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Stable periodic orbits induced by an extremely-weak diffusive connection in a pair of coupled chaotic oscillators

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Abstract—This paper investigates an interesting phenomenon in a pair of coupled chaotic oscillators. Each individual oscillator behaves chaotically and has an unstable periodic orbit that dominates the chaotic attractor. We experimentally observe that the stable periodic orbit, which is quite close to the unstable periodic orbit, can be induced by an extremely-weak diffusive connection in the Shinriki-Mori circuits. This phenomenon is examined by the bifurcation analysis.

1. Introduction

Control of chaos, which is a stabilization of unstable periodic orbits (UPOs) embedded within a chaotic attractor, has received considerable attention for more than two decades in nonlinear science. Various control methods have been proposed and applied to real systems, including electronic circuits, mechanical systems, and chemical reactions [1, 2, 3]. The two typical control methods are well known: the state feedback control method and the delayed feedback control method. The state feedback control method, which was proposed by Ott, Grebogi, and Yorke, has been recognized as the OGY method [4]; the delayed feedback control (DFC) method, which was proposed by Pyragas, has received considerable attention, not only in the field of nonlinear science [5], but also in control theory [6].

These methods are meant for controlling a single chaotic system; however, from a practical viewpoint, it is important to control spatiotemporal chaos in coupled systems [7, 8]. There are two solutions to this control problem. The first solution is that the OGY method or the DFC method is modified for coupled systems: the decentralized OGY or DFC controllers can stabilize the unstable states in coupled systems [7, 8, 9, 10, 11]. Since this solution requires many decentralized controllers, it is costly to implement them in practical situations. The second solution is that *time delays* are used in the connections [12, 13]. The main advantage of this solution is that there is no need to use controllers. Fiedler *et al.* showed that an UPO in the coupled normal forms for Hopf bifurcation can be stabilized; however, this method requires an additional no-delay connection to maintain the ani-phase state [12]. Choe *et al.* reported that the UPO in the coupled forms can be stabilized by the delay connection, but the connection has to rotate the signal phase [13]. These features of the second solution indicate that a simple diffusive delay connection cannot stabilize it. Furthermore, the two solutions can be used only for the UPO in the normal form; thus, the delay connections cannot stabilize UPOs in general chaotic systems, since the shapes of such UPOs are different from that of the normal form.

Zhan *et al.* demonstrated on numerical simulations that a simple weak-diffusive no-delay connection, which is the simplest connection, can induce a stable periodic orbit in vicinity to the UPO embedded within the single chaotic system [14]. The main advantage of this method is that the implementation cost of the connection is quite low compared with the above two solutions. The reasons are as follows: there is no need to use the controllers and the delay connection; as coupling strength is extremely weak, the signals in the connection are extremely small. On the other hand, its disadvantages are as follows: the range of coupling strength for stabilization is narrow and the desired UPO has to dominate the chaotic attractor.

This connection has potential ability to be widely used in practical coupled chaotic systems; however, there is no experimental verification of such stable periodic orbit. Furthermore, the reason why this phenomenon arises is still not clear. The main purpose of this work is to provide an experimental verification of this phenomenon by using the Shinriki-Mori circuits [15]. In addition, we investigate the detailed phenomenon on the basis of the bifurcation analysis.

2. Coupled chaotic circuits

Two Shinriki-Mori circuits (i.e, circuits a and b) are coupled by a resistor as shown in Fig. 1: they are

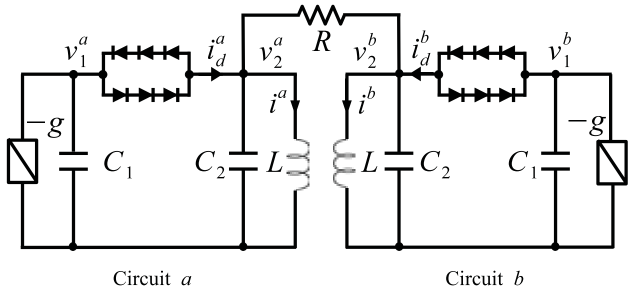


Figure 1: Shinriki-Mori circuits coupled by resistor R.

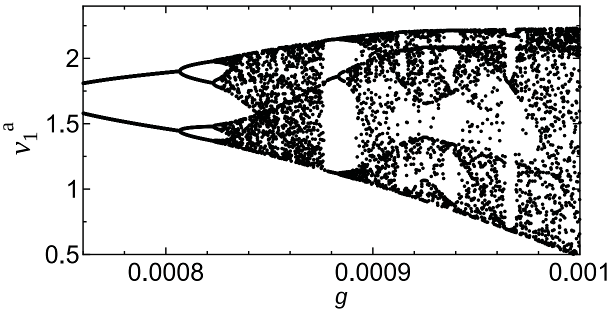


Figure 2: Bifurcation diagram of a chaotic oscillator without connection $(1/R = 0)$: Poincare section $\Sigma := \{(i_1, v_1, v_2) : v_2 = 0, v_1 < 2.2\}$.

governed by

$$\begin{cases} L\dfrac{\mathrm{d}i^{a,b}}{\mathrm{d}t} & = & v_2^{a,b} \\ C_1\dfrac{\mathrm{d}v_1^{a,b}}{\mathrm{d}t} & = & gv_1^{a,b} - i_d^{a,b} \\ C_2\dfrac{\mathrm{d}v_2^{a,b}}{\mathrm{d}t} & = & i_d^{a,b} - i^{a,b} + \dfrac{1}{R}\left(v_2^{b,a} - v_2^{a,b}\right) \end{cases}, \quad (1)$$

where the currents through the nonlinear part are

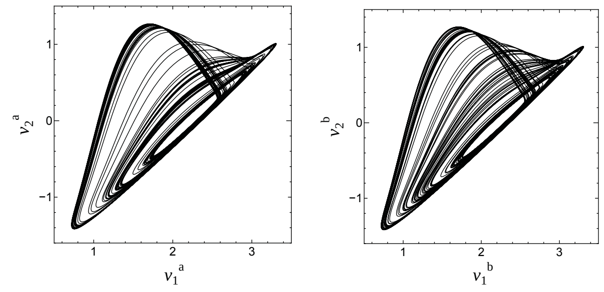
$$i_d^{a,b} = 5.0 \times 10^{-6} \times \left(v_1^{a,b} - v_2^{a,b}\right)^{15}.$$

Throughout this paper, the circuit parameters are fixed at

$$C_1 = 0.068\mu\mathrm{F},\ C_2 = 0.047\mu\mathrm{F},\ L = 10\mathrm{mH}.$$

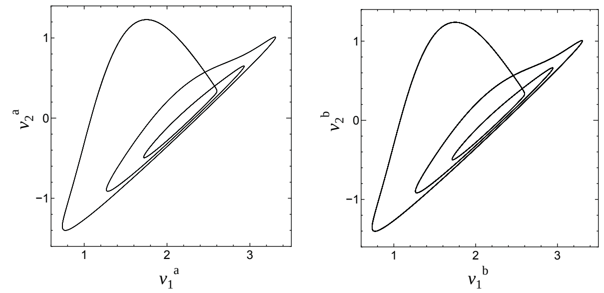
The coupling strength is described by $1/R$.

Figure 2 shows the bifurcation diagram of the chaotic oscillator without connection $(1/R = 0)$. The parameter g is set to $g = 0.893 \times 10^{-3}$S, where the period-three unstable orbit dominates the chaotic attractor. The circuits behavior without connection $(1/R = 0)$ on numerical simulations is illustrated in Fig. 3. These circuits are coupled by the extremely weak connection $(1/R = 3.5 \times 10^{-5})$; the circuits behavior is shown in Fig. 4, where the period-three stable



(a) oscillator a (b) oscillator b

Figure 3: Circuits behavior without connection $(1/R = 0)$ (numerical simulation).



(a) oscillator a (b) oscillator b

Figure 4: Circuits behavior with connection $(1/R = 3.5 \times 10^{-5})$ (numerical simulation).

orbits are observed. These orbits are extremely close to an UPO embedded within the single chaotic circuit, but they are slightly different from it.

In order to experimentally verify the above numerical results, the two circuits are implemented by popular-priced circuit devices, which have an error of several percent. The parameter g of each circuit is adjusted such that the period-three unstable orbit dominates the chaotic attractor[1]. The circuits behavior without connection $(1/R = 0)$ and with connection $(1/R = 1.9 \times 10^{-5})$ are shown in Figs. 5 and 6 respectively. It can be seen that the stable periodic orbits are experimentally induced by the extremely weak connection. These numerical and experimental results imply that the stable periodic orbit induced by the extremely weak connection occurs in the coupled Shinriki-Mori circuits.

[1] g is fixed close to the period-three window.

(a) oscillator a (b) oscillator b

Figure 5: Circuits behavior without connection ($1/R = 0$) (circuit experiments). Horizontal axis: v_1 (0.5 V/div); vertical axis: v_2 (0.5 V/div).

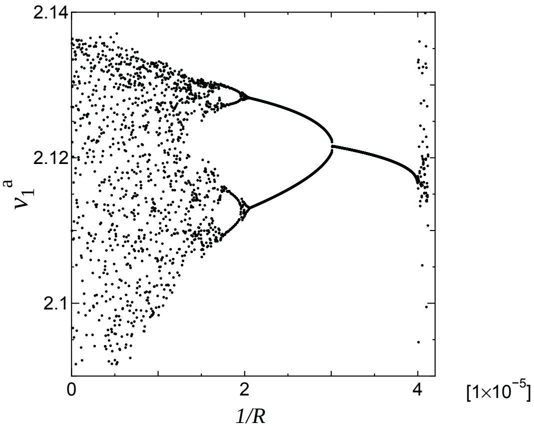
(a) oscillator a (b) oscillator b

Figure 6: Circuits behavior with connection ($1/R = 1.9 \times 10^{-5}$) (circuit experiments). Horizontal axis: v_1 (0.5 V/div); vertical axis: v_2 (0.5 V/div).

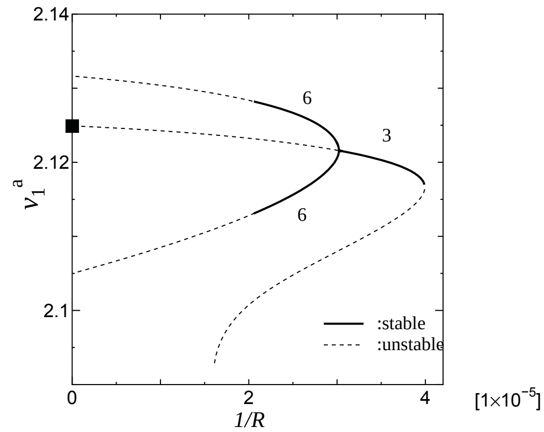
3. Discussion

This section investigates the extremely-weak connection induced stable periodic orbits. Figure 7(a) shows the bifurcation diagram ($v_1^a \in [2.09, 2.14]$) of the coupled circuits against the coupling strength $1/R$. The similar diagrams are observed for $v_1^a \in [1.70, 1.95], [1.04, 1.12]$; thus, we see the period-three stable orbit for $1/R \in [3.01\times10^{-5}, 3.99\times10^{-5}]$ and the period-six stable orbit for $1/R \in [2.05 \times 10^{-5}, 3.01 \times 10^{-5}]$. Figure 7(b) illustrates the stable and unstable periodic orbits which are numerically derived using the software BUNKI[2]. From these figures, it can be seen that, without connection ($1/R = 0$), the period-three unstable orbit corresponding to ■ in Fig. 7(b) is embedded within the chaotic attractor. The period-three orbit is unstable for $1/R \in (0, 3.01 \times 10^{-5})$. At $1/R \simeq 3.01 \times 10^{-5}$, the unstable orbit becomes stable via the period-doubling bifurcation. The period-three orbit is stable for $1/R \in [3.01 \times 10^{-5}, 3.99 \times 10^{-5}]$. It merges with the period-three unstable orbit at $1/R \simeq 3.99 \times 10^{-5}$; then, they disappear via the saddle-node bifurcation.

[2]http://bunki.sat.iis.u-tokyo.ac.jp/BUNKI/



(a) Bifurcation diagram



(b) Stable/unstable periodic orbits

Figure 7: Bifurcation diagram and stable/unstable periodic orbits (v_1^a vs. $1/R$) on Σ: (a) bifurcation diagram; (b) stable/unstable periodic orbits.

4. Conclusion

This paper experimentally showed that the stable periodic orbit extremely close to the UPO is induced by the simple weak-diffusive no-delay connection in the well known Shinriki-Mori circuits. Furthermore, this phenomenon was investigated by the bifurcation analysis.

Acknowledgments

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