2010 International Symposium on Nonlinear Theory and its Applications
NOLTA2010, Krakow, Poland, September 5-8, 2010

Analysis of Several Spatio-Temporal Phase Patterns in Coupled Chaotic Maps by Varying Coupling Strength

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Abstract—Coupled chaotic map systems are attracted as a good model for representation of several phenomena in the real world. In the previous studies related with several coupled network models, the value of the coupling strength was almost a constant value and fixed. In this study, the value of the coupling strength is varying in dependence on state which is provided by a Gaussian function. Several spatio-temporal phase patterns by their complex dynamics could be confirmed. Two types of coupled network system were considered, and pattern dynamics was investigated.

1. Introduction

Pattern dynamics and mechanism of organization in several complex system attract many researchers' attention as a good model which can realize the complicated phenomena in the real world. Coupled chaotic system and its dynamics can yield a wide variety of complex and strange phenomena. The coupled systems existing in nature exhibit great variety of phenomena such as complex mechanisms for all of the systems in the natural fields or in the universe. These phenomena can be found in a metabolic network, a human society, the process of a life, self-organization of neuron, a biological system, an ecological system and so many nonlinear systems. Among the studies on such coupled systems, many interesting researches relevant to the spatio-temporal chaos phenomena on the coupled chaotic systems have been studied until now, e.g. mathematical model in one- or two-dimensional network investigated earnestly by Kaneko[1]-[3], and found in physical circuit model[4]. Moreover, research of complicated phenomena and emergent property in the coupled cubic maps on 2-dimensional network system has been also reported[5]. The studies of coupled map lattice (CML), globally coupled maps (GCM) and so many studies related with such complex systems provided us tremendous interesting phenomena. We had also reported the research on spatio-temporal phase patterns in coupled maps using a fifth-power function[6]-[8], in which it has been carried out in the unique case. We had reported a research for complex network by non-uniform coupling strength as one of examples[9]. However many coupled chaotic systems have wide variety of features and furthermore its dynamics is also expected to be applied much engineering applications, there are many problems which should be solved in large scale coupled network systems by their complexity.

In this study, analysis of several spatio-temporal chaotic behavior in coupled chaotic maps with varying coupling strength by state of neighbors will be presented. The chaotic map which has been governed by a third power polynomial function is properly selected as a chaotic cell. We consider the model which chaotic cells are mutually connected to neighbors as a ring or 2-dimensional network with an arbitrary coupling strength. In the almost previous studies related with CML and GCM, the value of the coupling strength was a unique value and fixed, and also using the same coupling strength. In this study, contrary to the previous them, we adopt a value of the coupling strength which is provided in dependence on each state by a Gaussian function as a non-uniform network. Several phase patterns made from complex dynamics will be shown. Then, we show some phenomena which spatio-temporal chaos, complex behavior and several phase patterns can be confirmed in the proposed coupled systems.

2. Chaotic Maps

Chaotic maps are generally used for several approaches to investigate complex dynamics and several phenomena on coupled network systems. Especially, the logistic map and other types of chaotic maps such as a cut map, a circle map, a tent map, a cubic map are well known and popular. Obviously, it is necessary to have a lot of equilibrium points with the complex phenomena that corresponds to the natural world. Let us consider an n-th order polynomial function. The n-th order polynomial function is normally written as follows.

$$f(x) = \sum_{k=1}^{n} a_k x^k \tag{1}$$

where a_k is the characteristic parameter which can determine for their chaotic feature. If it is needed to adopt the map with respect to the origin, odd-numbered coefficients a_k are only set suitable values in (1). In other words, even-numbered coefficients are set as all zero. Then, we can easily confirm that it generates chaos in this function.

In this study, we consider a simple network using the chaotic map as a subsystem. We use the cubic map as the chaotic subcomponent in the following.

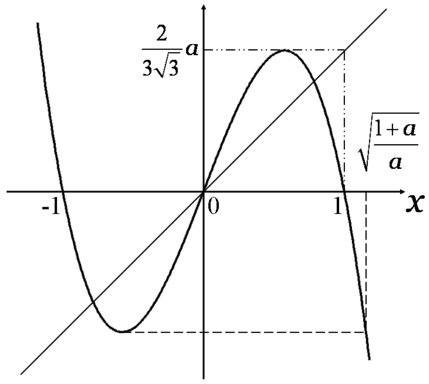

Figure 1: Diagram of a cubic map for $a = 2.6$

$$x(t+1) = a_3 x(t)^3 + a_1 x(t) \tag{2}$$

where a_k is a parameter which can determine the strength of nonlinear characteristic. We can easily confirm that chaos generates in this subsystem.

In order to simplify, consider the parameters $-a_3$ and a_1 is the same value a, then we hereafter use the cubic map in the following.

$$x(t+1) = ax(t)\left(1 - x(t)^2\right) \tag{3}$$

It is well known that we can confirm *a crisis of chaos*, if the parameter a is greater than around 2.6, which it is obtained from Fig. 1. Hence the chaotic map can move both positive and negative area when the parameter a is larger than a rigorous value $3\sqrt{3}/2(\simeq 2.6)$. Further, if $a > 3$, i.e. $\frac{2a}{3\sqrt{3}} > \sqrt{\frac{1+a}{a}}$, we can also confirm that the system diverges to infinity.

In order to evaluate the function (1), Lyapunov exponent can be calculated by the following formula.

$$\lambda = \lim_{N\to\infty} \sum_{k=1}^{N} \log \left| \frac{df(x)}{dx} \right| \tag{4}$$

Lyapunov exponent is a very important measurement often used to show the existence of chaos. Lyapunov exponents with bifurcation diagram by changing one parameter a for Eq. (3) are shown in Fig. 2. These are typical results which can be obtained from computer calculation. In case of polynomial functions, period doubling and tangent bifurcation can be confirmed. Therefore chaotic maps possessing several equilibrium points can yield various wide interesting behavior.

3. Several Phase Patterns in Coupled Chaotic Maps

In this section, we consider two types of coupled chaotic network system as shown in Fig. 3, which each cell means a chaotic map as a subsystem of the network. It can be considered easily that coupled chaotic systems have wide variety of phase patterns or spatio-temporal features. The term

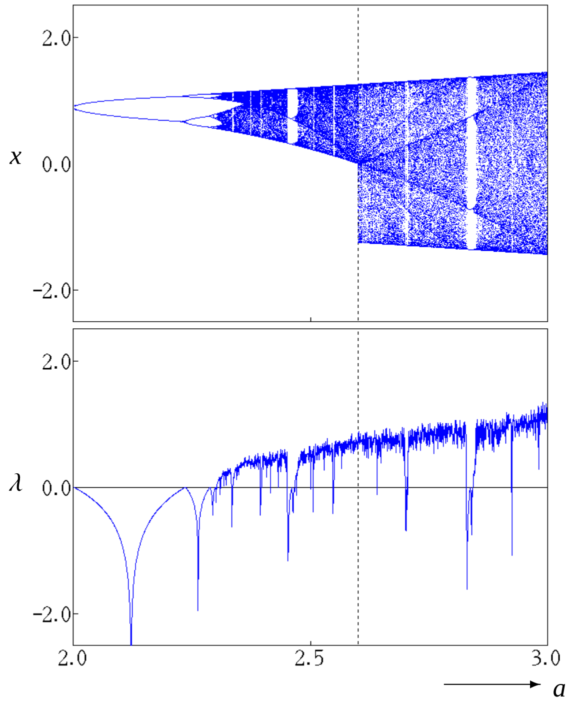

Figure 2: Bifurcation diagram and Lyapunov exponent for changing a

"spatio-temporal" is extensively used for irregular dynamical behavior observed from large scale complex systems of the relevant to both time and space.

In order to confirm spatio-temporal phenomena or phase patterns, consider a coupled model of the chaotic maps which are connected to neighbors. Each chaotic cell is connected to neighbors by arbitrary coupling strength ε. The whole system of CML is represented as

$$\begin{aligned} x_k(t+1) = \; & (1-\varepsilon) f\left(x_k(t)\right) \\ & + \frac{\varepsilon}{2}\left(f\left(x_{k-1}(t)\right) + f\left(x_{k+1}(t)\right)\right), \\ & \qquad (k = 1, 2, \cdots, N) \end{aligned} \tag{5}$$

where t is an iteration, k is an index number of the cell which follows the cyclic rule, and N is a size of coupled cells, respectively.

On the other hand, the 2-dimensional network system is represented as

$$x_{ij}(t+1) = (1-\varepsilon) f\left(x_{ij}(t)\right) + \frac{\varepsilon}{4} \sum_{kl\in\Xi} f\left(x_{kl}(t)\right) \tag{6}$$

where $\{i, j\}$ is an index number of cell, and Ξ means four neighbor cells.

In the previous studies related with CML network, the coupling strength is used to be a constant and an unique value. In this study, we propose the coupled chaotic system model which the coupling strength will be used in the following Gaussian function.

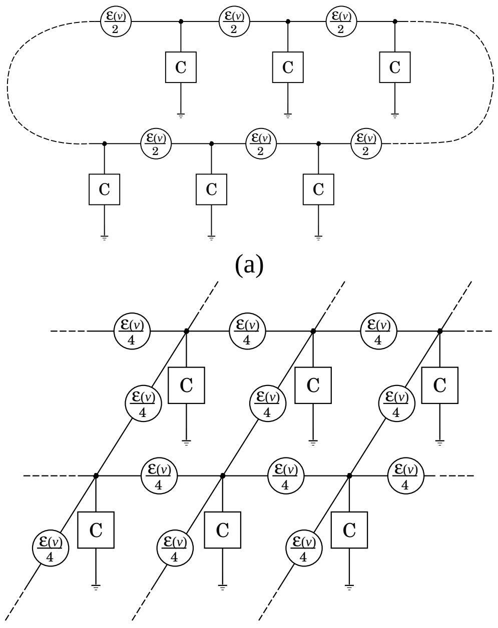

Figure 3: Coupled chaotic network

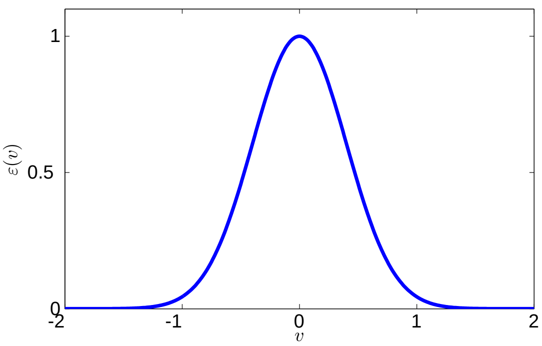

Figure 4: Nonlinear coupling strength $\varepsilon(v)$ for $\gamma = 1.0$ and $\sigma = 0.4$

$$\varepsilon(v) = \gamma e^{-\frac{v^2}{2\sigma^2}} \tag{7}$$

where $\varepsilon(v)$ is the coupling strength changing by their state v, in which it moves monotonously between 0 and γ. The diagram of Eq. (7) is shown in Fig. 4.

Then, the value of the coupling strength ε in Eqs. (5)–(6) is replaced to the function $\varepsilon(x)$ with $\gamma = 1$ and $\sigma = 0.4$. Thus, every time the coupling strength is changed by their state.

We show some numerical simulation results obtained from two types of coupled systems. First, we consider that the number of cells N is as 100 in Eq. (5) of a CML type network. Fig. 5 shows some simulation results of time-waveform for some cases with varying coupling strength. As we can see, a lot of interesting phenomena were confirmed though all the results cannot be represented more here. In general, it almost tends to become random or disorder phase patterns because the coupling strength depends on their state.

Similarly, we consider a 2-dimensional network system which each cell is coupled to four neighbors. Figure 6 shows some simulation results obtained from 2-dimensional system (6) at time $t = 1000$. This figure indicates a grade of synchronous state for phase difference with an average of four neighbors, which is illustrated with gray scale monotone colors between white □ and black ■ correspond to synchronous and asynchronous state, respectively. We can confirm that self-organizing formation advances gradually as the parameter grows. Although we cannot present all the simulation results, several phase patterns and spatio-temporal phenomena from its complex dynamics will be observed in such coupled systems.

4. Conclusions

In this study, we considered the coupled network using a cubic map as a chaotic cell, and investigated their dynamics. Some computer simulation results of spatio-temporal chaotic behavior several phase patterns in the coupled chaotic maps for ring and 2-dimensional network systems had been shown. Varying coupling strength provided by a Gaussian function is very important to solve several features in the real world. We consider that the coupled chaotic system is also a good model as a stochastic model such as natural patterns, self-organization, and so on. Furthermore, we would like to use these pattern dynamics as a stochastic model for several moving robots in near future.

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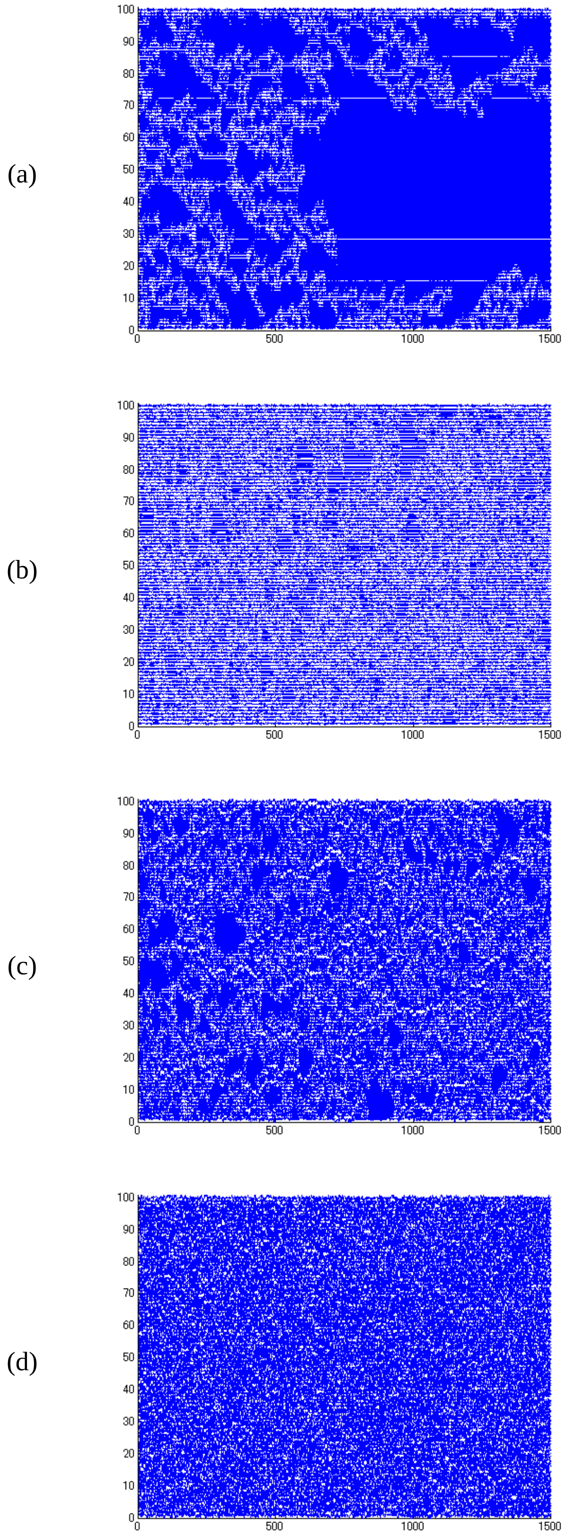

Figure 5: Some simulation results of time-waveform with non-uniform coupling strength $\varepsilon(v)$ for $\sigma = 0.4$. Horizontal axis is a time t, and vertical axis corresponds to waveform of each cell x_k. (a) $a = 2.60$, $\gamma = 0.6$, (b) $a = 2.58$, $\gamma = 1.0$, (c) $a = 2.60$, $\gamma = 0.6$, and (d) $a = 2.88$, $\gamma = 0.6$

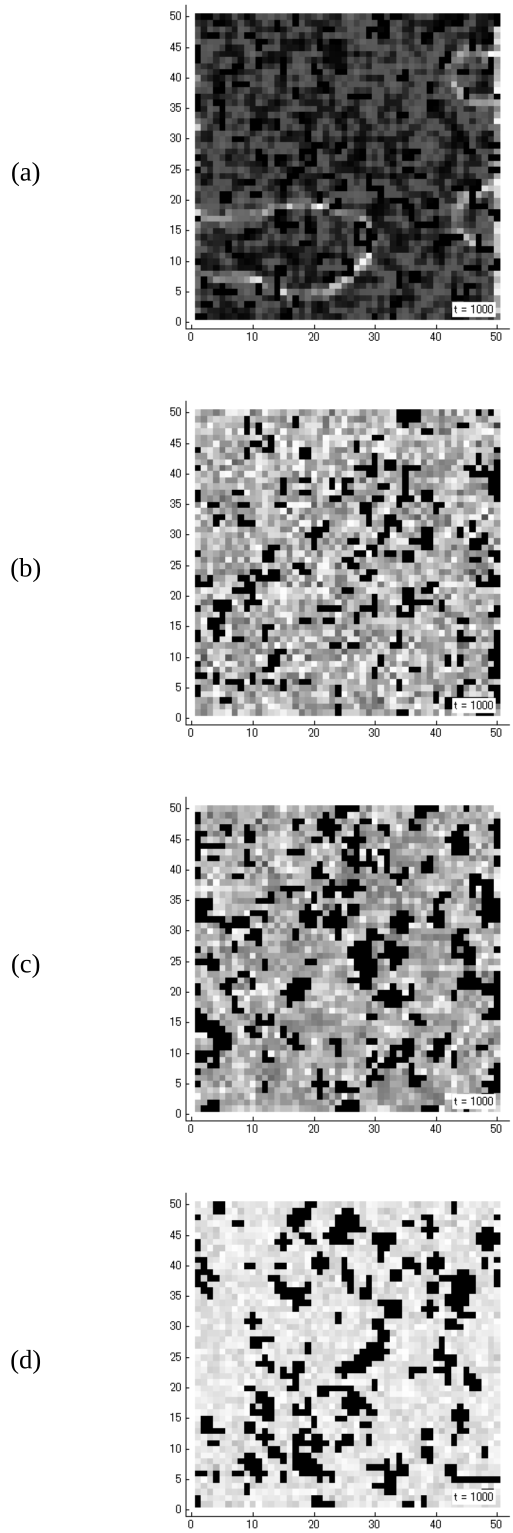

Figure 6: Snapshots of some simulation results of 2-dimensional 50×50 network at $t = 1000$. (a) $a = 2.60$, $\gamma = 0.60$, $\sigma = 0.20$, (b) $a = 2.59$, $\gamma = 1.0$, $\sigma = 0.40$, (c) $a = 2.69$, $\gamma = 0.80$, $\sigma = 0.40$, and (d) $a = 2.88$, $\gamma = 0.60$, $\sigma = 0.80$