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Bifurcation Analysis of Coupled Dictyostelium Oscillators

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Abstract—In recent years, a new rhythm synchronization has been reported. The generalization of the function for receptors in the Dictyostelium by a nonlinear coupled oscillators realizes the synchronization. After that, this phenomena were investigated qualitatively by the averaging method. However, a relationship between synchronization modes and bifurcation phenomena has not been studied. In this paper, we feature van der Pol equation into Nagano's model, and investigate bifurcation phenomena of the periodic solution and classify synchronization modes. We show bifurcation diagrams and some typical attractors.

1. Introduction

A synchronization is a research field that has been studied for many years and many research results[1, 2, 3, 4, 5] have been reported . Synchronization is observed everywhere, e.g., population luminescence of fireflies in nature or the workings of the human body such as brain waves. Then, in these years, new and very powerful synchronization method has been found from the internal dynamics of Dictyostelium[6, 7, 8, 9, 10]. Chemical substance called cAMP is secreted into extracellular to communicate between cells when Dictyosteliums are aggregated. However, aggregation of Dictyosteliums with each other is in time of starvation, and Dictyostelium try to save as much as possible cAMP. Thus, biological sensors to adjust the concentration of cAMP in Dictyostelium, called receptors, determine the concentration of extracellular cAMP. Receptors stop production of cAMP when concentration of extracellular is too high. Furthermore, receptors promote production of cAMP when concentration of extracellular is too low. In that way, receptors have been adjusted to a suitable concentration of cAMP. This function of receptors causes synchronization between cells. And, This synchronization is formulated as follows[7][8]:

$$\frac{dx_j}{dt} = X_j(x_j,\ y_j),\quad \frac{dy_j}{dt} = Y_j(x_j + \gamma_j \sum_l x_l,\ y_j), \tag{1}$$

where $j = 1, 2, \ldots, N$ is the number of coupled cells, P_j is product concentration of cell j and R_j is receptor activity. In addition, $\gamma > 0$ is a coupling factor. This formulation is a nonlinear coupling between oscillators.

In this paper, we apply two van der Pol oscillators into X_j and Y_j in Eq. (1), and investigate bifurcation phenomena. As a result, we have got a bifurcation diagram explaining synchronization modes of the system.

2. Bifurcation analysis

The coupled van der Pol oscillators are expressed by the following equations. They are the similar as that has been used in the paper [7],[8] and [11] .

$$\begin{cases} \dfrac{dx_j}{dt} = y_j = X_j(x_j,\ y_j), \\ \dfrac{dy_j}{dt} = -\omega_j^2 x_j + \epsilon(1 - x_j^2)y_j = Y_j(x_j,\ y_j), \end{cases} \tag{2}$$

where ω_j is the fundamental frequency of j-th oscillators, $\epsilon > 0$ is nonlinearity. The equations that two van der Pol oscillators by nonlinear coupled are expressed by the following equations corresponding to Eq. (1).

$$\begin{cases} \dfrac{dx_1}{dt} = y_1, \\ \dfrac{dy_1}{dt} = -\omega_1^2\{x_1 + \gamma(x_1 + x_2)\} \\ \qquad\quad +\epsilon[1 - \{x_1 + \gamma(x_1 + x_2)\}^2]y_1, \\ \dfrac{dx_2}{dt} = y_2, \\ \dfrac{dy_2}{dt} = -\omega_2^2\{x_2 + \gamma(x_1 + x_2)\} \\ \qquad\quad +\epsilon[1 - \{x_2 + \gamma(x_1 + x_2)\}^2]y_2. \end{cases} \tag{3}$$

From now, we will explain the procedure of bifurcation analysis. Firstly, we consider an n-dimensional autonomous system described as Eq. (4).

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{f}(\boldsymbol{x}, \lambda),\ t \in \boldsymbol{R},\ \boldsymbol{x} = (x_1, x_2, \ldots, x_n) \in \boldsymbol{R}^n \tag{4}$$

where λ is a parameter. The solution of Eq. 4 is described as follows:

$$\boldsymbol{x}(t) = \varphi(t, \boldsymbol{x}_0, \lambda),\ \boldsymbol{x}(0) = \boldsymbol{x}_0 = \varphi(0, \boldsymbol{x}_0, \lambda) \tag{5}$$

Assume that Eq. (4) has a limit cycle with the period L. Then, the Poincaré section is defined as follows:

$$\Pi = \{\boldsymbol{x} \in \boldsymbol{R}^n \mid q(\boldsymbol{x}) = 0\},\ q : \boldsymbol{R}^n \to \boldsymbol{R},\ \boldsymbol{x} \mapsto q(\boldsymbol{x}). \tag{6}$$

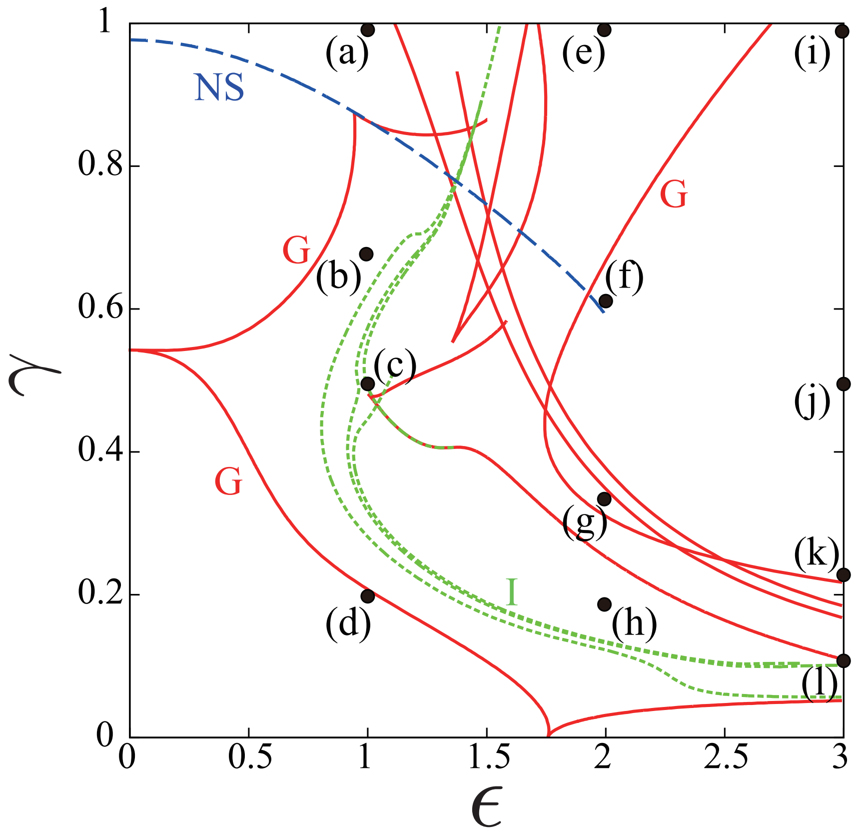

Figure 1: Bifurcation diagram of nonlinear coupled van der Pol oscillators

A condition of a periodic solution can be defined as follows:

$$\varphi(L, \boldsymbol{x}_0, \lambda) - \boldsymbol{x}_0 = 0. \tag{7}$$

Second, we consider the condition that local bifurcations occur. Since it occurs when the magnitude of multiplier is unity, we obtain multiplier. Characteristic equation for obtaining multiplier is as follows.

$$\chi(\mu) = \det\left(\left.\frac{\partial \varphi}{\partial \boldsymbol{x}_0}\right|_{t=L} - \mu I \right) = 0 \tag{8}$$

where μ is a multiplier. $\partial\varphi/\partial\boldsymbol{x}_0$ of Eq. (8) is a Jacobian matrix obtained by numerical integration of the variational equation from the initial value. Bifurcations can be computed by solving a two-point boundary value problem composed by coalition Eq. (7) and Eq. (8). Local bifurcation phenomena are tangent bifurcation, period-doubling bifurcation and Neimark-Sacker bifurcation.

3. Numerical

We will show results of bifurcation analysis. Figure 1 is the bifurcation diagram of nonlinear coupled oscillators Eq. (3). Solid lines are tangent bifurcations(G), dotted lines are period-doubling bifurcations(I) and long dashed lines are Neimark-Sacker bifurcation(NS). Figure 2 is the enlarged diagram of Fig. 1. A typical fish hook structure is found in Fig. 2.

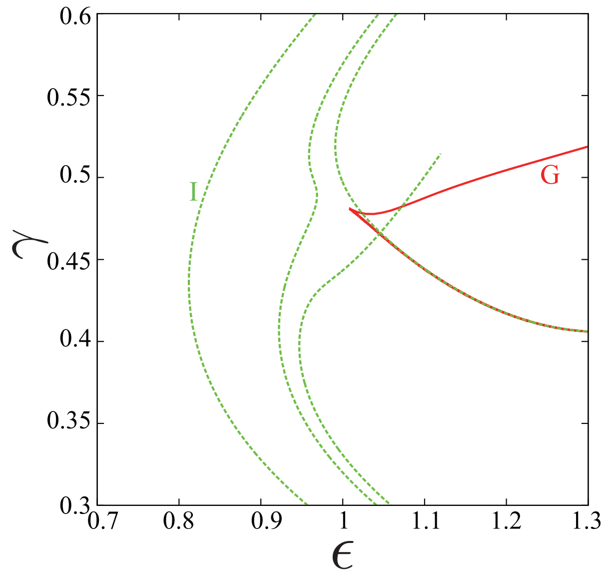

Figure 2: Fish hook structure

Figure 3–5 show attractors at (a)–(l) in Fig. 1. Figure 3 shows attractors at (a)–(d) which are $\epsilon = 1.0$. It is considered that the attractor in Fig. 3(a) has a tendency toward the

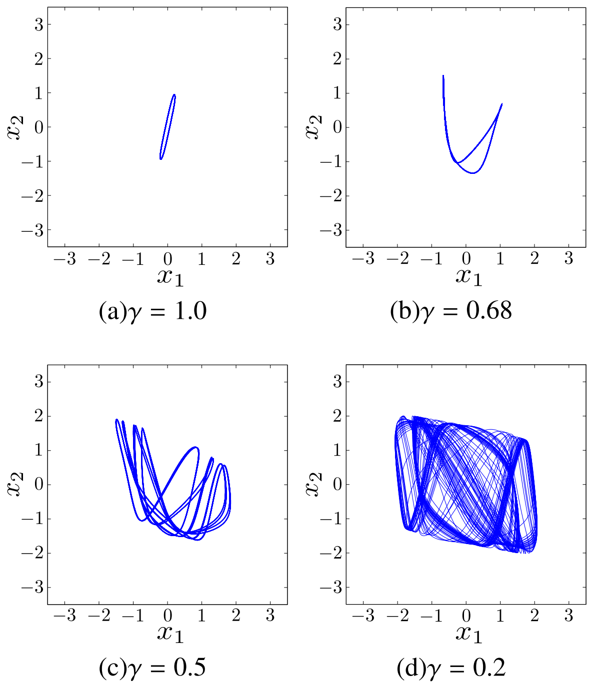

Figure 3: Attractors(ω_1 = 0.9, ω_2 = 1.6, ϵ = 1.0)

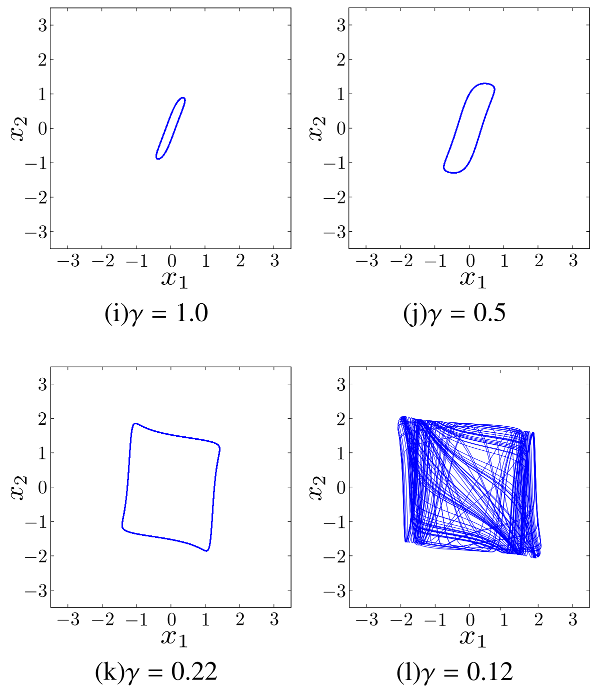

Figure 5: Attractors(ω_1 = 0.9, ω_2 = 1.6, ϵ = 3.0)

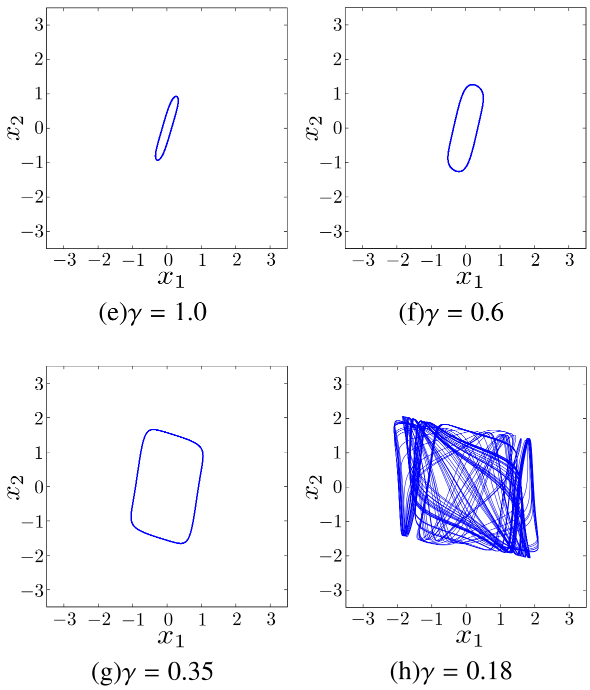

Figure 4: Attractors(ω_1 = 0.9, ω_2 = 1.6, ϵ = 2.0)

in-phase synchronization. If the parameter γ varies from (a) to (d), attractor shows the change in the anti-phase synchronization from the in-phase one. Figure 4 shows attractors at points(e)–(h) in Fig. 1. Changes in attractors of Fig. 4 are similar to that of Fig. 3. However, changes in attractor begin with smaller values of ϵ than Fig. 3. Figure 5 shows attractors at points(i)–(l) in Fig. 1. As with Fig. 4, changes in attractors of Fig. 5 is similar to that of Fig. 3. However, change is observed from the value of the smaller ϵ than Fig. 4. It can be concluded from the above observations that when the nonlinearity ϵ is increased, a coupling factor γ is shown a tendency toward the in-phase synchronization in smaller value. In order to estimate quantitatively the tendency toward the synchronization mode, we study the correlation coefficient of each others oscillators. The correlation coefficient takes a real value and ranges from −1 to 1, and can quantized the relationship of two waves. There is a positive and negative correlation if this value is close to 1 and −1, respectively. Also, the correlation is weak when it is close to 0. The correlation coefficient is shown as following:

$$\frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \overline{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \overline{y})^2}}, \tag{9}$$

where $\overline{x}$ and $\overline{y}$ are arithmetic means of each x and y. In this paper, we obtain a correlation coefficient of x_1 and x_2.

Figure 6 shows the correlation coefficient by varying γ. Weak negative correlation is observed when γ is small, but strong positive correlation is observed when γ is large. Thus, it can be said that attractor is changed to the in-phase synchronization from the anti-phase synchronization when γ is increased. In addition, if strong positive correlation is appeared quickly if value of ϵ is increased.

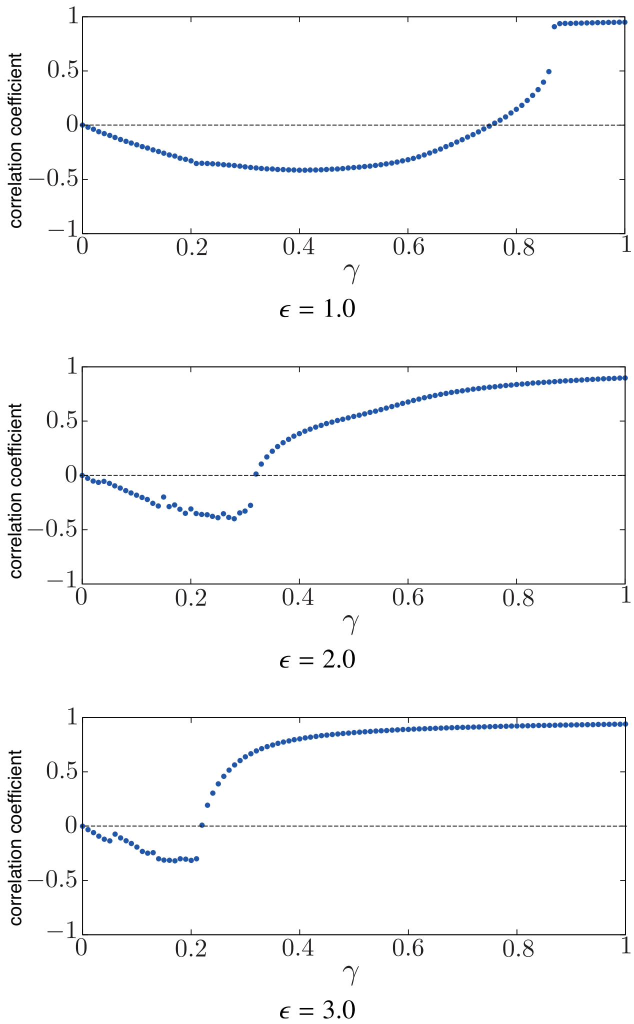

Figure 6: Correlation coefficient

4. Conclusion

We have investigated the bifurcation structure of two van der Pol oscillators that is connected by a non-linear coupling, and discussed a relationship the bifurcation and synchronization modes. From these results, synchronization modes of coupled oscillator are strongly depend on the coupling factor γ. Additionally, it is confirmed that the parameter γ, where oscillators are changed to the in-phase synchronization mode, becomes small value in the case that the nonlinearity ϵ is employed as the large value.

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