

# Constructing a lattice model supporting highly mobile discrete breathers

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**Abstract**—A lattice model that supports highly mobile discrete breathers (DBs) is investigated. In various nonlinear lattices, DBs have mobility. However mobile DBs with a constant velocity are not always realized. We propose a *symmetric lattice* with respect to shifting a displacement pattern to one lattice spacing. It is shown that DBs in the proposed symmetric lattices travels much better than those in Fermi-Past-Ulam (FPU) type lattice from an initial condition with a standing DB with arbitrary perturbations.

## 1. Introduction

Discrete breathers (DBs) or intrinsic localized modes (ILMs) have extensively attracted in nonlinear physics since the first report by Sievers and Takeno [1]. DBs are spatial localized modes that are excited in nonlinear lattices. Recently, observations of DB in various physical systems [2, 3, 4, 5] have been reported. It is also expected that DBs play critical roles in physical processes such as structure change in crystals [6].

It is usually observed that the DB travels in various lattices. A center of the spatial localization moves from a lattice site to a neighboring site with vibration frequency out of phonon band. Although a mobile DB is easily obtained in a numerical simulation, theoretical understanding of mobile DB have not been clarified. Yoshimura has reported that mobile DBs have finite tail in Fermi-Pasta-Ulam- $\beta$  (FPU- $\beta$ ) lattice [7]. The finite tail of mobile DBs has also been reported in nonlinear Klein-Gordon [8] and Salerno lattice [9].

In Ref.[7], the relation between a symmetry of an interaction potential and existence of the tail of mobile DB has been pointed out. Moreover it has been reported that the mobile DB with a arbitrary velocity can be constructed in a four particle symmetric lattice that corresponds to a four particle FPU- $\beta$  lattice [10]. Therefore symmetric lattices have importance for better understandings of dynamics of mobile DBs. In the present paper, we propose a symmetric lattice with *N* particles. Then we show numerical results of mobile DB in the proposed symmetric lattice.

#### 2. Symmetric Lattice

Let us consider a N-particle nonlinear lattice system.

$$H = \sum_{n=1}^{N} \frac{p_n^2}{2} + \Phi(\mathbf{q}),$$
 (1)

where  $p_n$  is the linear momentum and  $\Phi(\mathbf{q})$  is the potential defined by  $\mathbf{q} = (q_1, q_2, ..., q_N)$  or positions of particles.

$$\Phi(\mathbf{q}) = \frac{1}{4} \sum_{n=1}^{N} (q_{n+1} - q_n)^4.$$
(2)

This is the FPU- $\beta$  lattice without linear interactions. Consider a variable transformation from the physical space  $\{q_n\}$  to the complex normal mode space  $\{U_m\}$ 

$$q_n = \frac{(-1)^n}{\sqrt{N}} \sum_{m=-N/2+1}^{N/2} U_m \exp\left(i\frac{2\pi m}{N}n\right).$$
 (3)

Since  $\Phi(\mathbf{U})$  is invariant with respect to a uniform shift, the total momentum is a first integral. Therefore, we can assume  $U_{N/2} = \dot{U}_{N/2} = 0$  and neglect them. Substituting (3) into (2), we obtain a potential in terms of  $\mathbf{U} = (U_{-N/2+1}, U_{-N/2+2}, ..., U_{N/2-1})$ ,

$$\Phi(\mathbf{U}) = \frac{1}{4N} \sum_{\substack{i,j,k,l=-N/2+1}}^{N/2-1} \omega_i \omega_j \omega_k \omega_l U_i U_j U_k U_l \Delta(i+j+k+l)$$
  
$$- \frac{1}{4N} \sum_{\substack{i,j,k,l=-N/2+1}}^{N/2-1} \omega_i \omega_j \omega_k \omega_l U_i U_j U_k U_l$$
  
$$\times (\Delta(i+j+k+l-N) + \Delta(i+j+k+l+N)),$$
  
(4)

where  $\omega_m = 2 \cos\left(\frac{\pi m}{N}\right)$  (m = -N, -N + 1, ..., N/2 - 1). The function  $\Delta(p)$  is defined as follows,

$$\Delta(p) = \begin{cases} 1 & \text{if } p = 0, \\ 0 & \text{others.} \end{cases}$$
(5)

Let us consider a map  $\mathcal{T}_{\lambda}$ :  $U_m \mapsto U_m \exp(-im\lambda)$  with a real parameter  $\lambda$ . This map with  $\lambda = \pi/N$  corresponds to shifting a displacement pattern to one lattice spacing and reversing the phase in the physical space. The first term



Figure 1: Strength of connection between *d*-th neighbor lattice sites. Dashed line indicates the approximated function  $b_d/b_1 = d^{-2}$ .

of Eq.(4) is symmetric with respect to the map  $\mathcal{T}_{\lambda}$ . The second term of Eq.(4), on the other hand, is asymmetric with respect to the map  $\mathcal{T}_{\lambda}$ .

We define a symmetric lattice that has potential  $\Phi$  containing only symmetric terms with respect to the map  $\mathcal{T}_{\lambda}$ . The symmetric lattice can be obtained by eliminating the asymmetric terms in the potential in terms of  $\{U_m\}$  of Eq.(4). Hamiltonian of the symmetric lattice in terms of U can be written as

$$H = \frac{1}{2} \sum_{m=-N+1}^{N-1} \dot{U}_m \dot{U}_{-m} + \Phi_{\rm s}(\mathbf{U}), \tag{6}$$

where

$$\Phi_{\rm s}(\mathbf{U}) = \frac{1}{4N} \sum_{i,j,k,l=-N/2+1}^{N/2-1} \omega_i \omega_j \omega_k \omega_l \\ \times U_i U_j U_k U_l \Delta(i+j+k+l).$$
(7)

However, it is difficult to obtain a physically reasonable lattice in physical space  $\{q_n\}$  in general, since inverse transformation of Eq.(3) might reproduce unphysical terms in  $\{q_n\}$ .

Let us consider a following lattice system for even N,

$$\Phi_{\rm l}(\mathbf{q}) = \frac{1}{4} \sum_{d=1}^{N/2} \sum_{n=1}^{N} b_d (q_{n+d} - q_n)^4.$$
(8)

where  $b_d$  indicates strength of connection between *d*-th neighbor lattice sites. We can obtain a physically reasonable type of symmetric lattice for this model by choosing an appropriate set of  $\{b_d\}$  of Eq.(8). Fig. 1 shows the strength of connection  $\{b_d\}$  with various *N*. In the case of small *d*,  $b_d$  is approximately given by

$$b_d = b_1 d^{-2}.$$
 (9)

It is also found that  $b_{N/2}$  converges a finite value that is function of *N*.



Figure 2: Temporal evolution of site energy of perturbed DB in (a)FPU type lattice (2) and (b)symmetric lattice (8).

# 3. Numerical Results

We perform numerical simulations of temporal evolution of a perturbed odd DB in the FPU-type (2) and the symmetric lattice (8) with N=128 as shown in Fig. 2. In the case of FPU-type lattice, we choose a set of parameters of  $b_1 = 1$ and  $b_d = 0$  ( $d \neq 1$ ). In the case of the symmetric lattice, we choose a set of parameters of  $b_d \neq 0$  for all indices in order to eliminate terms in the transformed potential  $\Phi_1(\mathbf{U})$ of Eq. (8), that is asymmetric with respect to map  $\mathcal{T}_d$ .

Perturbations are added to momentum of the neighboring sites of the site with the maximum amplitude. It is observed that the kicked DB in the FPU-type lattice looses its velocity and energy. This is because the mobile DB in the FPU-type lattice is affected by the discreteness of the lattice. On the other hand, the kicked DB in the symmetric lattice starts to traveling with a constant velocity. This result indicates that the proposed model support highly mobile DBs.

## 4. Conclusion

We propose the lattice model supporting highly mobile DB by considering symmetry of the interaction potential. The proposed model has all-to-all connections with a certain connection strength that eliminates asymmetric term with respect to rotation map in the complex normal mode coordinate. In numerical simulation, DB in the proposed lattice can move with a constant velocity even in the condition that the DB in the FPU lattice looses its velocity and energy. It is expected that the proposed model is a useful model for investigation on mobile properties of DBs.

# Acknowledgments

This work was partially supported by JSPS KAKENHI Grant Number 24560074.

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