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Delayed feedback control for dynamical systems with jumping characteristics

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Abstract—The delayed feedback control is a one of controlling chaos and can be achieved by providing only the period of the unstable periodic orbit embedded in a chaotic attractor. When we apply delayed feedback control to switched dynamical systems with jumping operations, we notice that a small mismatch of the state and the memorized orbit info results a very large amplitude impulse in control ability. In this paper, we propose a method that is controlled impulse and show effective of proposal method.

1. Introduction

The chaotic phenomena are observed in a lot of fields. The chaos movements is a movement that although is generated from deterministic system, it shows an irregular and complex behavior and we cannot predict the state in the future. The features of chaos movement are sensitivity of initial conditions and existence of infinitely many unstable periodic orbit(UPO). However, especially, the chaos movement that shows these features is often avoided and required to control this in the engineering field.

There is a controlling chaos in an effective technique for controlling the chaos movement. The controlling chaos is a concept announced in 1990 [1]. This method is stabilized chaos orbits to UPO by using the fact that the chaos orbit is set of UPO that exists infinitely. Therefore, various methods have been proposed and researched actively [2]. Delayed feedback control(DFC) is methods for one of the controlling chaos that Pyragas [3] proposed. This method is controlled chaos in continuous dynamical systems and it is stabilized the desired UPO by feeding back the difference between the current state and the delayed state of the desired UPO. The advantage of DFC are that the analysis of the periodical trajectory that we want to stabilize is unnecessary, essential parameter of controlling is only period τ of the desired UPO and it may be provided easily by using the hardware memory. Therefore, we can confirm the computer simulation etc whether the control show an enough performance.

In our study, to adopt the DFC to switched dynamical systems with jump operation, we notice that a small mismatch of the state and the memorized orbit info results a very large amplitude impulse in control ability. In this paper, we show application example for adopting DFC by using Izhikevich model.

2. Delayed Feedback Control

DFC is method of controlling chaos in dynamical systems and it stabilizes the desired UPO by feeding back the difference between the current state and the delayed state of the desired UPO. Figure 1 shows a block diagram of DFC. We assume that the target dynamical system is expressed in the following equation:

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{f}(t,\, \boldsymbol{x}),\;\; \boldsymbol{x} \in \boldsymbol{R}^n \tag{1}$$

where, we assume the solution of Eq. (1) as $\boldsymbol{x}(t) = \boldsymbol{\varphi}(\tau,\, \boldsymbol{x}_0)$. In general, the UPO that exists in the chaos attractor is expressed in the following equation:

$$\boldsymbol{x}_0 = \boldsymbol{\varphi}(0,\, \boldsymbol{x}_0) = \boldsymbol{\varphi}(\tau,\, \boldsymbol{x}_0), \tag{2}$$

where τ denotes the period of this UPO. The conventional DFC to Eq. (1) is expressed in the following equation:

$$\begin{aligned} \frac{d\boldsymbol{x}}{dt} &= \boldsymbol{f}(t,\, \boldsymbol{x}) + \boldsymbol{z}(t) \\ \boldsymbol{z}(t) &= K(\boldsymbol{x}(t-\tau) - \boldsymbol{x}(t)) \end{aligned} \tag{3}$$

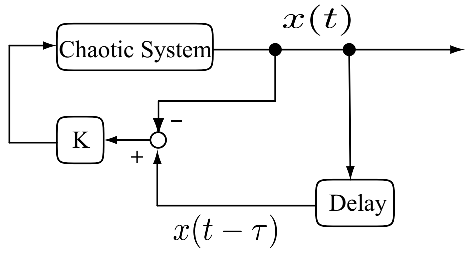

Figure 1: Block diagram of DFC

where K is a constant $n \times n$ matrix. If $z(t)$ converges to 0 by adopting an appropriate K, Eq. (3) become $\boldsymbol{z}(t) = 0$ and become congruent with Eq. (1). In addition, the orbit of the control system at that time become periodic orbit of τ-period. The advantage of DFC are that the analysis of the periodical trajectory that we want to stabilize is unnecessary, essential parameter of controlling is only period τ of the desired UPO and it may be provided easily by using the hardware memory. Therefore, we can confirm the computer simulation etc whether the control show an enough performance. The demerit of DFC is that the theoretic analysis on stability by the calculation of the characteristic exponents is very difficult. Because the control system totality become infinite dimension systems expressed in the differential difference equation. As a result, we must pick K by trial and error. In addition, if two or more UPO of period τ that exist in chaos attractor of controlled system exists, we do not know to stabilize which UPO. To adopt DFC to dynamical systems with jumping characteristic, the following problem happens. The $z(t)$ becomes like the impulse of a large amplitude by delaying slightly the timing of reset as shown in Fig. 2. We notice that this $z(t)$ negatively affects the control performance.

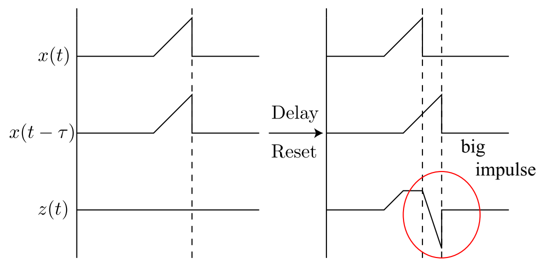

Figure 2: controlled variable caused by delay of timing of reset

3. Izhikevich

We use two model that Izhikevich model in this paper. Izhikevich Model is the mathematical model as the ignition phenomenon of the neuron and is expressed in the following equation:

$$\begin{aligned} \dot{v} &= 0.04v^2 + 5v + 140 - u + I \\ \dot{u} &= a(bv - u) \end{aligned} \tag{4}$$

The reset after spiking is expressed in the following equation:

$$\text{if } v \geq 30\text{mV}, \text{ then} \begin{cases} v \leftarrow c \\ u \leftarrow u + d \end{cases} \tag{5}$$

Where a, b, c, d and I are parameter and it is known to be able to reproduce a variety of neuron ignition patterns by selecting the parameter [4]. When we define a, b, c, d and I as $a = 0.2$, $b = 2$, $c = -56$, $d = -16$ and $I = -99$, we can observe the chaos attractor. Figure 3 shows the chaos attractor in this model. Figure 3 shows the desired UPO of Izhikevich model in this experiment.

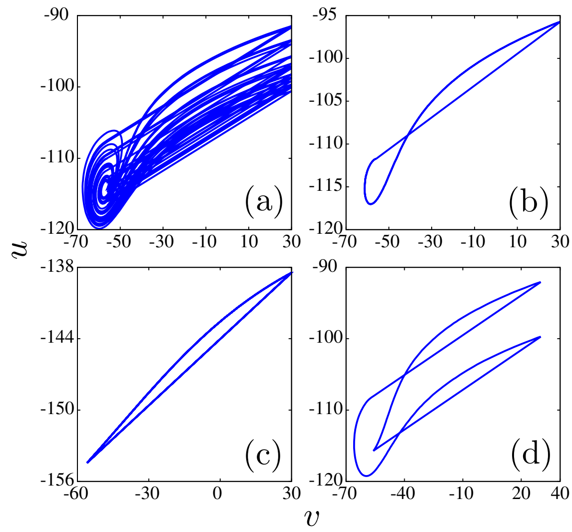

Figure 3: (a)chaos atrtractor:$a = 0.2$, $b = 2$, $c = -56$, $d = -16$, $I = -99$ (b):1-period $\lambda = -2.19$, fixed point $(v_0, u_0) = (-56.0, -111.734371)$ (c):1-period $\lambda = 1.01$, fixed point $(v_0, u_0) = (-56.0, -154.450720)$ (d):2-period $\lambda = -5.04$, fixed point $(v_0, u_0) = (-56.0, -108.062467)$, $(-56.0, -115.721659)$

4. Proposed method

In this section, we try stabilizing by impulse deletion method as method of controlling controlled variable. The impulse deletion method adjusts controlled variable to zero toward reset as shown in Fig. 4. The condition of reset is state of v in Izhikevich model that we use in this study. Thus we configure threshold before it. If its value is more than condition of $v(t)$ or $v(t-\tau)$, we adjust controlled variable to zero. It is expressed in the following equation:

$$\omega(t) = \begin{cases} 0 & \text{if } x(t) \geq L \text{ or } x(t-\tau) \geq L \\ z(t) & \text{else} \end{cases}$$

where L shows threshold and we define $x(t)$ as $v(t)$ in this paper. In this experimentation, we define $K = \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix}$, $k_2 = 0$. As a result, equation that we apply controlled variable is expressed in the following equation:

$$\begin{aligned} \dot{v} &= 0.04v^2 + 5v + 140 - u + I + \omega(t) \\ \dot{u} &= a(bv - u) \end{aligned} \tag{6}$$

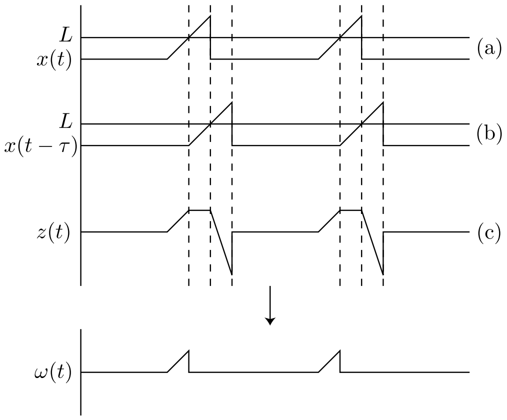

Figure 4: Impulse deletion method

If UPO that we want to stabilize is more than one period, we add the following condition. Whenever UPO of the controlled system is reset, we divide τ into τ_1, τ_2, $\cdots$. We adopt arbitrarily divided period τ_i and control only during adopted period. If UPO that we want to control is 2-period in this time, we divide τ into τ_1 and τ_2. We adopt arbitrarily period from there, and we control during adopted period. Figure 6–7 show execution results of 1-period. This figure is called basin boundary. It is a figure that we use different color by attractor converges initial value space. Figure 8–9 show transition of controlled variable when we compute parameter of each point in Fig. 6–7. Figure 8–9 show transition of controlled variable when we control during both τ_1 and τ_2 and controlled only during τ_1 by using PDFC. It is thought it is stabilized to UPO from controlled variable in Fig. 8–9 (a) stabilize value that closes illimitably to 0. We notice that the control performance is better only during τ_1 when we draw a comparison between Fig. 8 and Fig. 9. However we could not stabilize when we control only during τ_2. As a result, we notice that we cannot stabilize in some periods as controlled systems insofar as PDFC. Hence, it is not always true that PDFC does

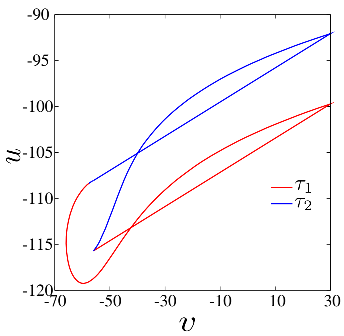

Figure 5: Partial delayed feedback control

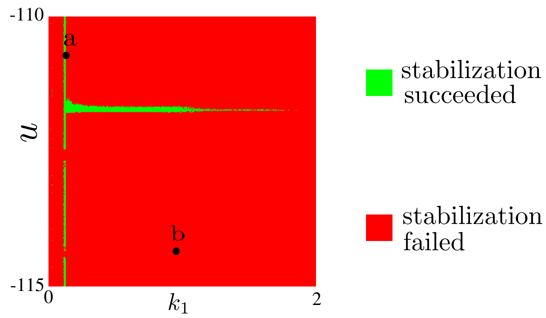

Figure 6: basin boundary of $\lambda = -2.19$, $L = 20.0$

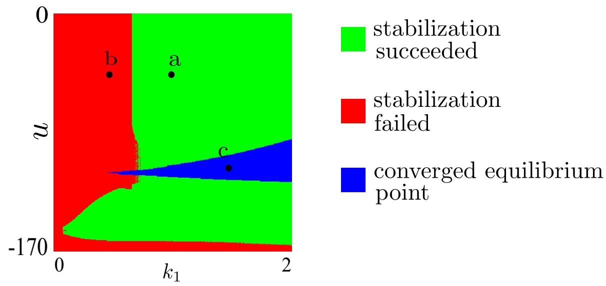

Figure 7: basin boundary of $\lambda = 1.01$, $L = 20.0$

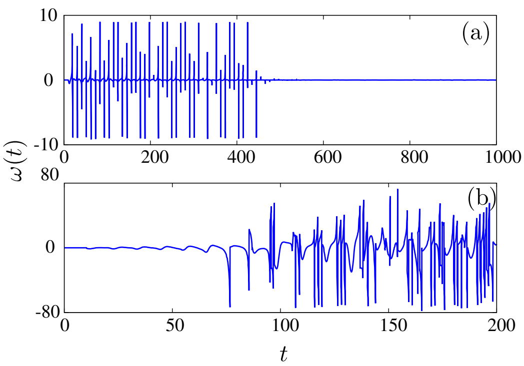

Figure 8: transition of controlled variable of Fig. 6 (a):$k = 0.12$, $(v_0, u_0) = (-56.0, -110.5)$ (b):$k = 1.0$, $(v_0, u_0) = (-56.0, -114.0)$

not always hold Izhikevich model.

5. Conclusion

In this paper, we showed problem when we adopt DFC to switched dynamical systems with jumping characteristics and proposed the Impulse deletion method as personal cure of this problem. As a result, we can stabilize UPO of 1-period and 2-period. This shows that it is thought that the proposed method works on dynamical systems with jumping characteristics. In addition, we propose PDFC. This method is

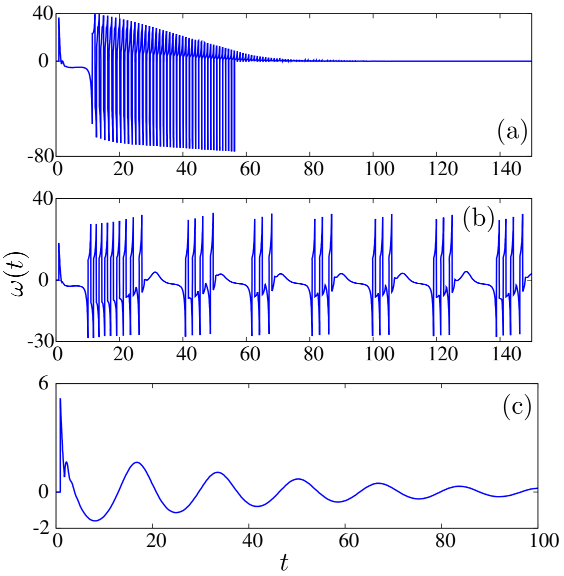

Figure 9: transition of controlled variable of Fig. 7 (a):$k = 0.5$, $(v_0,\ u_0) = (-56.0, -50.0)$ (b):$k = 1.0$, $(v_0,\ u_0) = (-56.0, -50.0)$ (c):$k = 1.5$, $(v_0,\ u_0) = (-56.0, -110.0)$

divided period in which case UPO that we want to stabilize is more than one period, and controlled during adopted arbitrarily period. As a result, range that can be stabilized runs or we could not stabilize by adopting period. We must pick K by trial and error. However, in case of PDFC, it is important that we adopt period as controlled systems.

Acknowledgment

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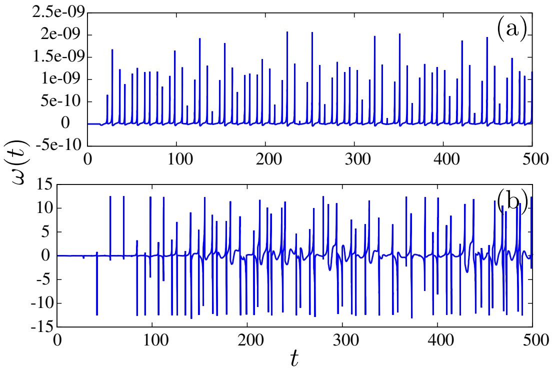

Figure 10: DFC:transition of controlled variable (a):$k = 0.155$, $(v_0,\ u_0) = (-56.0, -108.062467)$ (b):$k = 0.155$, $(v_0,\ u_0) = (-56.0, -108.064467)$

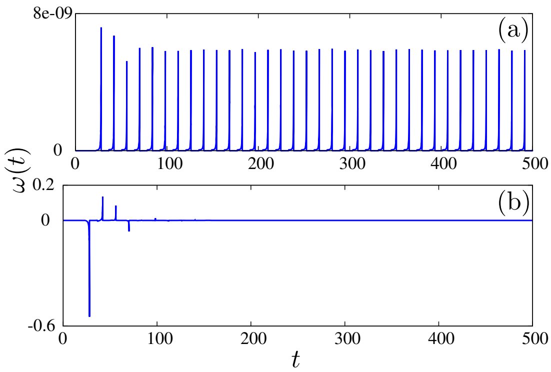

Figure 11: PDFC:transition of controlled variable (a):$k = 0.155$, $(v_0,\ u_0) = (-56.0, -108.062467)$ (b):$k = 0.155$, $(v_0,\ u_0) = (-56.0, -108.064467)$

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