

Study on Electromagnetic Scattering Problem of Periodic Cylinder Array with Additional Cylinder

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Abstract

This paper consider the electromagnetic scattering problem of a periodic cylinder array with an additional cylinder, and present expressions of the scattering factors that is proposed in Nakayama's shadow theory. The formulation is based on the pseudo-periodic Fourier transform with the use of the recursive transition-matrix algorithm.

Keywords: electromagnetic scattering, imperfectly periodic structure, pseudo-periodic Fourier transform, shadow theory

1. Introduction

When a plane-wave illuminates a perfectly periodic structure, the Floquet theorem asserts that the scattered fields are pseudo-periodic and the scattered fields have discrete spectra in the wavenumber space. The field components can be therefore expressed in the generalized Fourier series expansions, and the analysis region can be reduced to only one periodicity cell. Then most of the approaches for periodic structures are based on the Floquet theorem. However, when the structural periodicity is locally broken, the Floquet theorem is no longer applicable.

This paper considers an approach in spectral-domain to the scattering problem of a periodic cylinder array with an additional cylinder. The fields in imperfectly periodic structures have continuous spectra, and an artificial discretization is necessary on numerical computation. When perfectly periodic structures are illuminated by incident fields with continuous spectra, the spectra of scattered fields have infinite number of non-smooth points in the wavenumber space, which are called the Wood anomalies. They do not vanish if the structural periodicity is locally collapsed, and should be taken into account on the discretization in the wavenumber space. The present approach is based on the pseudo-periodic Fourier transform (PPFT) [1]. PPFT converts an arbitrary function into a pseudo-periodic one, and the conventional formulations for perfectly periodic structures based on the Floquet theorem can be applied for the scattering problem of imperfectly periodic structures. The transformed function has also a periodic property in terms of the transform parameter, which is related to the wavenumber, and the analysis region in the spectral domain is reduced to the Brillouin zone. Therefore, the discretization scheme in terms of the transform parameter can be considered inside the Brillouin zone. Recently, Nakayama [2] proposed the shadow theory of grating, in which he suggested to use the scattering factors instead of the reflection coefficients. This paper shows expressions of the scattering factors with the help of the multilayer technique and the recursive transition-matrix algorithm (RTMA).

2. Settings of the Problem

We consider time-harmonic electromagnetic fields assuming a time-dependence in $e^{-i\omega t}$ and the scattering problem from a circular cylinder located near a periodic array of circular cylinders schemat-

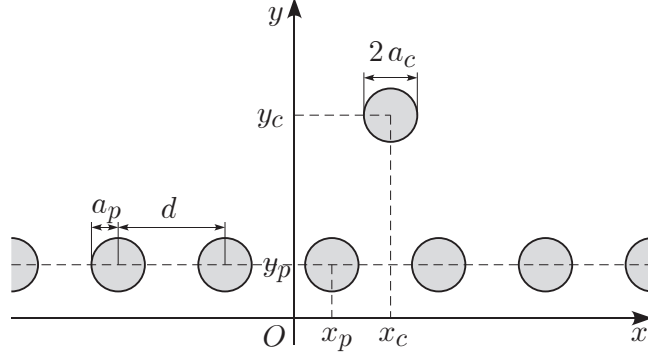


Figure 1: Geometry under consideration.

ically shown in Fig. 1. All the cylinders are infinitely long in the z -direction and situated parallel to each other. The periodic cylinder array consists of identical cylinders with homogeneous and isotropic medium described by the permittivity ε_p and the permeability μ_p , and the radius is a_p . One cylinder in the periodic array is located at $(x, y) = (x_p, y_p)$ and the other cylinders are periodically spaced with a common distance d in the x -direction. The additional cylinder with the permittivity ε_c , the permeability μ_c , the radius a_c is located at the center position $(x, y) = (x_c, y_c)$, and the surrounding region is a lossless, homogeneous, and isotropic material with the permittivity ε_s and the permeability μ_s . For $r = p, c$, we also denote $y_r + a_r$ and $y_r - a_r$ by $h_{r,1}$ and $h_{r,2}$, and the wavenumber and the characteristic impedance in each region are respectively given by $k_r = \omega \sqrt{\varepsilon_r \mu_r}$ and $\zeta_r = \sqrt{\mu_r / \varepsilon_r}$. The parameters are chosen not to overlap each other and we suppose $h_{p,1} < h_{c,2}$. The electromagnetic fields are supposed to be uniform in the z -direction and two-dimensional scattering problem is here considered. Two fundamental polarizations are expressed by TM and TE, in which the electric and the magnetic fields are respectively parallel to the z -axis. Here, we denote the z -component of electric field for the TM-polarization and the z -component of magnetic field for the TE-polarization by $\psi(x, y)$, and show the formulation. The incident field is supposed to illuminate the scatterers from the upper or lower regions and there exists no source inside the scatterer region $h_{p,1} \leq y \leq h_{c,2}$.

3. Outline of Formulation

The present approach uses the pseudo-periodic Fourier transform (PPFT) [1] to consider the discretization scheme in the wavenumber space. PPFT of the field and its inverse are formally given by

$$\bar{\psi}(x; \xi, y) = \sum_{m=-\infty}^{\infty} \psi(x - m d, y) e^{i m d \xi}, \quad \psi(x, y) = \frac{1}{k_d} \int_{-k_d/2}^{k_d/2} \bar{\psi}(x; \xi) d\xi \quad (1)$$

where ξ is a transform parameter, and $k_d = 2\pi/d$ is the inverse lattice constant. PPFT converts an arbitrary function into a pseudo-periodic one in terms of the spatial parameter x . Since the field outside the cylinders satisfies the Helmholtz equation for the surrounding medium, the transformed field can be expressed in the plane-wave expansion as

$$\bar{\psi}(x; \xi, y) = \mathbf{f}^{(-)}(x, y - y'; \xi)^t \bar{\boldsymbol{\psi}}^{(-)}(\xi, y') + \mathbf{f}^{(+)}(x, y - y'; \xi)^t \bar{\boldsymbol{\psi}}^{(+)}(\xi, y') \quad (2)$$

where the basis functions of plane-wave expansion are here given by column matrices $\mathbf{f}^{(\pm)}(x, y; \xi)$, in which the n th-component is given as

$$\left(\mathbf{f}^{(\pm)}(x, y; \xi) \right)_n = e^{i(\alpha_n(\xi) x \pm \beta_n(\xi) y)}, \quad \alpha_n(\xi) = \xi + n k_d, \quad \beta_n(\xi) = \sqrt{k_s^2 - \alpha_n(\xi)^2}. \quad (3)$$

The superscripts (+) and (-) indicate the column matrices corresponding to the plane-waves propagating in the positive and the negative y -direction, respectively, and $\bar{\boldsymbol{\psi}}^{(+)}(\xi, y')$ and $\bar{\boldsymbol{\psi}}^{(-)}(\xi, y')$ denote the column matrices of the amplitudes at $y = y'$.

Following RTMA manipulation, the amplitudes of incoming and outgoing plane-waves for the periodic cylinder array and the additional cylinder are related as follows:

$$\begin{pmatrix} \bar{\psi}^{(+)}(\xi, h_{p,1}) \\ \bar{\psi}^{(-)}(\xi, h_{p,2}) \end{pmatrix} = \begin{pmatrix} \bar{\mathbf{S}}_{p,11}(\xi) & \mathbf{F}(0, 2a_p; \xi) + \bar{\mathbf{S}}_{p,12}(\xi) \\ \mathbf{F}(0, 2a_p; \xi) + \bar{\mathbf{S}}_{p,21}(\xi) & \bar{\mathbf{S}}_{p,22}(\xi) \end{pmatrix} \begin{pmatrix} \bar{\psi}^{(-)}(\xi, h_{p,1}) \\ \bar{\psi}^{(+)}(\xi, h_{p,2}) \end{pmatrix} \quad (4)$$

$$\begin{pmatrix} \bar{\psi}^{(+)}(\xi, h_{c,1}) \\ \bar{\psi}^{(-)}(\xi, h_{c,2}) \end{pmatrix} = \begin{pmatrix} \mathbf{0} & \mathbf{F}(0, 2a_c; \xi) \\ \mathbf{F}(0, 2a_c; \xi) & \mathbf{0} \end{pmatrix} \begin{pmatrix} \bar{\psi}^{(-)}(\xi, h_{c,1}) \\ \bar{\psi}^{(+)}(\xi, h_{c,2}) \end{pmatrix} + \frac{1}{k_d} \int_{-k_d/2}^{k_d/2} \begin{pmatrix} \bar{\mathbf{S}}_{c,11}(\xi, \xi') & \bar{\mathbf{S}}_{c,12}(\xi, \xi') \\ \bar{\mathbf{S}}_{c,21}(\xi, \xi') & \bar{\mathbf{S}}_{c,22}(\xi, \xi') \end{pmatrix} \begin{pmatrix} \bar{\psi}^{(-)}(\xi', h_{c,1}) \\ \bar{\psi}^{(+)}(\xi', h_{c,2}) \end{pmatrix} d\xi' \quad (5)$$

with

$$\bar{\mathbf{S}}_{p,11}(\xi) = \mathbf{F}(-x_p, a_p; \xi) \mathbf{B}^{(+)}(\xi)^t (\mathbf{T}_p^{-1} - \mathbf{L}(\xi))^{-1} \mathbf{A}^{(-)}(\xi)^t \mathbf{F}(x_p, a_p; \xi) \quad (6)$$

$$\bar{\mathbf{S}}_{p,12}(\xi) = \mathbf{F}(-x_p, a_p; \xi) \mathbf{B}^{(+)}(\xi)^t (\mathbf{T}_p^{-1} - \mathbf{L}(\xi))^{-1} \mathbf{A}^{(+)}(\xi)^t \mathbf{F}(x_p, a_p; \xi) \quad (7)$$

$$\bar{\mathbf{S}}_{p,21}(\xi) = \mathbf{F}(-x_p, a_p; \xi) \mathbf{B}^{(-)}(\xi)^t (\mathbf{T}_p^{-1} - \mathbf{L}(\xi))^{-1} \mathbf{A}^{(-)}(\xi)^t \mathbf{F}(x_p, a_p; \xi) \quad (8)$$

$$\bar{\mathbf{S}}_{p,22}(\xi) = \mathbf{F}(-x_p, a_p; \xi) \mathbf{B}^{(-)}(\xi)^t (\mathbf{T}_p^{-1} - \mathbf{L}(\xi))^{-1} \mathbf{A}^{(+)}(\xi)^t \mathbf{F}(x_p, a_p; \xi) \quad (9)$$

$$\bar{\mathbf{S}}_{c,11}(\xi, \xi') = \mathbf{F}(-x_{c,1}, a_c; \xi) \mathbf{B}^{(+)}(\xi)^t \mathbf{T}_c \mathbf{A}^{(-)}(\xi')^t \mathbf{F}(x_c, a_c; \xi') \quad (10)$$

$$\bar{\mathbf{S}}_{c,12}(\xi, \xi') = \mathbf{F}(-x_{c,1}, a_c; \xi) \mathbf{B}^{(+)}(\xi)^t \mathbf{T}_c \mathbf{A}^{(+)}(\xi')^t \mathbf{F}(x_c, a_c; \xi') \quad (11)$$

$$\bar{\mathbf{S}}_{c,21}(\xi, \xi') = \mathbf{F}(-x_{c,1}, a_c; \xi) \mathbf{B}^{(-)}(\xi)^t \mathbf{T}_c \mathbf{A}^{(-)}(\xi')^t \mathbf{F}(x_c, a_c; \xi') \quad (12)$$

$$\bar{\mathbf{S}}_{c,22}(\xi, \xi') = \mathbf{F}(-x_{c,1}, a_c; \xi) \mathbf{B}^{(-)}(\xi)^t \mathbf{T}_c \mathbf{A}^{(+)}(\xi')^t \mathbf{F}(x_c, a_c; \xi') \quad (13)$$

$$(\mathbf{F}(x, y; \xi))_{n,m} = \delta_{n,m} e^{i(\alpha_n(\xi)x + \beta_n(\xi)y)} \quad (14)$$

$$(\mathbf{A}^{(\pm)}(\xi))_{n,m} = \left(\frac{i\alpha_n(\xi) \pm \beta_n(\xi)}{k_s} \right)^m, \quad (\mathbf{B}^{(\pm)}(\xi))_{n,m} = \frac{2}{d\beta_m(\xi)} \left(\frac{-i\alpha_m(\xi) \pm \beta_m(\xi)}{k_s} \right)^n \quad (15)$$

$$(\mathbf{T}_r)_{n,m} = \delta_{n,m} \begin{cases} \frac{\zeta_s J_n(k_s a_r) J'_n(k_r a_r) - \zeta_r J'_n(k_s a_r) J_n(k_r a_r)}{\zeta_r H_n^{(1)}(k_s a_r) J_n(k_r a_r) - \zeta_s H_n^{(1)}(k_r a_r) J'_n(k_s a_r)} & \text{for TM-polarization} \\ \frac{\zeta_r J_n(k_s a_r) J'_n(k_r a_r) - \zeta_s J'_n(k_s a_r) J_n(k_r a_r)}{\zeta_s H_n^{(1)'}(k_s a_r) J_n(k_r a_r) - \zeta_r H_n^{(1)}(k_s a_r) J'_n(k_r a_r)} & \text{for TE-polarization} \end{cases} \quad (16)$$

$$(\mathbf{L}(\xi))_{n,m} = \sum_{l=1}^{\infty} H_{m-n}^{(1)}(l k_s d) \left[(-1)^{m-n} e^{i l d \xi} + e^{-i l d \xi} \right] \quad (17)$$

where $\delta_{n,m}$ is the Kronecker delta and $r = p, c$.

Here, considering the periodicity in terms of the transform parameter ξ , we take L sample points $\{\xi_l\}_{l=1}^L$ in the first Brillouin zone $-k_d/2 < \xi \leq k_d/2$, and Eqs. (4) and (5) are assumed to be satisfied only at the sample points. Also, the integration in Eq. (5) is approximated by an appropriate numerical integration scheme. Then, Eqs. (4) and (5) are rewritten as follows:

$$\begin{pmatrix} \tilde{\psi}^{(+)}(h_{p,1}) \\ \tilde{\psi}^{(-)}(h_{p,2}) \end{pmatrix} = \begin{pmatrix} \tilde{\Sigma}_{p,11} \tilde{\mathbf{D}} - \mathbf{I} & \tilde{\Sigma}_{p,12} \tilde{\mathbf{D}} \\ \tilde{\Sigma}_{p,21} \tilde{\mathbf{D}} & \tilde{\Sigma}_{p,22} \tilde{\mathbf{D}} - \mathbf{I} \end{pmatrix} \begin{pmatrix} \tilde{\psi}^{(-)}(h_{p,1}) \\ \tilde{\psi}^{(+)}(h_{p,2}) \end{pmatrix} \quad (18)$$

$$\begin{pmatrix} \tilde{\psi}^{(+)}(h_{c,1}) \\ \tilde{\psi}^{(-)}(h_{c,2}) \end{pmatrix} = \begin{pmatrix} \tilde{\Sigma}_{c,11} \tilde{\mathbf{D}} - \mathbf{I} & \tilde{\Sigma}_{c,12} \tilde{\mathbf{D}} \\ \tilde{\Sigma}_{c,21} \tilde{\mathbf{D}} & \tilde{\Sigma}_{c,22} \tilde{\mathbf{D}} - \mathbf{I} \end{pmatrix} \begin{pmatrix} \tilde{\psi}^{(-)}(h_{c,1}) \\ \tilde{\psi}^{(+)}(h_{c,2}) \end{pmatrix} \quad (19)$$

with

$$\tilde{\psi}^{(\pm)}(y) = \begin{pmatrix} \bar{\psi}^{(\pm)}(\xi_1, y) \\ \vdots \\ \bar{\psi}^{(\pm)}(\xi_L, y) \end{pmatrix}, \quad \tilde{\mathbf{D}} = \begin{pmatrix} \mathbf{D}(\xi_1) & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{D}(\xi_L) \end{pmatrix}, \quad (\mathbf{D}(\xi))_{n,m} = \delta_{n,m} \frac{2\beta_n(\xi)}{k_s} \quad (20)$$

$$\tilde{\Sigma}_{p,nm} = \left[\begin{pmatrix} \bar{\mathbf{S}}_{p,nm}(\xi_1) & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \bar{\mathbf{S}}_{p,nm}(\xi_L) \end{pmatrix} + \delta_{n,m} \mathbf{I} + (1 - \delta_{n,m}) \tilde{\mathbf{F}}(2a_p) \right] \tilde{\mathbf{D}}^{-1} \quad (21)$$

$$\tilde{\Sigma}_{c,nm} = \left[\begin{pmatrix} \frac{w_1}{k_d} \bar{S}_{c,nm}(\xi_1, \xi_1) & \cdots & \frac{w_L}{k_d} \bar{S}_{c,nm}(\xi_1, \xi_L) \\ \vdots & \ddots & \vdots \\ \frac{w_1}{k_d} \bar{S}_{c,nm}(\xi_L, \xi_1) & \cdots & \frac{w_L}{k_d} \bar{S}_{c,nm}(\xi_L, \xi_L) \end{pmatrix} + \delta_{n,m} \mathbf{I} + (1 - \delta_{n,m}) \tilde{\mathbf{F}}(2a_c) \right] \tilde{\mathbf{D}}^{-1} \quad (22)$$

$$\tilde{\mathbf{F}}(y) = \begin{pmatrix} \mathbf{F}(0, y; \xi_1) & \mathbf{0} \\ \vdots & \vdots \\ \mathbf{0} & \mathbf{F}(0, y; \xi_L) \end{pmatrix} \quad (23)$$

where $\{w_l\}_{l=1}^L$ denotes the weight factor and \mathbf{I} denotes the identity matrix. The entries of $\tilde{\Sigma}_{nm}$ ($n, m = 1, 2$) are the scattering factors proposed in Nakayama's shadow theory [2], and prospected to be possible to calculate even when the Wood-Rayleigh anomalies. From Eqs. (18) and (19), we finally obtain the scattering relation for the entire structure in the following form:

$$\begin{pmatrix} \tilde{\psi}^{(+)}(h_{c,1}) \\ \tilde{\psi}^{(-)}(h_{p,2}) \end{pmatrix} = \begin{pmatrix} \tilde{\Sigma}_{11} \tilde{\mathbf{D}} - \mathbf{I} & \tilde{\Sigma}_{12} \tilde{\mathbf{D}} \\ \tilde{\Sigma}_{21} \tilde{\mathbf{D}} & \tilde{\Sigma}_{22} \tilde{\mathbf{D}} - \mathbf{I} \end{pmatrix} \begin{pmatrix} \tilde{\psi}^{(-)}(h_{c,1}) \\ \tilde{\psi}^{(+)}(h_{p,2}) \end{pmatrix} \quad (24)$$

and the scattering factors the entire structure is given by

$$\tilde{\Sigma}_{11} = \tilde{\Sigma}_{c,11} + \tilde{\Sigma}_{c,12} \tilde{\mathbf{D}} \tilde{\mathbf{W}} \left(\tilde{\Sigma}_{p,11} \tilde{\mathbf{D}} - \mathbf{I} \right) \tilde{\mathbf{F}}(h_{c,2} - h_{p,1}) \tilde{\Sigma}_{c,21} \quad (25)$$

$$\tilde{\Sigma}_{12} = \tilde{\Sigma}_{c,12} \tilde{\mathbf{D}} \tilde{\mathbf{W}} \tilde{\Sigma}_{p,12} \quad (26)$$

$$\tilde{\Sigma}_{21} = \tilde{\Sigma}_{p,21} \tilde{\mathbf{D}} \tilde{\mathbf{F}}(h_{c,2} - h_{p,1}) \left[\mathbf{I} + \left(\tilde{\Sigma}_{c,22} \tilde{\mathbf{D}} - \mathbf{I} \right) \tilde{\mathbf{W}} \left(\tilde{\Sigma}_{p,11} \tilde{\mathbf{D}} - \mathbf{I} \right) \tilde{\mathbf{F}}(h_{c,2} - h_{p,1}) \right] \tilde{\Sigma}_{c,21} \quad (27)$$

$$\tilde{\Sigma}_{22} = \tilde{\Sigma}_{p,21} \tilde{\mathbf{D}} \tilde{\mathbf{F}}(h_{c,2} - h_{p,1}) \left(\tilde{\Sigma}_{c,22} \tilde{\mathbf{D}} - \mathbf{I} \right) \tilde{\mathbf{W}} \tilde{\Sigma}_{p,12} + \tilde{\Sigma}_{p,22} \quad (28)$$

$$\tilde{\mathbf{W}} = \tilde{\mathbf{F}}(h_{c,2} - h_{p,1}) \left[\mathbf{I} - \left(\tilde{\Sigma}_{p,11} \tilde{\mathbf{D}} - \mathbf{I} \right) \tilde{\mathbf{F}}(h_{c,2} - h_{p,1}) \left(\tilde{\Sigma}_{c,22} \tilde{\mathbf{D}} - \mathbf{I} \right) \tilde{\mathbf{F}}(h_{c,2} - h_{p,1}) \right]^{-1}. \quad (29)$$

4. Concluding Remarks

This paper has presents a formulation of the two-dimensional electromagnetic scattering problem from a circular cylinder located near a periodic array of circular cylinders. The formulation is based on PPFT and the fields in homogeneous media are expressed in the plane-wave expansions. The scattering matrices of the periodic cylinder array and the additional cylinder are separately calculated by RTMA, and the plane-wave amplitudes are matched by the technique for multilayer structure. PPFT introduces a transform parameter ξ and we need to discretize it for practical computation. The transformed fields are periodic in terms of ξ and the discretization scheme can be considered inside the Brillouin zone. It is worth noting that, if the sample points of the transform parameter $\{\xi_l\}_{l=1}^L$ are taken with the constant interval and the weights $\{w_l\}_{l=1}^L$ are identical constants, the conventional scattering matrices converge very slowly in terms of the sample number L and the practical computation is impossible. This problem is due to the Wood anomalies and seems to be resolved by splitting the Brillouin zone at the anomalies. Then, the sample points and weights are decided by applying, for example, the Gauss-Legendre scheme for each subinterval. We have showed expressions of the scattering factor that is thought to weaken the singularity of Wood anomalies. This paper does not include the results of numerical experiments because of the page limitation, but they will be shown in the presentation.

References

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