Constructing a lattice model supporting highly mobile discrete breathers

Yusuke Doi† and Kazuyuki Yoshimura‡

†Department of Adaptive Machine Systems, Graduate School of Engineering, Osaka University
2-1 Yamadaoka, Suita, Osaka 565–0871, Japan
‡NTT Communication Science Laboratories, NTT Corporation
2-4 Hikaridai, Seika, Kyoto 619-0237, Japan
Email: doi@ams.eng.osaka-u.ac.jp, yoshimura.kazuyuki@lab.ntt.co.jp

Abstract—A lattice model that supports highly mobile discrete breathers (DBs) is investigated. In various nonlinear lattices, DBs have mobility. However mobile DBs with a constant velocity are not always realized. We propose a *symmetric lattice* with respect to shifting a displacement pattern to one lattice spacing. It is shown that DBs in the proposed symmetric lattices travels much better than those in Fermi-Past-Ulam (FPU) type lattice from an initial condition with a standing DB with arbitrary perturbations.

1. Introduction

Discrete breathers (DBs) or intrinsic localized modes (ILMs) have extensively attracted in nonlinear physics since the first report by Sievers and Takeno [1]. DBs are spatial localized modes that are excited in nonlinear lattices. Recently, observations of DB in various physical systems [2, 3, 4, 5] have been reported. It is also expected that DBs play critical roles in physical processes such as structure change in crystals [6].

It is usually observed that the DB travels in various lattices. A center of the spatial localization moves from a lattice site to a neighboring site with vibration frequency out of phonon band. Although a mobile DB is easily obtained in a numerical simulation, theoretical understanding of mobile DB have not been clarified. Yoshimura has reported that mobile DBs have finite tail in Fermi-Pasta-Ulam-β (FPU-β) lattice [7]. The finite tail of mobile DBs has also been reported in nonlinear Klein-Gordon [8] and Salerno lattice [9].

In Ref.[7], the relation between a symmetry of an interaction potential and existence of the tail of mobile DB has been pointed out. Moreover it has been reported that the mobile DB with a arbitrary velocity can be constructed in a four particle symmetric lattice that corresponds to a four particle FPU-β lattice [10]. Therefore symmetric lattices have importance for better understandings of dynamics of mobile DBs. In the present paper, we propose a symmetric lattice with N particles. Then we show numerical results of mobile DB in the proposed symmetric lattice.

2. Symmetric Lattice

Let us consider a N-particle nonlinear lattice system.

$$H = \sum_{n=1}^{N} \frac{p_n^2}{2} + \Phi(\mathbf{q}), \tag{1}$$

where p_n is the linear momentum and $\Phi(\mathbf{q})$ is the potential defined by $\mathbf{q} = (q_1, q_2, ..., q_N)$ or positions of particles.

$$\Phi(\mathbf{q}) = \frac{1}{4} \sum_{n=1}^{N} (q_{n+1} - q_n)^4. \tag{2}$$

This is the FPU-β lattice without linear interactions. Consider a variable transformation from the physical space $\{q_n\}$ to the complex normal mode space $\{U_m\}$

$$q_n = \frac{(-1)^n}{\sqrt{N}} \sum_{m=-N/2+1}^{N/2} U_m \exp\left(i\frac{2\pi m}{N} n\right). \tag{3}$$

Since $\Phi(\mathbf{U})$ is invariant with respect to a uniform shift, the total momentum is a first integral. Therefore, we can assume $U_{N/2} = \dot{U}_{N/2} = 0$ and neglect them. Substituting (3) into (2), we obtain a potential in terms of $\mathbf{U} = (U_{-N/2+1}, U_{-N/2+2}, ..., U_{N/2-1})$,

$$\begin{aligned}\Phi(\mathbf{U}) &= \frac{1}{4N} \sum_{i,j,k,l=-N/2+1}^{N/2-1} \omega_i\omega_j\omega_k\omega_l U_i U_j U_k U_l \Delta(i+j+k+l) \\ &- \frac{1}{4N} \sum_{i,j,k,l=-N/2+1}^{N/2-1} \omega_i\omega_j\omega_k\omega_l U_i U_j U_k U_l \\ &\quad \times \left(\Delta(i+j+k+l-N) + \Delta(i+j+k+l+N)\right),\end{aligned} \tag{4}$$

where $\omega_m = 2\cos\left(\frac{\pi m}{N}\right)$ $(m = -N, -N+1, ..., N/2-1)$. The function $\Delta(p)$ is defined as follows,

$$\Delta(p) = \begin{cases} 1 & \text{if} \quad p = 0, \\ 0 & \text{others.} \end{cases} \tag{5}$$

Let us consider a map $\mathcal{T}_\lambda : U_m \mapsto U_m \exp(-im\lambda)$ with a real parameter λ. This map with $\lambda = \pi/N$ corresponds to shifting a displacement pattern to one lattice spacing and reversing the phase in the physical space. The first term

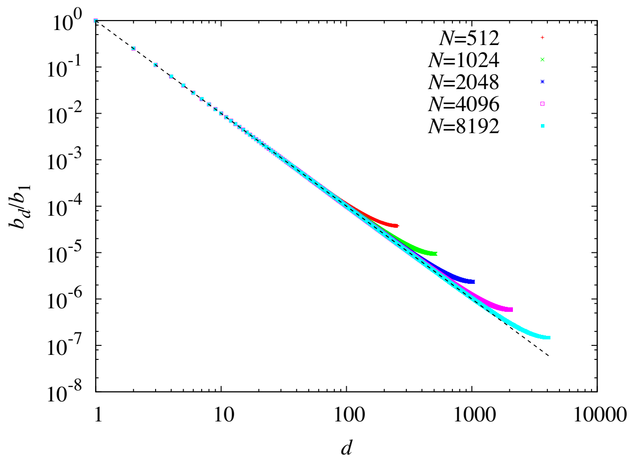

Figure 1: Strength of connection between d-th neighbor lattice sites. Dashed line indicates the approximated function $b_d/b_1 = d^{-2}$.

of Eq.(4) is symmetric with respect to the map $\mathcal{T}_\lambda$. The second term of Eq.(4), on the other hand, is asymmetric with respect to the map $\mathcal{T}_\lambda$.

We define a symmetric lattice that has potential Φ containing only symmetric terms with respect to the map $\mathcal{T}_\lambda$. The symmetric lattice can be obtained by eliminating the asymmetric terms in the potential in terms of $\{U_m\}$ of Eq.(4). Hamiltonian of the symmetric lattice in terms of $\mathbf{U}$ can be written as

$$H = \frac{1}{2}\sum_{m=-N+1}^{N-1} \dot{U}_m \dot{U}_{-m} + \Phi_s(\mathbf{U}), \tag{6}$$

where

$$\Phi_s(\mathbf{U}) = \frac{1}{4N}\sum_{i,j,k,l=-N/2+1}^{N/2-1} \omega_i\omega_j\omega_k\omega_l \times U_iU_jU_kU_l\Delta(i+j+k+l). \tag{7}$$

However, it is difficult to obtain a physically reasonable lattice in physical space $\{q_n\}$ in general, since inverse transformation of Eq.(3) might reproduce unphysical terms in $\{q_n\}$.

Let us consider a following lattice system for even N,

$$\Phi_l(\mathbf{q}) = \frac{1}{4}\sum_{d=1}^{N/2}\sum_{n=1}^{N} b_d(q_{n+d} - q_n)^4. \tag{8}$$

where b_d indicates strength of connection between d-th neighbor lattice sites. We can obtain a physically reasonable type of symmetric lattice for this model by choosing an appropriate set of $\{b_d\}$ of Eq.(8). Fig. 1 shows the strength of connection $\{b_d\}$ with various N. In the case of small d, b_d is approximately given by

$$b_d = b_1 d^{-2}. \tag{9}$$

It is also found that $b_{N/2}$ converges a finite value that is function of N.

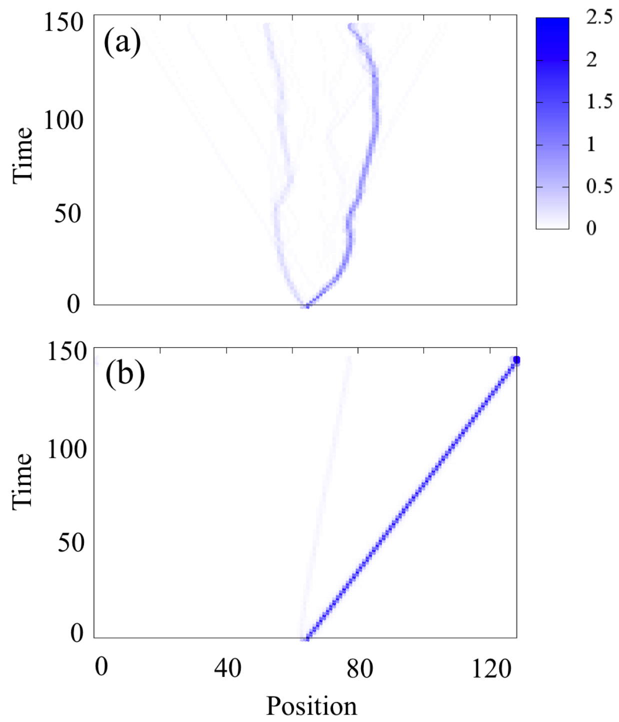

Figure 2: Temporal evolution of site energy of perturbed DB in (a)FPU type lattice (2) and (b)symmetric lattice (8).

3. Numerical Results

We perform numerical simulations of temporal evolution of a perturbed odd DB in the FPU-type (2) and the symmetric lattice (8) with N=128 as shown in Fig. 2. In the case of FPU-type lattice, we choose a set of parameters of $b_1 = 1$ and $b_d = 0$ $(d \neq 1)$. In the case of the symmetric lattice, we choose a set of parameters of $b_d \neq 0$ for all indices in order to eliminate terms in the transformed potential $\Phi_l(\mathbf{U})$ of Eq. (8), that is asymmetric with respect to map $\mathcal{T}_\lambda$.

Perturbations are added to momentum of the neighboring sites of the site with the maximum amplitude. It is observed that the kicked DB in the FPU-type lattice looses its velocity and energy. This is because the mobile DB in the FPU-type lattice is affected by the discreteness of the lattice. On the other hand, the kicked DB in the symmetric lattice starts to traveling with a constant velocity. This result indicates that the proposed model support highly mobile DBs.

4. Conclusion

We propose the lattice model supporting highly mobile DB by considering symmetry of the interaction potential. The proposed model has all-to-all connections with a certain connection strength that eliminates asymmetric term with respect to rotation map in the complex normal mode coordinate. In numerical simulation, DB in the proposed lattice can move with a constant velocity even in the condition that the DB in the FPU lattice looses its velocity and energy. It is expected that the proposed model is a useful model for investigation on mobile properties of DBs.

Acknowledgments

This work was partially supported by JSPS KAKENHI Grant Number 24560074.

References

[1] A.J. Sievers and S. Takeno, "Intrinsic localized modes in anharmonic crystals," *Phys. Rev. Lett.*, vol.61, pp.970–973, 1988.

[2] M. Sato, B. E. Hubbard, A. J. Sievers, B. Ilic, D.A. Czaplewski and H. G. Craighead, "Observation of locked intrinsic localized vibrational modes in a micromechanical oscillator array," *Phys. Rev. Lett.*, vol.90, 044102, 2003.

[3] M. E. Manley, D. L. Abernathy, N. I. Agladze and A. J. Sievers, "Symmetry-breaking dynamical pattern and localization observed in the equilibrium vibrational spectrum of NaI," *Sci. Rep.*, vol.1, 4, 2011.

[4] H. S. Eisenberg, Y. Silberberg, R. Morandotti, A. R. Boyd and J. S. Aitchison, "Discrete spatial optical solitons in waveguide arrays," *Phys. Rev. Lett.*, vol.81, pp.3383–3386, 1998.

[5] G. Kalosakas and S. Aubry, "Polarobreathers in a generalized Holstein model," *Physica D*, vol.113, pp.228–232, 1998.

[6] T. Shimada, D. Shirasaki and T. Kitamura, "Stone-wales transformation triggered by intrinsic localized modes in carbon nanotubes," *Phys. Rev. B*, vol.81, 035401, 2010.

[7] K. Yoshimura and Y. Doi, "Moving discrete breathers in nonlinear lattice: resonance and stability," *Wave Motion*, vol.45, pp.83–99, 2007.

[8] S. Aubry, T. Cretegny, "Mobility and reactivity of discrete breathers," *Physica D*, vol.119, pp.34-46, 1998.

[9] J. Goḿez-Gardenẽs, F. Falo, L.M. Floría, "Mobile localization in nonlinear Schrödinger lattices," *Phys. Lett. A*, vol.332, pp.213–219, 2004.

[10] Y. Doi and K. Yoshimura, "Translational asymmetry controlled lattice and numerical method for moving discrete breather in four particle system," *J. Phys. Soc. Jpn.*, vol.78, 034401, 2009.