2010 International Symposium on Nonlinear Theory and its Applications
NOLTA2010, Krakow, Poland, September 5-8, 2010

Detecting system state transitions in environmental time-series using non linear time series analysis

T.E. Karakasidis, A. Fragkou and A. Liakopoulos

Laboratory of Hydromechanics and Environmental Engineering,
Department of Civil Engineering, University of Thessaly, Volos, Greece
Email: thkarak@uth.gr, fthanos@uth.gr, aliakop@uth.gr

Abstract– Environmental systems and time series emanating from such systems present a particular interest. System state transitions can occur in time and/or in space and the detection of such transitions can be particularly useful in the design related to such systems. Since the majority of physical systems present non linear behavior the use of appropriate tools is necessary. Recurrence Plots (RPs) and Recurrence Quantitative Analysis (RQA) along with Cross Recurrence Plots (CRPs) are some tools that permit to extract the underlying system dynamics [1-3].

In the present work we analyze time series from two different environmental systems. In the first case we study records of daily values of the Nestos river (Greece) water level at various measurement stations. Transitions from "periodicity" to "chaos" and "chaos" to "laminarity" are identified in time. In the second part we analyze temperature fluctuations in a horizontal round heated turbulent jet where instantaneous temperature time series were recorded at several points on the jet cross section. The temperature time series are analyzed in a first place using RQA. The variation of RQA measures is related with and interpreted via the transitions of the physical state of the fluid from the fully-turbulent flow near the jet centerline to the transitional flow near the boundary of the jet [9]. In a second phase the CRP analysis reveals correlations between the various parts of the jet.

1. Introduction

The analysis of time-series is of particular importance in understanding the dynamics of various environmental systems and in forecasting their behavior. The majority of methods for the analysis of environmental time series and forecast presuppose the linearity of the underlying dynamical system and the presence of stochastic noise. Classical linear methods (autocorrelation function, cross-correlation function, spectral analysis, etc.) have been proved exceptionally successful in the analysis of time series. However, they fail to distinguish chaotic behavior resulting from nonlinear deterministic systems. Moreover, they fail to detect correlations between two signals. For the detection of chaotic deterministic behavior and its discrimination from stochastic behavior, so as for correlations between two time series, nonlinear methods have been proposed which are based on the conceptual framework of time-lag embedding. Nonlinear methods include correlation-dimension-based characterization, average mutual information Recurrence Quantification Analysis and Cross Recurrence Quantification Analysis [1-3]. In this study, we focus on Recurrence Quantification Analysis and Cross Recurrence Quantification Analysis.

In a first place RP and RQA are applied on a time series of Nestos river water level daily measurements, recorded over a period of 16 years and 4 months.. From the global inspection of the RP as well as from the use of epoch RQA analysis different areas of transition were identified among "laminarity", "chaos" and "periodicity" states.

In the second part we consider a horizontal round heated turbulent jet. We analyze the macroscopic local temperature fluctuation measurements along a horizontal line, located in a cross section of the jet, so that some of our time series correspond to conditions of fully-developed turbulence (these are the time series obtained close to the centerline of the jet) while other time series have both laminar and turbulent flow characteristics (intermittency, etc.) i.e., fluctuations measured close to the boundary between the heated jet and ambient water. Considering the interrelations between the time series by means of CRP analysis we observe strong indication of non linear relationship between the data near the center line of the jet in contrast with the boundaries of the jet.

2. Theoretical Background

2.1. Recurrence plots

Recurrence Plot is a graphical tool introduced by Eckmann et al. [1] in order to extract qualitative characteristics of time series. It is a powerful tool with the great advantage it can be applied on non-stationary data. It is based on the reconstruction of phase space and on the recurrence of states (eqn. 1).

$$R_{i,j}^{m,\varepsilon_i} = \Theta(\varepsilon_i - \|\vec{x}_i - \vec{x}_j\|) \qquad i,j = 1,...N \qquad (1)$$

$R_{i,j}$ is the recurrence matrix where the corresponding RP is based on, m the embedding dimension, ε the cutoff distance for the points considered to be recurrent and Θ is the Heaviside function. If the points are located at smaller distances than the considered distance ε are recurrent and Θ=1, else Θ=0. The RP is obtained by plotting the recurrence matrix using different colours for its binary entries, e.g. plotting a black dot at coordinates *(i,j)* if the corresponding element of the recurrence matrix is $R_{i,j} = 1$ and a white dot for $R_{i,j} = 0$. By definition $R_{i,j} = 1$ for every *i* thus the RP has a black main diagonal line called line of identity. Moreover RPs are symmetric by definition with respect to the diagonal $R_{i,j} = R_{j,i}$ .When computing an RP a norm must be chosen. The most widely employed norms are the L_1 norm, the L_2 norm (Euclidean norm) and the L_∞ norm (maximum or supremum norm) [3]. In the present study the Euclidean norm was used since it results in an intermediate number of neighbours compared to L1 and L2 norms [3].

2.2. Recurrence Quantification Analysis

Webber & Zbilut [2, 4] and Marwan et al. [5] extended the idea of Recurrence Plots and defined a number of measurable quantities that can be extracted from the RPs giving rise to Recurrence Quantification Analysis (RQA). We present briefly some of the RQA indices that have been proposed [2, 5] and we have employed in the present study:

2.2.1. %Recurrence

It gives the ratio of the number of recurrence points (pixels) to the total number of points (pixels) of the plot.

$$\%REC = \frac{\sum_{i,j=1}^{N} R_{i,j}}{N^2} \qquad (2)$$

2.2.2 %Determinism

It represents the ratio of the number of recurrence points forming upward diagonal lines to the total number of recurrence points

$$\%DET = \frac{\sum_{l=l_{\min}}^{N} lP(l)}{\sum_{l=1}^{N} lP(l)} \qquad (3)$$

2.2.3 MaxLine

Maxline is the length of the longest diagonal line segment in the plot, excluding the main diagonal line of identity. This is a very important recurrence variable because it is related to the Lyapunov exponents [1, 6]

$$L_{\max} = \max(\{l_i; i = 1,..., N_l\}) \qquad (4)$$

2.2.4. Trapping Time

It shows the average length of the vertical lines.

$$TT = \frac{\sum_{\upsilon=\upsilon_{\min}}^{N} \upsilon P(\upsilon)}{\sum_{\upsilon=\upsilon_{\min}}^{N} P(\upsilon)} \qquad (5)$$

Trapping Time represents the average time that the system has been trapped in the same state.

2.3. Cross Recurrence Plots

In order to analyze the dependencies between two different systems by comparing their states the idea of Cross Recurrence Plots was proposed [8, 10]. This concept, based on Recurrence Plots considers a phase space with two trajectories x_i and y_j of length N_x and N_y respectively.

$$CR_{i,j}^{m,\varepsilon_i} = \Theta(\varepsilon_i - \|\vec{x}_i - \vec{y}_j\|) \quad \vec{x}_i, \vec{y}_j \in \Re^m \quad i = 1,...N_x, j = 1,...N_y \qquad (6)$$

Analogous to the RPs the philosophy of the CRPs is that, if the points of the two trajectories are close enough, a black point will be marked onto the CR matrix at *(i,j)* location. In CRP instead of the Line of Identity, the Line of Synchronization makes its appearance.

2.4. Cross Recurrence Quantification Analysis

Zbilut et. al. [8] gave an extension in quantification of CRPs, by introducing the Cross Recurrence Quantification Analysis. This analysis based on the fact that long diagonal lines in the CRP reveal similar time evolution of the trajectories of both processes. This similarity is quantified by introducing some quantitative measures which consider the frequency distributions of the diagonal line lengths $P_k^{\varepsilon}(l) = \{l_i; i = 1...N_l\}$ for each diagonal parallel to the main diagonal $CR_{i,j}^{m,\varepsilon}(i - j = k)$. Considering that for $k = 0$ the line is the LOI, $k > 0$ the diagonals above and $k < 0$ the diagonals below the LOI, the quantitative measures are the following.

2.4.1. Recurrence Rate (RR_k)

$$RR_k = \frac{1}{N-k} \sum_{l=1}^{N-k} lP_k^{\varepsilon}(l) \qquad (7)$$

It represents the probability of occurrence of similar states in both systems with a certain delay $t = k\Delta t$.

2.4.2 Determinism (DET_k)

The proportion of recurrence points forming long diagonal lines to all recurrence points. However, it is constrained to the considered diagonal.

$$DET_k = \frac{\sum_{l=l_{\min}}^{N-k} l P_k^{\varepsilon}(l)}{\sum_{l=1}^{N-k} l P_k^{\varepsilon}(l)} \quad (8)$$

2.4.3. Averaged Line Length (L_k)

It quantifies the duration of having two deterministic processes similar time evolution in the phase space

$$L_k = \frac{\sum_{l=l_{\min}}^{N-k} l P_k^{\varepsilon}(l)}{\sum_{l=l_{\min}}^{N-k} P_k^{\varepsilon}(l)} \quad (9)$$

So, finding high values of *RR* means there is a high probability to occur same states in both processes [7]. Moreover, high values of *DET* and *L,* shows similar dynamics for a long duration in time.

3. Results and Discussion

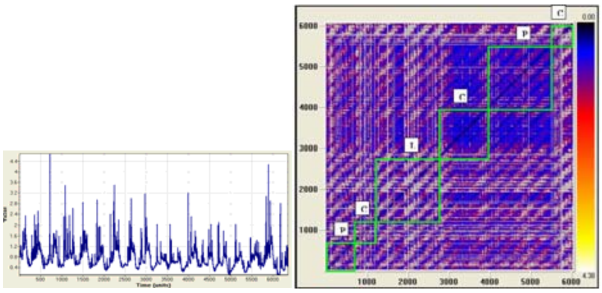
Time series analyses were performed using CRP toolbox [11], Visual Recurrence Analysis (VRA) [12], and TISEAN [13] packages.

3.1. RP and RQA in Nestos environmental system time series

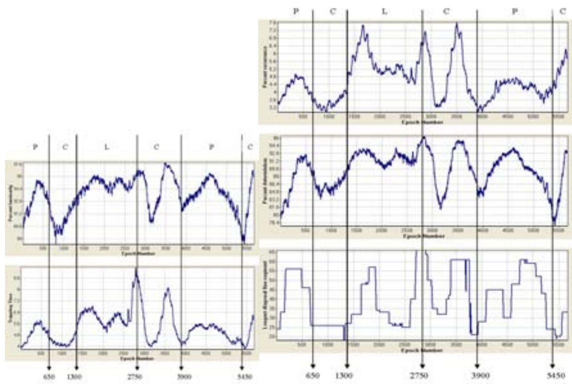
The corresponding time series of water level appears in Fig.1A and the corresponding RP in Fig. 1B. From the global inspection of the RP of Nestos river, 16 lines parallel to the main diagonal are observed (periodicity) so as the Airplane Structure of the RP (sign of trend in our time series). Moreover from a closely inspection of the RP and the RQA (Fig.1C) we can observe transitions from Periodicity to Chaos, Chaos to Laminarity, Laminarity to Chaos, Chaos to periodicity and Periodocity to Chaos. We note that these terms do not have the same meaning as in fluid dynamics. In “Chaotic” regions (650-1300), (2750-3900), (5450-6000) deterministic lines are very small thus we observe that Maxline drops down to 25 and sometimes even to zero. In Periodic areas (0-650), (3900-5450) values of %determinism and maxline are very high, 92.8% and 62 respectively. In Laminar region (1300-2750) we observe high values of %determinism, almost 94%, but no large deterministic lines (parallel to the main diagonal) appear, so the maximum line is low (25 to 30). State remains “trapped” in time, Trapping time has high values (6.6 and 6.7), while in the other regions the values are smaller

3.2. CRP and CRQA in a round heated jet

In order to find the center line of the jet in the second environmental system, instead of recurrence Plots and Recurrence Quantification Analysis methods of spectrum analysis and mutual information were applied [9]. According to all above methods, we found that the center of the jet is located at x=17.5 cm. So the CRP analysis gives the correlations between x=17.5cm and the other parts of the jet. Figure 3 shows CRPs of x=17.5 and x=16.5cm (near the center line of the jet) and CRPs of x=17.5cm and x=9.5cm (left boundary of the jet). It can be clearly seen that as we approach the boundary of the jet (x=9.5cm) the lines parallel to the main diagonal tend to became very small. On the other hand CRP which is close to the center line seems to have lines parallel to the main diagonal big enough to reveal the correlation between the states.



(A) (B)



(C)

Figure1. (A) Recurrence plot and timeseries from the first environmental system. The observed transitions are placed in green rectangles. (B) Recurrence Quantification Analysis with Epochs. Transitions are separated by solid lines.

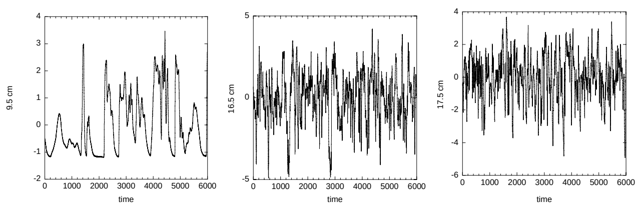
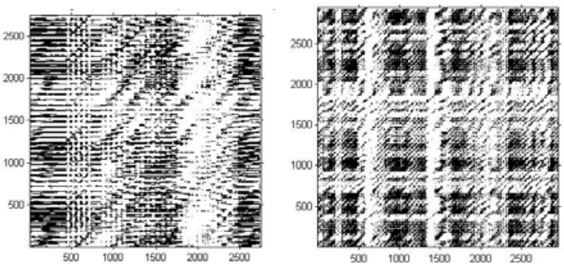


Fig.2 Temperature time series at x=9.5cm (a) x=16.5cm (b) and x=17.5cm (c).

From the Cross Recurrence Quantification Analysis the measures of RR_+/L_+ and RR_-/L_- were selected. As it can be clearly seen in Fig. 4 states (x=16.5cm and x=18.5cm) close to the center line of the jet (x=17.5cm), give maxima around 16 for RR_+/L_+. Moreover, the same states give maxima around 12 (x=16.5cm) and 18 (x=18.5cm) for RR-/L-. Those maxima for + and – measures are a strong indication of a non linear relationship between the data [3] The solid lines between the maximum lags of RR and L measures shows the differences between those lags. So as we approaching the boundaries (x=9.5cm and x=23.5cm) those differences seem to become larger. The quantification analysis of CRPs was able to detect nonlinear relations between time series.



(A) (B)

Fig.3 Cross Recurrence Plots of x=17.5cm with (A) x=16.5cm and (B) x=9.5cm.

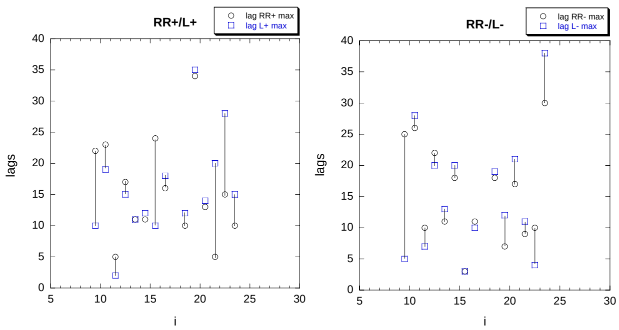


Fig.4 Cross Recurrence Quantification Analysis of x=17.5cm with all the states across the jet from x=9.5cm to x=23.5cm (left and right boundaries of the jet)

4. Conclusions

From the analysis of the first environmental system we can conclude that characteristic times of the dynamics of the system were revealed both from the visual inspection, as well as from the quantitative analysis of RQA. The system presents transition among “periodicity”, “chaotic” and “laminar” states. We believe that these transitions are related to the variation of other environmental time series, mainly total precipitation. However, river water levels and stream flow are naturally characterized by smoother variations in time compared to the driving process of precipitation. RQA-Epoch analysis identifies multiannual periods where specific characteristics appear persistently.

The Cross Recurrence Quantification Analysis of the second environmental system shows interdependencies between the states of the system which gives as information about the turbulence effect on the flow state and is suitable in order to find the nonlinear relation between the considered data series. Further research is under progress in order to extract characteristic features of the system.

Acknowledgments

Part of this research has been funded by the program «PYTHAGORAS-I», grant uth-51908.04, jointly funded by the European Community and the Greek Ministry of Education.

References

[1] J.-P Eckmann, S.O. Kamphorst and D. Ruelle, "Recurrence Plots Of Dynamic-Systems", *Europhys. Lett.* vol.4, pp. 973-977. 1987.

[2] C.L. Jr, Webber, J.P. Zbilut, "Dynamical Assessment Of Physiological Systems And States Using Recurrence Plot Strategies", *J. Appl. Physiol.*, vol. 76, pp. 965-973, 1994.

[3] N. Marwan, M.C. Romano, M. Thiel and J. Kurths, Recurrence plots for the analysis of complex systems", *Physics Reports*, vol. 438, pp. 237-329, 2007.

[4] Zbilut, J.P., Webber, C.L., Jr. "Embeddings and delays as derived from quantification of recurrence plots", *Physics Letters A* Vol.171, pp. 199-203, 1992

[5] Marwan, N., Wessel, N., Meyerfeldt, U., Schirdewan, A., Kurths, J. "Recurrence-plot-based measures of complexity and their application to heart-rate-variability data", *Phys. Rev. E* 66, 26702, 2002

[6] Trulla L.L., Giuliani A., Zbilut J. , Webber C.L. Jr., "Recurrence quantification analysis of the logistic equation with transients", *Physics Letters A* Vol. 223, pp. 255-260, 1996

[7] Marwan, N. Ph.D. Thesis, University of Potsdam, 2003

[8] J.P. Zbilut, A. Giuliani, C.L. Webber Jr., "Detecting deterministic signals in exceptionally noisy environments using cross-recurrence quantification", *Phys. Lett. A* 246 (1–2) 122–128, 1998

[9] T.E. Karakasidis, A. Liakopoulos, A. Fragkou, P. Papanicolaou, "Recurrence Quantification Analysis of Temperature Fluctuations in a Horizontal Round Heated Jet", *International Journal of Bifurcation and Chaos*, vol. 19, pp. 2487-2498, 2009.

[10] N. Marwan, J. Kurths, "Nonlinear analysis of bivariate data with cross recurrence plots", *Phys. Lett. A* 302 (5–6) (2002) 299–307

[11] Cross Recurrence Plot Toolbox for Matlab, ver 5.12, release25

[12] Kononov E, Visual Recurrence Analisis, 4.9

[13] Hegger R. , Kantz H., and Schreiber T., "*Nonlinear Time Series Analysis, TISEAN 3.0.1.* ", 2007