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Application of Independent-Minded Particle Swarm Optimization to Parameter Search in Switched Dynamical Systems

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Abstract—Our previous study proposed an application of the particle swarm (PSO) optimization to find desired parameters for multi-objective problems (MOPs) in switched dynamical systems. This study proposes an application of the independent-minded PSO (IPSO) to the MOP. Various simulation results show that the application of IPSO to the MOP is effective and IPSO can obtain better results than PSO.

1. Introduction

The Particle Swarm Optimization (PSO) [1]–[2] is an algorithm to simulate the movement of flocks of birds. Each particle of swarm tries to find a better solution according to its personal best position and the swarm best position. The many real/potential applications have been proposed, including design of artificial neural networks and power electronics [3], [4].

Our previous study proposed an application of the PSO to multi-objective problems (MOP) [5]. We considered the application to an example of the switched dynamical systems (SDS) which relates to simplified model of photovoltaic (PV) systems such that the input is a single solar cell and is converted to the output via a boost converter [6]. Our SDS includes a peace wise linear (PWL) current-controlled voltage source (CCVS) that is a simplified model of the solar cell and the switching role is variant of peak-current-controlled switching. We derived two equations that give period-doubling bifurcation set and the maximum power point (MPP) for the parameter. The two equations wear transformed into an the MOP described by the hybrid fitness function consisting of two functions evaluating the validity of parameters and criteria. The proposed method permits deteriorate of some component below the criterion, and it helped an improvement of trade-off problems in existing the MOP solvers. Simulation results showed that the efficiency of the proposed algorithm is confirmed by measuring in terms of accuracy, computation amount and robustness.

On the other hand, we have also proposed new PSO, independent-minded PSO (IPSO) [7]. The most important feature of IPSO is that it is decided stochastically that each particle depends on *gbest* or becomes independent from the swarm and moves depending only on *pbest*. In other words, the particles are not always connected each other, and they act with self-reliance. IPSO was applied to some benchmarks used widely in the literature, and it has been confirmed that IPSO is effective for multimodal problems with numerous local optima.

This study propose an application of IPSO to the MOP and applies the proposed algorithm to find desired parameters for the MOP in SDS. Various simulation results show that IPSO is effective not only for the uni-objective benchmarks but also for the MOP. Furthermore, we confirm that the application method of PSO to MOP proposed in our previous study is effective not only for the standard PSO but also for other improved PSOs.

2. Multi-Objective Optimization for Circuit model of the boost converter with a solar cell.

This study consider a optimization problem which relates to the stability and MPP detection of the SDS, as an application example of IPSO to the MOPs.

2.1. Circuit model

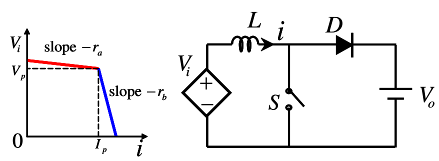

Figure 1: Circuit model of the boost converter with a solar cell.

Figure 1 shows the SDS where the CCVS is characterized by the 2-segment PWL function [6]. The dimensionless circuit equation is described by

$$\frac{dx}{d\tau} = \begin{cases} \gamma y(x), & \text{for State 1} \\ \gamma(y(x) - q), & \text{for State 2,} \end{cases} \tag{1}$$

$$y(x) = \begin{cases} -\beta(x-1)+1, & \text{for } x > 1 \\ -\alpha(x-1)+1, & \text{for } x \le 1, \end{cases} \tag{2}$$

$$\text{SW Rule: } \begin{array}{l} \text{State 2} \rightarrow \text{State 1: when } x = X_- > 0. \\ \text{State 1} \rightarrow \text{State 2: at } \tau = n \text{ and } x > X_-, \end{array} \tag{3}$$

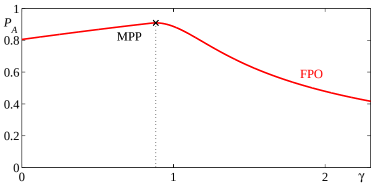

Figure 2: Average power characteristics for $\alpha = 0.5$, $\beta = 9.0$, $q = 1.6$ and $X_- = 0.7$. FPO shows characteristics of the stable/unstable FPO.

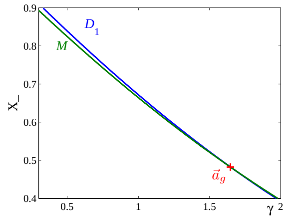

Figure 3: Parameter sets of bifurcation and MPP for $\alpha = 0.5$, $\beta = 0.9$ and $q = 1.6$. D_1: the period doubling bifurcation set. M: ridge of the average power of FPO.

where the SW rule is a variant to peak-current control. The dimensionless variables and parameters are defined by

$$\begin{aligned} &\tau = \tfrac{t}{T},\ x = \tfrac{i}{I_p},\ y(x) = \tfrac{V_i(I_p x)}{V_p},\ \alpha = \tfrac{r_a I_p}{V_p} \\ &\beta = \tfrac{r_b I_p}{V_p},\ q = \tfrac{V_o}{V_p},\ \gamma = \tfrac{T V_p}{L I_p},\ X_- = \tfrac{J_-}{I_p}. \end{aligned} \tag{4}$$

The dimensionless 5 parameters can be classified into two categories: (α, β, q), which characterizes "solar cell and load", and (γ, X_-) which characterizes "switching control".

In order to consider the power characteristics, the dimensionless instantaneous and average powers are defined as

$$p_{in}(\tau) = \frac{i(t)}{I_p}\frac{V_i(t)}{V_p},\ P_A = \frac{1}{N_p}\int_0^{N_p} p_{in}(\tau)d\tau \tag{5}$$

where $N_p = T_p/T$ is the dimensionless period of a stable periodic orbit (SPO) or an unstable periodic orbit (UPO) for dimensionless time τ.

In order to describe the objective equation, we define the phase map. Let τ_n denotes the n-th switching time at the lower threshold X_-, and let θ_d be a border time such that a trajectory started from (θ_d, X_-) reaches (1,1). Since τ_n determines τ_{n+1}, we can define the return map from positive reals to itself. Using the exact piecewise solution, the one-dimensional map can be described explicitly :

$$F(\tau_n) = \begin{cases} f_1(f_2(f_3(f_4(\tau_n)))), & \text{for } 0 \le \tau_n < \theta_d \\ g_1(g_2(\tau_n)), & \text{for } \theta_d \le \tau_n < 1 \end{cases} \tag{6}$$

where

$$\begin{aligned} &\tau_{n+1} = f_1(\tau_{s2}),\ \tau_{s2} = f_2(x_{s1}),\ x_{s1} = f_3(\tau_{s1}),\ \tau_{s1} = f_4(\tau_n) \\ &\tau_{n+1} = g_1(x_{s2}),\ x_{s2} = g_2(\tau_n),\ g_2(\theta_d) = 1. \end{aligned}$$

Let a phase variable $\theta_n = \tau_n \bmod 1$. The phase map f from the unit interval $I \equiv [0, 1)$ to itself;

$$\theta_{n+1} = f(\theta_n) \equiv F(\theta_n) \bmod 1,\ \text{for } \theta_n \in I. \tag{7}$$

A point p is said to be a k-periodic point if $p = f^k(p)$ and $p \neq f^l(p)$ for $1 \le l < k$ where $f^l(x_p) = f(f^{l-1}(p))$ and $f^0(p) \equiv p$. A 1-periodic point is referred to as a fixed point. A periodic point p is said to be unstable and stable for initial value if $|Df^k(p)| > 1$ and $|Df^k(p)| < 1$, respectively, where $Df^k(p)$ is the slope of f^k at p. The stable and unstable periodic points correspond to the SPO and UPO of the SDS, respectively.

Figure 2 shows the average power P_A of a fundamental periodic orbit (FPO) which corresponds to the fixed point p_1. As γ reaches the first period doubling bifurcation set D_1 and decreases, the FPO is changed from SPO to UPO and the P_A has the peak (i.e., MPP) at $\gamma = 0.884$, namely the MPP for γ ($\partial P_A/\partial\gamma = 0$). The M is one-to-one on the γ versus X_- plane and gives the ridge of the P_A characteristics shown as Fig. 3.

2.2. Multi-Objective function

The MOP is defined for finding desired parameters. Since bifurcation analysis in the 5-dimensional parameter space (corresponding to particle position $\vec{a}_i$ in the PSO) is extremely hard, we focus on 2-dimensional parameters $\vec{a} \equiv (a_1, a_2) \equiv (\gamma, X_-)$ which control the switching. As the target parameter value of the MOP, we consider the intersection of the parameter set M (the MPP for γ) and the period doubling bifurcation set D_1 (border of stability of the FPO). In other words, we define the intersection $(a_1, a_2) = (\gamma, X_-) = (1.605, 0.506)$ with $P_A = 0.832$ shown in Fig. 3 as the desired parameter set which is the objective solution. The search space Ds is defined as $0 < a_1 \le 2$ and $0.3 < a_2 \le 0.9$. Note that P_A increases as γ decreases and as X_- increases. Now, therefore, the MPP among the search space Ds is in the area with the smaller γ and larger X_-. However, γ depends on the clock period T and device speed as Eq. (4). Therefore, because the circuit with small γ is hard to realize, it is not enough to simply find the parameter with the large P_A.

Two objective functions are follows. If we can minimize both F_1 and F_2, we can obtain the realizable output having desired power.

The first one is about the MPP for γ. Whether $\vec{a}$ is the MPP is obtained by calculating of the slope on present position as

$$\begin{aligned} &f_1(\vec{a}) \equiv \frac{\partial P_a}{\partial \gamma} = 0,\ F_1(\vec{a}) = |f_1(\vec{a})| \\ &F_1(\vec{a}) = \left|\frac{\partial P_A(\vec{a})}{\partial \gamma}\right| = \left|\frac{\partial P_A(\vec{a})}{\partial a_1}\right|. \end{aligned} \tag{8}$$

$F_1(\vec{a}) = 0$ means that $\vec{a}$ generates the maximum average power on γ, namely a_1.

The second objective function evaluates whether $\vec{a}$ is the period doubling bifurcation set according to

$$f_2(\vec{a}) = Df(p_1) - 1 = 0,\ F_2(\vec{a}) = |f_2(\vec{a})| \\ F_2(\vec{a}) = |Df(p_1) + 1|. \tag{9}$$

$F_2(\vec{a}) = 0$ means that $\vec{a}$ gives the period doubling bifurcation set D_1 shown in Fig. 3.

3. Independent-Minded PSO for MOPs

We propose the application of IPSO to the MOP. This section explains how to apply IPSO to the Sect. 2.

In the algorithm of PSO, multiple potential solutions called "particles" coexist. Each particle has two informations; position and velocity. At each time step, each particle flies toward its own past best position (*pbest*) and the best position among all particles (*gbest*). In other words, they always influence each other. However, the particles of IPSO have independence, thus, it is decided stochastically whether they are connected to others at every step. In other words, they are not always affected by *gbest* and their *pbest* does not always affect the swarm.

The position vector of i-th particle at discrete time n and its velocity vector are represented by $\vec{a}_i(n) \equiv (a_{i1}, a_{i2}, \cdots, a_{iD}) \in \Re^D$ and $\vec{v}_i(n) \equiv (v_{i1}, v_{i2}, \cdots, v_{iD}) \in \Re^D$, respectively, where $(i = 1, 2, \cdots, N)$. The position corresponds to the parameter $\vec{a}$ in the MOPs. *pbest* of the particle i and *gbest* are denoted as $\vec{a}_{p_i}$ and $\vec{a}_g$. The $\vec{a}_g$ is the potential solution at time n.

Step 1 (Initialization): Let a discrete generation step $n = 0$. Randomly initialize the particle position $\vec{a}_i(n)$ in the search space $Ds \subset \Re^D$, and initialize other variables; velocity $\vec{v}_i(n) = 0$, $\vec{a}_{p_i} = \vec{a}_i(n)$ and $\vec{a}_g = \vec{a}_1(n)$.

Step 2 (Evaluation): Terminate the algorithm if

$$F_1(\vec{a}_g) < Cr_1 \text{ AND } F_2(\vec{a}_g) < Cr_2, \tag{10}$$

If not, go to Step 3.

Step 3 (Connecting): Decide whether each particle i is connected to the others according to r_{3i} which is a random value ($\in$ (0,1)) for the particle i. If $r_{3i} \le C$, the particle i is connected to other particles. If not, the particle i is isolated from the swarm, then, it and others does not affect each other. C is a constant cooperativeness coefficient which is the independence probability of the particles.

Step 4 (Updating): Update $\vec{v}_i$ and $\vec{a}_i$ of each particle i,

$$\vec{v}_i(n+1) = \begin{cases} w\vec{v}_i(n) + \vec{r}_1\rho_1(\vec{a}_{p_i} - \vec{a}_i(n)) + \vec{r}_2\rho_2(\vec{a}_g - \vec{a}_i(n)), & r_{3i} \le C \\ w\vec{v}_i(n) + \vec{r}_1\rho_1(\vec{a}_{p_i} - \vec{a}_i(n)), & r_{3i} > C \end{cases} \\ \vec{a}_i(n+1) = \vec{a}_i(n) + \vec{v}_i(n+1), \tag{11}$$

where w is the inertia weight determining how much of the previous velocity of the particle is preserved. ρ_1 and ρ_2 are two positive acceleration coefficients. $\vec{r_1}$ and $\vec{r_2}$ are D-dimensional uniform random number vectors from $U(0, 1)$. These equations mean that whether each particle is affected by *gbest* is decided at random with the cooperativeness C. When $C = 0$, all the dimensions of all the particles move depending only on own *pbest*, and when $C = 1$, the algorithm is completely the same as the standard PSO.

Step 5 (Hybrid fitness): Renew $pbest_i$ if the fitness is improved or satisfies the criteria. Let $\vec{a}_{p_i} = \vec{a}_i(n+1)$ if

$$\Big(F_1(\vec{a}_i(n+1)) < F_1(\vec{a}_{p_i}) \text{ OR } F_1(\vec{a}_i(n+1)) < Cr_1\Big), \\ \text{AND} \\ \Big(F_2(\vec{a}_i(n+1)) < F_2(\vec{a}_{p_i}) \text{ OR } F_2(\vec{a}_i(n+1)) < Cr_2\Big), \tag{12}$$

is satisfied for all the objective functions.

Renew *gbest* as $\vec{a}_g = \vec{a}_{p_i}$, where i is connected to others, namely $r_{3i} \le C$, and is a particle whose $pbest_i$ satisfies

$$\Big(F_1(\vec{a}_{p_i}) < F_1(\vec{a}_g) \text{ OR } F_1(\vec{a}_{p_i}) < Cr_1\Big), \\ \text{AND} \\ \Big(F_2(\vec{a}_{p_i}) < F_2(\vec{a}_g) \text{ OR } F_2(\vec{a}_{p_i}) < Cr_2\Big). \tag{13}$$

If more than one particle satisfies Eq. (13), the particle i with the smallest index is chosen.

Step 6 Let $n = n + 1$, return to Step 2 and repeat until the maximum time limit $n_{\max}$.

4. Numerical experiments

For numerical experiments, we use the following parameters; $N = 40$, $w = 0.729$, $\rho_1 = \rho_2 = 1.494$, $n_{\max} = 1500$, $C = 0.1$, $Cr_1 = 5.0 \times 10^{-4}$, $Cr_2 = 1.5 \times 10^{-3}$.

Table 1 shows the performances of the standard PSO and IPSO in terms of the mean termination generation and achievement rate over 100 trials. We can see that IPSO greatly improved the performance of PSO. Although the termination generation of IPSO is shorter than PSO, the achievement rate of IPSO is better than PSO.

Table 1: Average results of PSO and IPSO over 100 trials.

	PSO	IPSO
Termination generation	459.96	242.31
Achievement rate	70%	99%

Figures 4 and 5 show typical changes of the average power P_A of *gbest* and typical fitness functions in the searching process, of the standard PSO and IPSO, respectively. From these figures, we can observe that the two fitness functions converged, not only decreasing, but also increasing. This effect was caused by the two criteria in Eqs. (12) and (13). Each increase of the two fitness helps

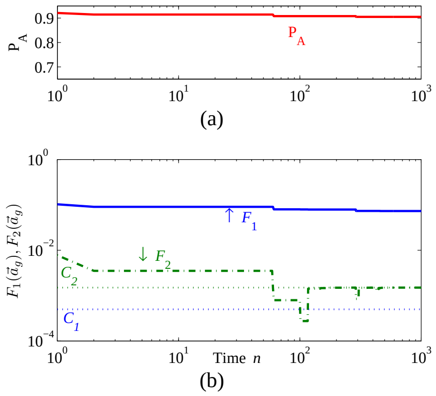

Figure 4: Search process by the standard PSO. (a) Change of the average power corresponding to *gbest*. $P_A = 0.9051$. (b) Search process of $F_1(\vec{a}_g)$ and $F_2(\vec{a}_g)$. Termination time is $n = 1000 = n_{\max}$. The results are $(F_1, F_2) = (0.0733, 0.0015)$ with $(a_1, a_2) = (\gamma, X_-) = (0.990, 0.6892)$.

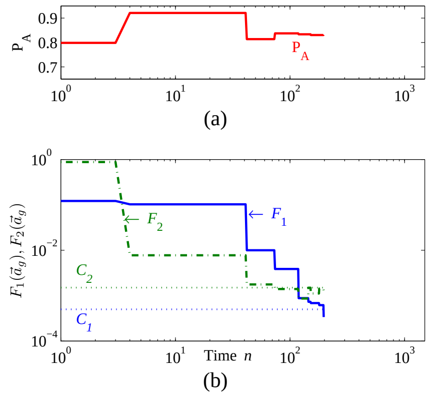

Figure 5: Search process by IPSO. (a) Change of the average power corresponding to *gbest*. $P_A = 0.8321$. (b) Search process of $F_1(\vec{a}_g)$ and $F_2(\vec{a}_g)$. Termination time is $n = 197$. The results are $(F_1, F_2) = (3.38 \times 10^{-4}, 0.013)$ with $(a_1, a_2) = (\gamma, X_-) = (1.605, 0.506)$.

other decrease, and this effect leads the P_A to the maximum average power with the realizable parameters. However, PSO was unable to succeed in escaping from the local optima even with the criterion effect. In fact, indeed F_2 reached the termination criterion Cr_2, but F_1 did not converge to Cr_1. On the other hand, IPSO converged more rapidly than PSO, and even though it trapped at local optima, it was able to escape. We can conclude that this is due not only to the criterion effect but also to the capability of IPSO.

Figure 6 shows the mean termination generation and achievement rate over 100 runs in different cooperativeness C. Note that the standard PSO used $C = 1.0$ for all the simulations, namely, IPSO with $C = 1.0$ is exactly the same as PSO. We should be noted that IPSO with $C = 0.02$–0.9 obtained better results than when it was fully-connected ($C = 1.0$). From this result, we can understand that this the MOP is the multimodal problem [7] and the parameter-dependence of IPSO is weak because IPSO kept better results than PSO long range of C. Furthermore, this result mean that that the particle diversity is more important for the multimodal functions than the quick communication, and the particles of IPSO is more diverse than PSO.

From these results, we can conclude that IPSO is effective not only for the uni-objective benchmarks but also for the MOP. It also means that the proposed application method [5] of PSO to the MOP is effective not only for the standard PSO but also for other improved PSOs as IPSO.

5. Conclusions

This study has proposed the application of IPSO to the MOP for finding desired parameter of the SDS. Performing basic numerical experiments, we have confirmed that IPSO is effective not only for the uni-objective benchmarks but also for the MOP.

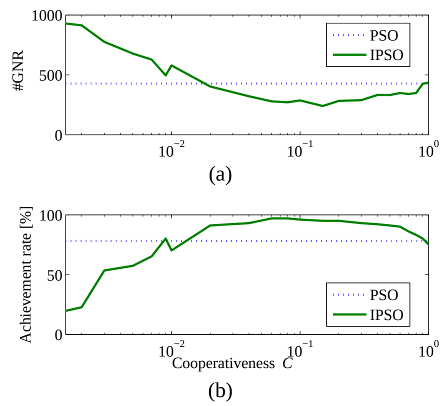

Figure 6: Influence of the Cooperativeness C on the performance of IPSO. (a) Average of termination generations 100 trials. (b) Average of the achievement rate over 100 trials.

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