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Stabilization of a DC Bus Network System with Delayed Feedback Control Based on an Equivalent Circuit

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Abstract—This study demonstrates that a single delayed feedback controller can be used to stabilize a direct-current bus network system. The dynamics of the network system is often complex because the network consists of numerous heterogeneous subsystems. However, if the resistance-to-inductance ratios of the power lines of all subsystems are identical, the network system can be represented by a simple equivalent circuit. The equivalent circuit is beneficial in analyzing the stability of the system with a delayed feedback controller because changes in the subsystems can be reduced to changes in the parameters of the equivalent circuit.

1. Introduction

Direct-current (DC) microgrids supply DC power generated by multiple distributed power sources to DC loads. Recently, DC microgrids have received increasing attention owing to their high efficiencies and reliability [1]. The importance of DC microgrids is expected to increase with the increasing use of information and communication devices, the integration of renewable energy, and the expansion of the electric vehicle market. However, many DC loads employ DC/DC converters for voltage level conversion, which operate as constant power loads (CPLs) with negative current-voltage slope characteristics. These characteristics destroy the stability of DC microgrids; thus, many studies have proposed methods to eliminate the instability caused by CPLs [2, 3, 4].

Delayed feedback control (DFC) [5] is a well-studied scheme in nonlinear dynamics [6, 7]. The control signal of DFC is proportional to the difference between the current and past states of the target system, which stabilizes the system without requiring knowledge of the system's operating point. In addition, DFC requires less energy to maintain control because the control signal approaches zero after stabilization.

DFC can stabilize a simplified circuit of a DC bus system consisting of one power supply and one CPL [8]. To

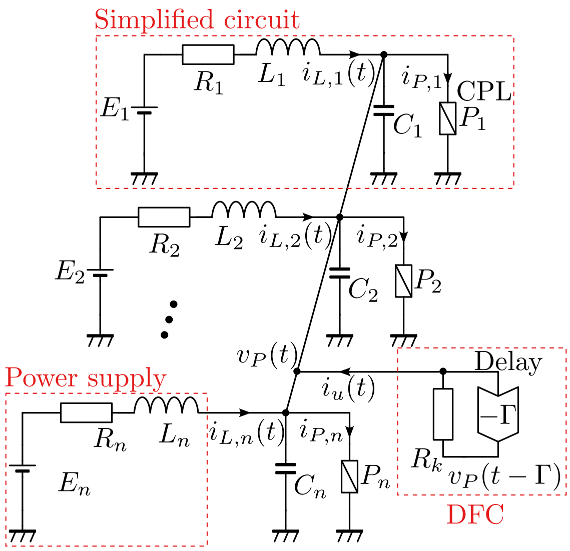

Figure 1: DC bus network system with DFC.

confirm the stability analysis, we conducted circuit experiments [9]. In addition to the local stability of the operating point of a simplified circuit with DFC, the basin of the operating point was also investigated [10]. Furthermore, a previous study [11] showed that a network system with multiple identical simplified circuits could be stabilized using multiple delayed feedback controllers. However, typical power systems comprise several different sources and loads. The different sources and loads increase the number of variables and render the analytical methods of the previous study [11] inapplicable.

If the resistance-to-inductance ratios of the power lines of all subsystems are identical, the network system can be represented by a single simplified equivalent circuit [12, 13]. This equivalent circuit is useful for analyzing the stability of the system because changes in the subsystems of the network can be reduced to changes in the parameters of the equivalent circuit. This study demonstrates that a single delayed feedback controller can be used to stabilize a DC bus network system with multiple heterogeneous subsystems by utilizing this equivalent circuit concept.

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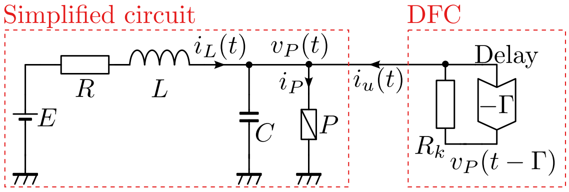

Figure 2: Equivalent circuit.

2. Equivalent system

This study focuses on a network system (Fig. 1) consisting of n simplified DC bus system circuits. A bus line connects all these simplified circuits, and the resistance and inductance of the bus line are assumed to be negligible. This network configuration describes a situation in which the DC power from multiple power supplies in different areas is transmitted to collected CPLs in a single location. The simplified circuit $j \in \{1, 2, \ldots, n\}$ contains a voltage source E_j [V], a resistance R_j [Ω], and an inductance L_j [H]. The current of circuit j is $i_{L,j}(t)$ [A]. Circuit j has an ideal CPL with power consumption P_j [W]. CPL current $i_{P,j}$ [A] is determined by

$$i_{P,j}(t) = \frac{P_j}{v_P(t)}, \tag{1}$$

where $v_P(t)$ [V] is the voltage of the bus line. A capacitor C_j [F] is connected in parallel to the CPL. The network system has only one delayed feedback controller to stabilize the operating point of the system. The controller consists of a delay unit with delay time Γ [s] and a feedback resistor R_k [Ω].

The total capacitance and total power consumption are denoted by

$$C = \sum_{j=1}^{n} C_j, \quad P = \sum_{j=1}^{n} P_j, \tag{2}$$

respectively. With these quantities, the network system has the form

$$\begin{cases} C\dfrac{\mathrm{d}v_P(t)}{\mathrm{d}t} = -\dfrac{P}{v_P(t)} + \displaystyle\sum_{j=1}^{n} i_{L,j}(t) + i_u(t), \\ L_1\dfrac{\mathrm{d}i_{L,1}(t)}{\mathrm{d}t} = -v_P(t) - Ri_{L,1}(t) + E_1, \\ \qquad\vdots \\ L_n\dfrac{\mathrm{d}i_{L,n}(t)}{\mathrm{d}t} = -v_P(t) - Ri_{L,n}(t) + E_n, \end{cases} \tag{3}$$

where $i_u(t)$ [A] is the control current of DFC

$$i_u(t) = \frac{1}{R_k}\left[v_P(t-\Gamma) - v_P(t)\right]. \tag{4}$$

It is evident that one voltage variable and n current variables are required to describe the dynamics (3). Thus, with many simplified circuits, stability analysis becomes challenging, even in numerical simulation. This study demonstrates that the network system can be reduced to an equivalent circuit with fewer variables, as shown in Fig. 2.

If the resistance-to-inductance ratios are identical for all simplified circuits,

$$\frac{R_1}{L_1} = \frac{R_2}{L_2} = \cdots = \frac{R_n}{L_n}, \tag{5}$$

then, the dynamics of the system (3) can be reduced to that of a single circuit [12, 13]. Equation (5) is often satisfied because the resistance and inductance of the transmission lines typically increase with length. Another way to satisfy this condition is to use the virtual resistance in DC power supplies [14]. The equivalent inductance is described by

$$\frac{1}{L} = \sum_{j=1}^{n} \frac{1}{L_j}. \tag{6}$$

In addition, the other equivalent parameters are described by

$$R = \frac{R_j}{L_j}L, \quad E = L\sum_{j=1}^{n} \frac{E_j}{L_j}. \tag{7}$$

DC power supplies are typically designed to operate with a positive current $i_{L,j} > 0$. Because large voltage differences between power supplies E_j can cause negative currents, the differences should be maintained small. The line current $i_L(t)$ [A] in the equivalent circuit is the sum of simplified circuits in the network,

$$i_L(t) = \sum_{j=1}^{n} i_{L,j}(t). \tag{8}$$

Thus, the equivalent circuit is described by the system using two variables,

$$\begin{cases} C\dfrac{\mathrm{d}v_P(t)}{\mathrm{d}t} = -\dfrac{P}{v_P(t)} + i_L(t) + i_u(t), \\ L\dfrac{\mathrm{d}i_L(t)}{\mathrm{d}t} = -v_P(t) - Ri_L(t) + E. \end{cases} \tag{9}$$

The circuit configuration in Fig. 2 and the dynamics (9) are equivalent to the simplified circuit with DFC proposed in [8]. Thus, the stability analysis in [8] can be used to analyze the stability of the network system (3). The stability of the operating point is governed by the roots $s \in \mathbb{C}$ of characteristic equation,

$$s^2 + \gamma_1 s + \gamma_2 + (\eta_1 s + \eta_2)(1 - e^{-sT}) = 0, \tag{10}$$

where

$$\begin{aligned} \gamma_1 &:= b - a^*, & \gamma_2 &:= b - a^* b, \\ \eta_1 &:= \frac{R}{R_k}, & \eta_2 &:= \eta_1 b, \\ a^* &:= \frac{4a}{(1+\sqrt{1-4a})^2}, & a &:= \frac{RP}{E^2}, \\ b &:= \frac{R^2 C}{L}, & T &:= \frac{\Gamma}{RC}. \end{aligned} \tag{11}$$

3. Numerical simulations

3.1. Verification of equivalent circuit

This subsection presents a numerical confirmation of the equivalence of network system (3) and equivalent circuit (9). We set circuit parameters* for the network system (3) with $n = 3$ as

$$\begin{array}{llll} L_1 = 20, & L_2 = 40, & L_3 = 60 & \text{mH}, \\ R_1 = 20, & R_2 = 40, & R_3 = 60 & \Omega, \\ E_1 = 20, & E_2 = 18, & E_3 = 22 & \text{V}, \\ C_1 = 15, & C_2 = 15, & C_3 = 15 & \mu\text{F}, \\ P_1 = 3.0, & P_2 = 3.0, & P_3 = 2.5 & \text{W}. \end{array} \tag{12}$$

The equivalent parameters are described as

$$\begin{array}{ll} L = 10.909\text{mH}, & R = 10.909\Omega, \\ E = 19.818\text{V}, & C = 45\mu\text{F}, \\ P = 8.5\text{W}. & \end{array} \tag{13}$$

The stability of the operating point $[v_P^*, i_L^*]$ = [12.25V, 0.6941A] is unstable without DFC. Based on the results in [8], we used the control parameters

$$R_k = 100\Omega, \quad \Gamma = 2.5\text{ms}, \tag{14}$$

which stabilize the operating point. The initial condition was given by

$$[v_P(0),\ i_{L,1}(0),\ i_{L,2}(0),\ i_{L,3}(0)] = [12.2\text{V}, 0.4\text{A}, 0.3\text{A}, 0\text{A}]. \tag{15}$$

The control started at $t = 0.01$s. Figure 3 shows the numerical results of the network system (3) and equivalent circuit (9) with parameters (12)–(14) and the initial condition (15). The solid lines represent the time responses of the network system, and the dots represent those of the equivalent circuit. It can be observed that the time responses of both systems are exactly equal. DFC designed for the equivalent circuit (9) stabilizes the network system (3); $v_P(t)$ and $i_L(t)$ converge to the operating point.

3.2. Effect of additional power supply on stability

This subsection describes the advantage of the equivalent circuit (9) in analyzing the stability for the case in which a power supply is added to a network system. In general, adding power supplies can increase the available power and enhance the stability of power systems. Adding a new power supply j to a network is equivalent to connecting a simplified circuit with zero capacitance $C_j \equiv 0$ and power consumption $P_j \equiv 0$, which consists of a voltage source E_j, resistor R_j, and inductor L_j, as shown in Fig. 1. Thus, the equivalent circuit of a network system with an added power supply enables an analysis of its stability.

*These circuit parameters were set in a small order to facilitate future experimental verification in our laboratory. The order does not affect the system's stability because only the ratios between the parameters appear in the characteristic equation (10).

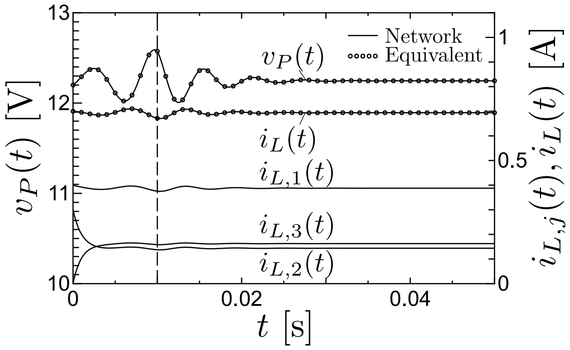

Figure 3: Behaviors of the network system (3) and equivalent circuit (9) with parameters (12)–(14) and initial condition (15). The control starts at $t = 0.01$s.

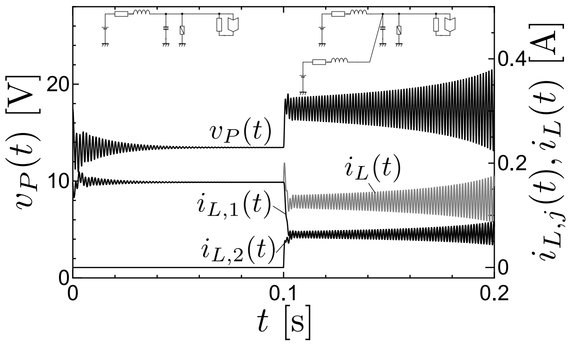

Figure 4: Behavior of the network system (3) with parameters (16) and initial condition (19). The power supply (18) is connected at $t = 0.1$s.

As a counter-intuitive example, we present a situation in which the addition of a power supply causes operating point instability. First, we consider a single ($n = 1$) DC bus system with circuit parameters

$$\begin{array}{lll} L_1 = 40\text{mH}, & R_1 = 40\Omega, & E_1 = 20\text{V}, \\ C_1 = 4.0\mu\text{F}, & P_1 = 2.2\text{W}. & \end{array} \tag{16}$$

The operating point of this single circuit is unstable without DFC. Thus, we apply DFC to this circuit with the following control parameters:

$$R_k = 100\Omega, \quad \Gamma = 1.12\text{ms}. \tag{17}$$

Second, we consider the addition of a new power supply with

$$\begin{array}{lll} L_2 = 40\text{mH}, & R_2 = 40\Omega, & E_2 = 20\text{V}, \\ C_2 = 0\text{F}, & P_2 = 0\text{W}, & \end{array} \tag{18}$$

to the stabilized circuit. The equivalent circuit (9) for the $n = 2$ network system reveals that its operating point is unstable when using the control parameters (17).

Figure 4 shows the simulation result using the following initial condition:

$$
\begin{aligned}
&[v_P(t),\ i_{L,1}(t),\ i_{L,2}(t)] \\
&= \begin{cases} [13.46\text{V}, 0.1634\text{A}, 0\text{A}] & t \in [-\Gamma, 0\text{s}), \\ [17.46\text{V}, 0.1634\text{A}, 0\text{A}] & t = 0\text{s}. \end{cases}
\end{aligned} \tag{19}
$$

The control starts at $t = 0$s. For $t \in [0, 0.1\text{s})$, the system consists of the circuit with Eq. (16) and DFC with Eq. (17). The voltage $v_P(t)$ and current $i_L(t)$ converge to its operating point. The new power supply is connected at $t = 0.1$s, and the position of the operating point and its stability change; the voltage and current oscillate. The simulation result agrees with the stability analysis based on the equivalent circuit (9).

4. Conclusions

This paper presents a stability analysis of a DC bus network system with multiple power supplies and CPLs. We derived an equivalent circuit consisting of a single power supply and a CPL when the ratios of resistance to inductance in the transmission lines are identical. The equivalent circuit is the same as the simplified circuit used in a previous study [8]. This allows the use of a single delayed feedback controller to stabilize the network system. In addition, the equivalent circuit is useful for stability analysis because changes in the subsystems of the network can be reduced to changes in the parameters of the equivalent circuit. We conducted numerical simulations in which the controller stabilizing the operating point of the network system fails to maintain stability when connecting a new power supply. In the future, we plan to investigate a robust delayed feedback controller for parameter changes.

Acknowledgments

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