directions in each path. Most of arbitrary two neurons can be linked through relaying neurons if paths are well selected.

2.4.1. Characteristic Path Length

As a method to evaluate the network characteristics, characteristic path length L is defined by the following equations:

$$L = \frac{1}{N}\sum_{i=1}^{N} L_i, \qquad L_i = \frac{1}{N-1}\sum_{j=1, j\neq i}^{N} d_{ij}, \quad (6)$$

where L_i is characteristic path length of #i neuron, N is the total number of the neurons, d_{ij} is the shortest path length from #i neuron to #j neuron.

2.4.2. Circular Path Length

To evaluate the feedback paths, we define circular path length C as the following equations:

$$C = \frac{1}{N}\sum_{i=1}^{N} C_i, \qquad C_i = \min_{all\ circular\ path} d_{ii}, \quad (7)$$

where C_i is minimum circular path length of #i neuron, N is the total number of the neurons, d_{ii} is the shortest path length from #i neuron to oneself with going through other neurons.

3. Simulation Results

3.1. Firing Pattern

Under moderate level of input frequency, synchronous firings are frequently observed without STDP (Fig.2 (a)). Asymmetric STDP tends to sharpen synchronous firings (Fig.2 (b)). By contrast, the synchronous firing disappears by symmetric STDP(Fig.2 (c)). These tendency are quantitatively evaluated by the synchrony coefficient (Fig.3). The synchrony coefficient $S(t_0)$ is defined by the following equations [8]:

$$S(t_0) = \max_{t'=0,1,\ldots,T-1} C(t_0 - t'; t_0), \quad (8)$$

$$C(t; t_0) = \frac{\frac{c}{N\tau_s}\sum_{i=1}^{N}\sum_l \sum_{t'=t-\tau_s/2}^{t+\tau_s/2} \delta(t' - t_i^l)}{\frac{1}{NT}\sum_{i=1}^{N}\sum_l \sum_{t'=t_0-T}^{t_0} \delta(t' - t_i^l),} \quad (9)$$

where c is a scaling parameter($c = \tau_s/T$ in this study), N is the total number of the neurons, T is the period of the input spatiotemporal pattern, t_i^l is the timing of the l th spike of the i th neuron, and τ_s is a temporal window epoch for synchrony evaluation. In the case of asymmetric STDP, firing neurons are clustered, as illustrated in Fig.4. As a result, feedforward networks will be self-organized.

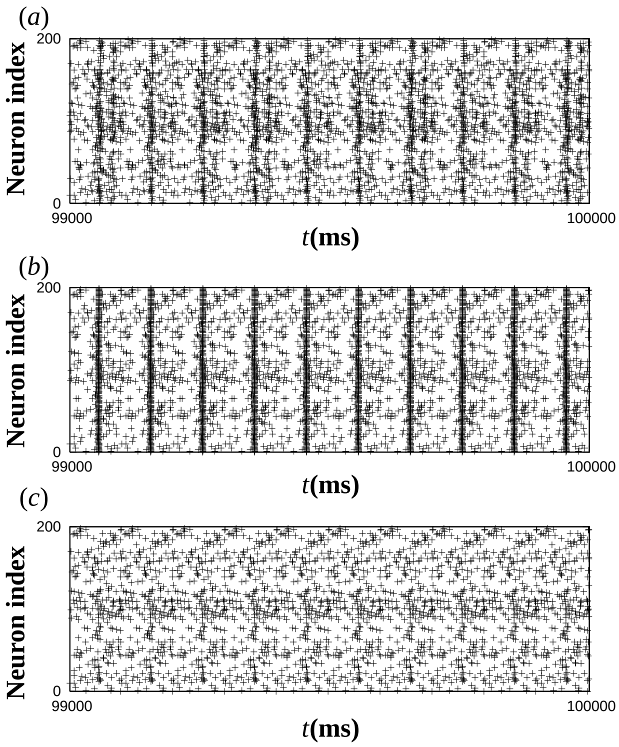

Figure 2: Typical examples of the firing pattern. (a)Without STDP (b)Asymmetric STDP (c)Symmetric STDP

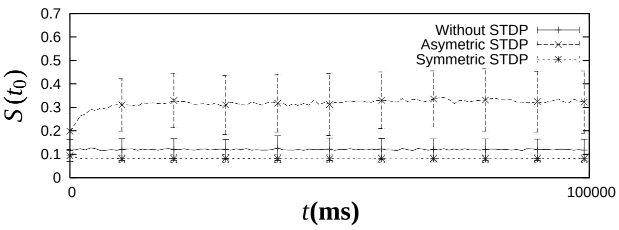

Figure 3: Synchrony coefficient. The line indicates mean synchrony coefficients over 100 simulations. The error bars are the standard deviation.

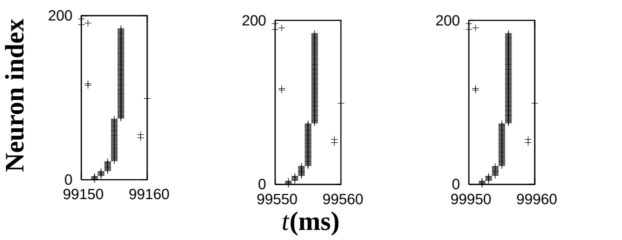

Figure 4: Firing neurons are clustered by asymmetric STDP (The neuron indices are sorted)

3.2. Network Diagram

In Fig.5 , neurons are arranged depending on the firing cluster in Fig.4. Feedback paths of network with asymmetric STDP are fewer than those of the initial state, as illustrated in Fig.5(b) . In contrast, the number of feedforward and feedback paths with symmetric STDP is almost the same as that of the initial state (Fig.5 (c)).

3.3. Characteristic Path Length

Figure 6 illustrates that asymmetric STDP increases gradually the characteristic path length. This suggests that arbitrary two neurons need more relay neurons between them. Assuming each relaying neuron included in a layer, we can consider that the layer becomes deep. In contrast, symmetric STDP does not change the characteristic path length so much.

3.4. Circular Path Length

Figure 7 illustrates that asymmetric STDP increases circular path length, and symmetric STDP does not change it so much. If arbitrary two neurons have feedforward and feedback paths, the circular path exists. The result suggests that recurrent paths decrease when asymmetric STDP is applied. This may be due to asymmetrical selection of paths in the network.

4. Discussion and Conclusion

The results may be due to the speciality of our model. Even if a simple integrate-and-fire neuron model is adopted, the results are almost the same as shown above. The dynamics of the model are represented by the following equations:

$$\tau \frac{dV(t)}{dt} = -V(t) + I(t), \tag{10}$$

$$V(t) := V_0 \qquad if \quad V(t) \geq V_1, \tag{11}$$

where $V(t)$ is membrane potential, τ is decay constant, $I(t)$ is input current, V_0 is initial value, V_1 is threshold.

In this study, we analyzed how the topology without pacemaker neuron is changed by asymmetric or symmetric STDP. We measured characteristic path length and circular path length. The result suggests that asymmetric STDP gets rid of feedback paths. As a result, the depth of the network increase, and feedback paths decrease, that is, asymmetric STDP turns a recurrent network into a feedforward network. In contrast, symmetric STDP leaves the feedback paths, namely, keeps the recurrent structure.

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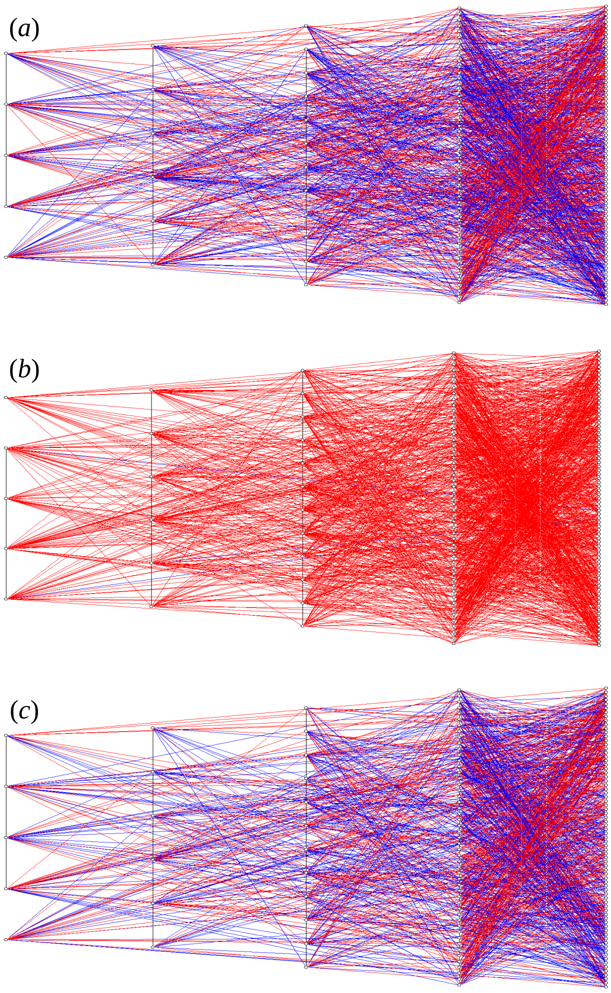

Figure 5: Typical examples of the network diagram. Red lines represent feedforward paths in a firing cluster, which is a firing sequence shown in Fig.4. Blue lines represent feedback paths (reverse directions to the red lines). The neurons, which do not belong to one of five firing cluster, are not illustrated. (a)Without STDP (b)Asymmetric STDP (c)Symmetric STDP

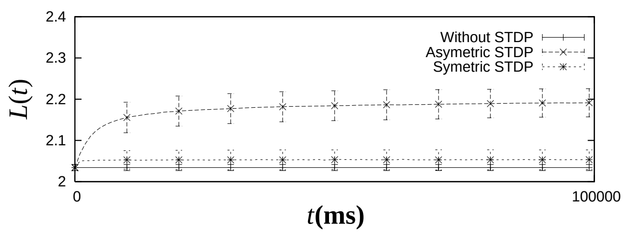

Figure 6: Characteristic path length. The line indicates mean characteristic path lengths over 100 simulations. The error bars are the standard deviation.

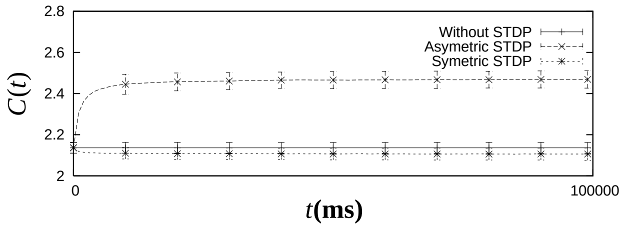

Figure 7: Circular path length. The line indicates mean circular path lengths over 100 simulations. The error bars are the standard deviation.

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