Nonlinear Localized Oscillations Excited in Forced Mass-Spring Chains

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Abstract– Localized oscillations in finite mass-spring chains driven sinusoidally at one end with the other fixed are studied numerically. The restoring force between neighbouring masses is assumed to be given by a piecewise-linear function of relative displacement and is anti-symmetric with respect to equilibrium point. Linear damping proportional to the velocity of the mass is taken into account. The mass at one end is forced to be displaced in the direction of the chains at a frequency slightly above the cut-off frequency of the linearized system. As the amplitude is large, localized oscillations are excited intermittently at the driving end and propagated down the chain at a constant speed.

1. Introduction

It is now known that the intrinsic localized modes (ILMs) or the discrete breathers (DBs) are generic in spatially periodic, discrete and nonlinear systems (see, for example [1-5].) It is also known that the moving ILMs can be excited numerically in a spatially semi-infinite system driven at one end sinusoidally at a frequency in a linear stopping band above the passing band [6-10].

In this paper, we consider finite mass-spring chains and study numerically forced oscillations at one end. Here it is assumed that all masses and springs are identical and the spring constant changes for a displacement greater than a threshold one. In other words, the restoring force is given by a piecewise-linear function of displacement and anti-symmetric with respect to equilibrium point (Fig. 1). This model may be regarded as a simple and good approximation to the Fermi-Pasta-Ulam model of beta type [11]. The linear damping is included in every chain. One end of the chains is forced to be displaced longitudinally and sinusoidally, where the other is fixed. We verify the existence of ILMs and study condition for excitation of ILMs and their properties by solving initial and boundary-value problems.

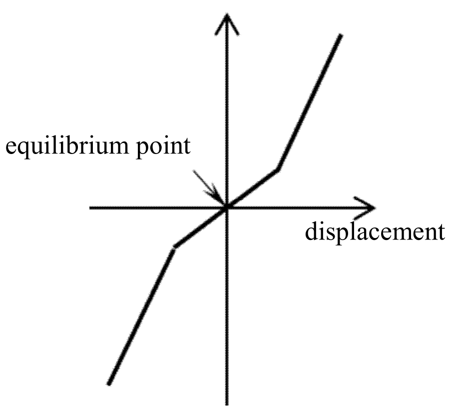

Figure 1: A piecewise linear force-displacement relation.

2. Numerical Analysis

Dynamical behaviors of the chains are described by N equations of motion for the masses and two boundary conditions. As is well known, there are N eignefrequecies in the linearized system, which lie below the cut-off frequency [11]. Letting the mass at one end be driven at a frequency above the cut-off frequency, we solve the equations of motions including the linear damping by the Runge-Kutta method.

The driving frequency is taken to be slightly higher than the cut-off one. It is found numerically that while the driving amplitude is small, the oscillations are evanescent and confined near the end (Fig. 2). As the driving

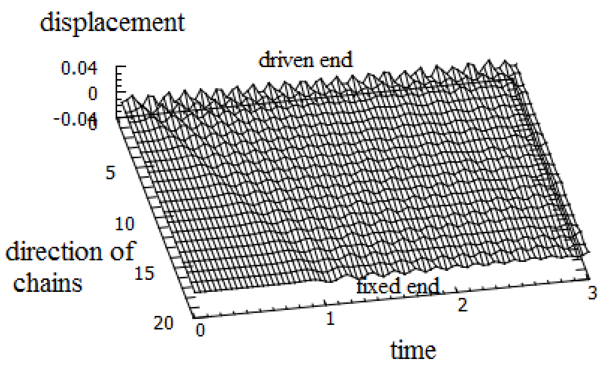

Figure 2: Spatial and temporal profile of the displacement confined near the driver due to the evanescent oscillations.

amplitude is larger and the displacement exceeds the threshold one at which the spring constant changes, the localized oscillations are excited intermittently at the driving end and propagated down the system at a constant speed (Fig. 3). When they hit the other end, they are reflected and propagated back and forth in the system, subject to nonlinear interactions between them (Figs. 4 and 5). The FFTs show that the frequencies of the oscillations lie in the linear stopping band. In this respect, the oscillations may be regarded as the moving ILMs.

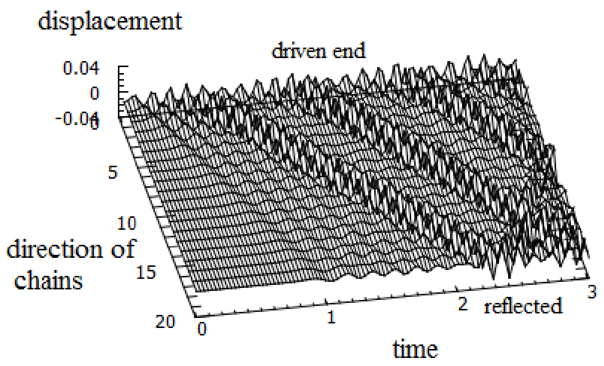

Figure 3: Spatial and temporal profile of the displacement in the mobile ILMs where localized oscillations are excited at the driver and are propagated down the system and reflected at the fixed end, subject to interactions each other.

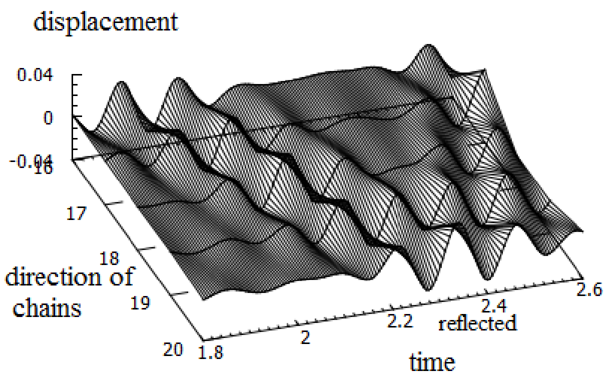

Figure 4: Reflection of the localized oscillations at the fixed end (blowup of Fig. 3).

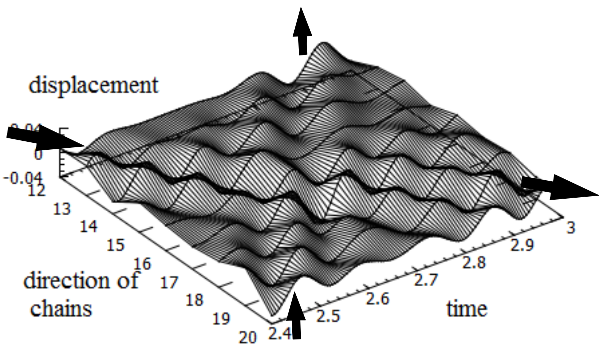

Figure 5: Interaction between two localized oscillations (blowup of Fig. 3).

3. Conclusions

Excitation and propagation of moving ILMs in the forced oscillations of the finite mass-spring chains with the restoring force which is a piecewise-linear function of the displacement have been examined numerically. No ILMs are generated if the displacement of the mass next to the driving end is smaller than the threshold one. In this respect, the present model is different from the FPU-β model, but it may be interesting to compare the ILMs in both cases and find differences, if any.

Acknowledgments

The authors thank Dr. K. Yoshimura for useful comments. This work was supported by JSPS KAKENHI Grant Numbers 24654124, 26400394.

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