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Interactions between Two Nonlinear Localized Oscillations in an Articulated Structure

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Abstract– This paper considers interactions between two localized oscillations of the same amplitude in a spatially periodic and articulated structure. It consists of identical rigid members which are connected to adjoining ones under the action of nonlinear restoring moments. The members extend straight in equilibrium and the displacements are restricted in a plane. The numerical solutions to the governing equations with free boundary conditions at both ends and the appropriate initial conditions show that intrinsic localized modes (ILMs) which have two movable localized oscillations can be excited and that they interact with each other.

1. Introduction

The ILMs or discrete breathers (DBs) are known as stable localized oscillations that occur in nonlinear, spatially discrete and perfectly periodic systems and now many investigations are still being made with theoretical and experimental interests, for example [1-4]. We proposed a simple model of a periodic and articulated structure for real mechanical systems [5]. It consists of a *finite* number of *rigid* members with couplers which yield hard nonlinear restoring moments against rotations. It should be noted the structure is periodic locally but not globally because of existence of both ends. Flexural (transverse) motions are of primary importance in this structure rather than longitudinal ones unlike as in lattice dynamics. Our calculations have shown that for adequate initial conditions ILMs occur in the structure [5]. It has been also shown that for asymmetrical initial conditions the localized oscillation generally moves in the structure [6]. These facts suggest if two or more localized oscillations could be simultaneously excited in the structure, they would interact with each other.

In this paper we examine the interactions between two localized oscillations of the same amplitude and symmetrical with regard to the center of structure.

2. Model for Analysis

The structure consists of N (>>2) identical, long and rigid members, such as beams or panels, which are of length l and of uniform density ρ per unit axial length, and connected to adjoining ones by couplers giving nonlinear restoring moment (Fig. 1). Here one member with

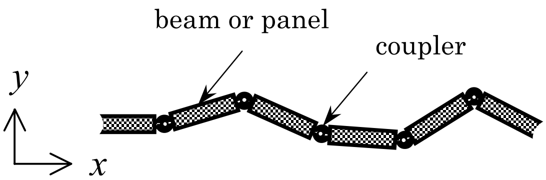

Fig. 1 Articulated structure of rigid members.

couplers on both ends is called one unit. Supposing that the number of the units N is large but finite, they are numbered consecutively by integer j ($1 \le j \le N$) and physical variables pertaining to the j th unit are denoted by attaching subscript j. Motions of the structure are restricted in the x-y plane where the x-axis is taken along the structure in equilibrium. In the j th unit, the position of the center of mass of the unit is denoted by $(x_j(t), y_j(t))$ and the angle of the centerline to the x-axis is denoted by $\phi_j(t)$, t being the time. Suppose that the restoring moment (torque) M_j, which is affected by the coupler at the left end of the j th unit, is given by a linear plus cubic function of difference in angle between two centerlines of the adjacent units in the following form:

$$M_j = K(\phi_j - \phi_{j-1}) + K_C(\phi_j - \phi_{j-1})^3,$$

for $2 \le j \le N$, K and K_C being positive constant.

Equations of motions are easily derived by applying the Newton's law of motions to each unit: the equations for translation and rotation about the center of mass. The conditions for continuity of displacement, i.e., for geometrical constraint are required on each junction. The variables in these equations and conditions are normalized as follows:

$$x_j \to l x_j,\ \ y_j \to l y_j \ \text{ and } \ t \to (\rho l^3 / K)^{1/2} t$$

with a parameter of nonlinearity $\kappa = K_C / K$ (> 0) [5, 6]. Both ends of the structure are assumed to be free. The total energy of the system is conserved of course.

3. Numerical Analysis

3.1. Mobile ILMs

The simultaneous equations derived in the previous papers are solved numerically [5, 6]. Initial conditions are chosen in such a way that the values for ϕ_j are given in the form of the π mode (zigzag mode with the shortest

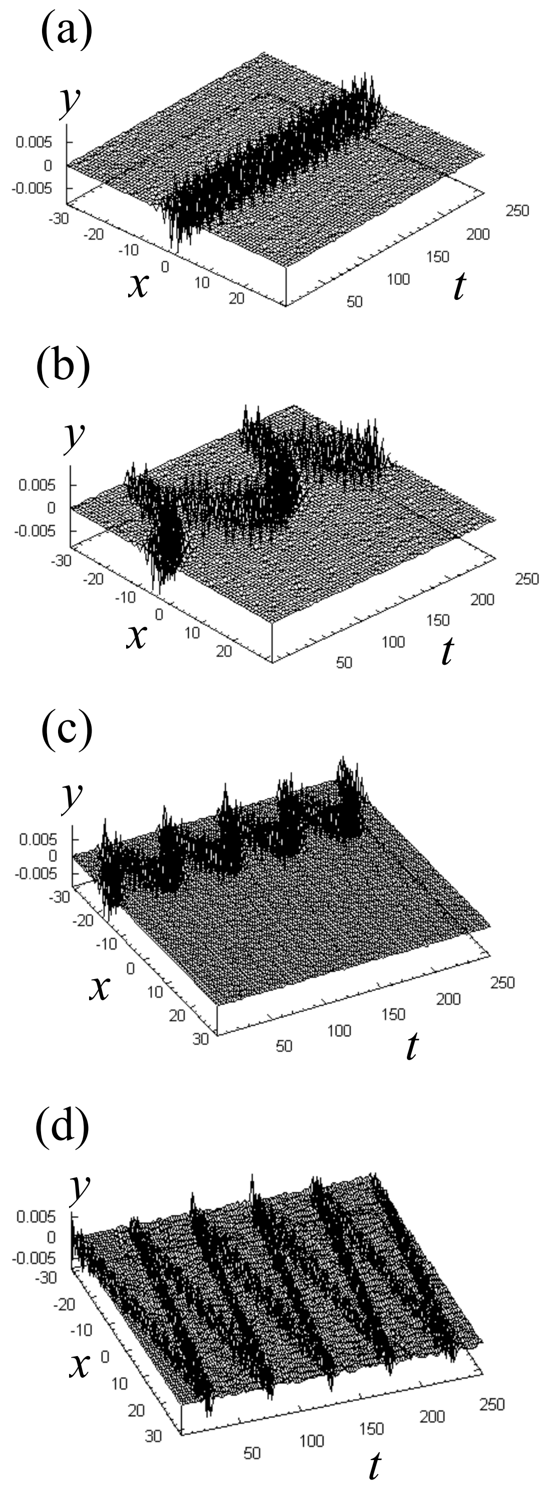

Fig. 2 Mobile ILMs in the structure for (a) $\sigma = 0$, (b) $\sigma = 4$, (c) $\sigma = 16$ and (d) $\sigma = 32$.

wavelength) whose envelope is modulated in the form of a pulse as

$$\phi_{\mathrm{j}} = (-1)^j A\,\mathrm{sech}[\alpha(j - c - \sigma)]$$

for $1 \le j \le N$, where A, α and σ are constants, c (= N/2 + 1/2) is the center of structure and σ ($0 \le |\sigma| \le N/2$) indicates the offset of the center position of modulation from c. The initial positions, $x_{\mathrm{j}}(0)$ and $y_{\mathrm{j}}(0)$, are determined by the conditions of constraints if one set of the values of $\phi_{\mathrm{j}}(0)$ for all j is prescribed. All initial velocities are taken to vanish, so that the system has no angular momentum, i.e. it does not rotate as a whole. We have already revealed that in the case $\sigma \neq 0$ the localized oscillations generally move in the structure depending on the value of $|\sigma|$, whereas they remain stationary at the center in the case $\sigma = 0$. Some typical solutions are displayed in Fig. 2 for N = 64, κ= 1400, A = π/180 and α = 0.6. Figure 2 (a) is a special case, compared with the cases (b) and (c), and shows a stationary type of ILM. Figures 2 (b) and (c) show the most general pattern in motion for the cases that $\sigma \neq 0$: excited localized oscillations are set in motion from their initial positions, head for their nearby end immediately and repeat reflecting there. (After bounding many times, they will be finally trapped there.) Figure. 2 (d) shows the special pattern. The localized oscillations continue to be propagating over the structure at a constant speed and reflecting at both ends repeatedly. This pattern appears in the case that the localized oscillations are excited at or very close to the one end, i.e. $|\sigma| \approx N/2$.

Because the ILMs are mobile, we study interactions among two or more localized oscillations excited in the structure. In the following sections, we will devote ourselves to the studies of interactions between two localized oscillations of the same amplitude and symmetrical with respect to the center of structure.

3.2. Interactions between two localized modes of oscillations

To excite ILMs with two localized oscillations of the same amplitude and symmetrical configuration as a whole, we adopt an initial condition in the following form:

$$\phi_{\mathrm{j}} = (-1)^j A\{\mathrm{sech}[\alpha(j - c + \sigma)] + \mathrm{sech}[\alpha(j - c - \sigma)]\}$$

with zero velocities for all j. For the same values of the parameters as the ones in previous section, the equations are solved by the standard 4th-order Runge-Kutta method, monitoring the total energy to be conserved.

Typical solutions are displayed in Fig. 3. It is seen that two identical localized oscillations are excited and they propagate with respective patterns depending on the value of σ. The configurations of structure maintain the line symmetry with respect to the center and the localized modes oscillate in phase with each other. For small value of σ (σ = 4), the two excited localized oscillations immediately come close to each other, coalesce at the center of structure and come to behave as a single stationary mode of oscillations (Fig. 3 (a)). For σ= 16, the two localized oscillations remain apart from each other and behave independently without any interactions (Fig. 3

(b)). For the special case that $\sigma = N/2$, it is seen that the two localized oscillations excited far away propagate toward the center to collide and pass through each other (Fig. 3 (c)). They continue to be propagated subject to the interactions at the center and reflections at the ends. Close observations show that at the center the two localized modes pass through transparently each other.

4. Conclusions

The interactions between two localized oscillations whose envelopes have the same amplitude and are symmetrical with respect to the center of the structure have been numerically studied. It is revealed that the interactions between the ones oscillating in phase are "attractive" and, as a result, they coalesce or pass through transparently each other.

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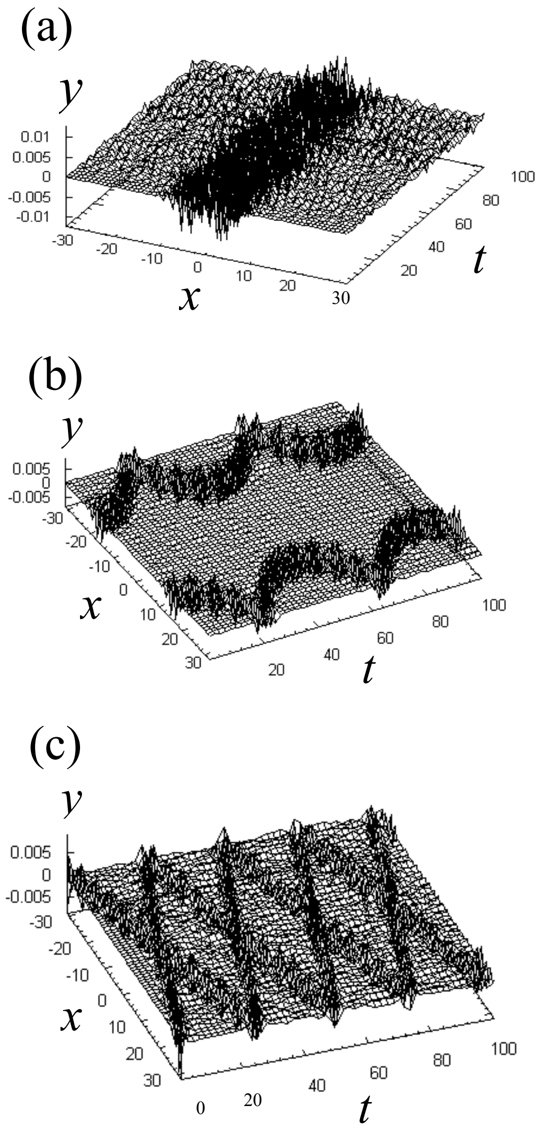

Fig. 3 Interactions between two identical localized oscillations for (a) σ= 4, (b) σ= 16 and (c) σ= 32.