Floquet-Mode Analysis of Two-Dimensional Photonic Crystal Waveguide with Triangular Lattice Formed by Circular Cylinder Array

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Abstract

This paper presents a formulation of two-dimensional straight photonic crystal waveguide consisting of circular cylinder array in triangular lattice. The fields are expressed in the generalized Fourier series expansions but we also use the cylindrical-wave expansions. The present formulation is derived by the technique for multilayer with infinite thickness.

key words: photonic crystal waveguide, Floquet-mode, eigenmode analysis

1. Introduction

The photonic crystals (PCs) are periodic structures that are designed to reject the propagation of electromagnetic waves at certain wavelength ranges, and waveguide structures are formed by introducing line defects into perfect PCs. The electromagnetic fields are strongly confined around the defects because any electromagnetic energy cannot escape through the surrounding PC. The structure of straight PC waveguide (PCW) is periodic in the propagation direction. The Floquet theorem asserts that the eigenmodes of the straight waveguide are pseudo-periodic (namely, each field component is a product of a periodic function and an exponential phase factor) in the propagation direction, and these eigenmodes are called the Floquet-modes [1].

The guided Floquet-modes of the straight PCW are often analyzed by the finite-difference time-domain (FDTD) method [2], [3] because of its simplicity and wide applicability. However, since the computation errors are comparatively easy to accumulate, FDTD method for periodic structures seems to require special techniques in accurate calculations. The electromagnetic fields in straight PCW can be expressed in the generalized Fourier series expansions because they are pseudo-periodic. Therefore, the conventional grating theory is applicable to describe the fields in straight PCW, and the reflection matrices of PC walls yield the dispersion equation for the Floquet-modes. The zeros of the equation correspond to the propagation constants of the Floquet-modes, and they are numerically founded by Müller's method. For two-dimensional problems, the reflection matrices of PC walls are derived by the scattering-matrix propagation algorithm (SMPA) [4] in many cases, and this approach makes us possible to obtain the guided-modes in high accuracy [5], [6]. SMPA is a recursive matrix algorithm for obtaining the scattering-matrix of multilayer structures with finite thickness. When PC wall is with finite thickness, the fields are not perfectly confined in the line defect and the propagation constants of the guided Floquet-modes become complex.

This paper presents a formulation of straight PCW consisting of circular cylinder array in triangular lattice. The fields are expressed in the generalized Fourier series expansions but we also use the recursive transition-matrix algorithm (RTMA) [7], which uses the cylindrical-wave expansions to deal with the boundary conditions on the cylinder surfaces adequately.

The dispersion equation is derived by the multilayer technique as same with the conventional formulation, but the reflection matrices of PC walls are obtained using the eigenvalue/eigenvector analysis of the transfer matrix for periodicity cell. This makes us possible to consider PC walls with infinite thickness.

2. Outline of Formulation

This paper considers the guided Floquet-modes propagating in PCW with triangular lattice schematically shown in Fig. 1. We consider time-harmonic electromagnetic fields assuming a time-dependence in $e^{-i\omega t}$. The structure consists of identical circular cylinders with infinite length, radius a, permittivity ε_c, and permeability μ_c. They are situated parallel to each other in a surrounding medium with permittivity ε_s and permeability μ_s, and the z-axis is taken parallel to the cylinder axis. The cylinders are arrayed periodically on a triangular lattice with period d, however the structure has a gap w along the x-axis. We suppose that the center of one cylinder in the cylinder array on $y = y_c = w/2 + \sqrt{3}d/4$ is located on the y-axis. We consider two-dimensional problem and the fields are decomposed into the transverse magnetic (TM) and the transverse electric (TE) polarization, in which the magnetic fields and the electric fields are perpendicular to z-axis, respectively.

The fields of the Floquet-mode are expressed by in the generalized Fourier expansion as

$$\psi(x, y) = \sum_{n=-\infty}^{\infty} \psi_n(y) e^{i\alpha_n x}, \qquad \alpha_n = \xi + nk_d \tag{1}$$

where ξ is the propagation constant of the Floquet-mode and $k_d = 2\pi/d$ is the inverse lattice constant. Since the fields in the surrounding medium satisfy the Helmholtz equation, they can be expressed in the plane-wave expansion as

$$\psi(x, y) = \boldsymbol{f}^{(+)}(x, y - y')^t \boldsymbol{\psi}^{(+)}(y') + \boldsymbol{f}^{(-)}(x, y - y')^t \boldsymbol{\psi}^{(-)}(y') \tag{2}$$

with

$$\left(\boldsymbol{f}^{(\pm)}(x, y)\right)_n = e^{i(\alpha_n x \pm \beta_n y)}, \qquad \beta_n = \sqrt{k_s^2 - \alpha_n^2} \tag{3}$$

where $k_s = \omega\sqrt{\varepsilon_s \mu_s}$ is the wavenumber in the surrounding medium, and $\boldsymbol{\psi}^{(\pm)}(y')$ are column matrices that is generated by the amplitudes of the plane-waves at $y = y'$.

Using RTMA with the lattice sums technique [8], the scattering by the periodic array located on $y = y_c$ is described by the following relation:

$$\begin{pmatrix} \boldsymbol{\psi}^{(+)}(y_c + 0) \\ \boldsymbol{\psi}^{(-)}(y_c - 0) \end{pmatrix} = \begin{pmatrix} \boldsymbol{S}_{c,11} & \boldsymbol{S}_{c,12} \\ \boldsymbol{S}_{c,21} & \boldsymbol{S}_{c,22} \end{pmatrix} \begin{pmatrix} \boldsymbol{\psi}^{(-)}(y_c + 0) \\ \boldsymbol{\psi}^{(+)}(y_c - 0) \end{pmatrix} \tag{4}$$

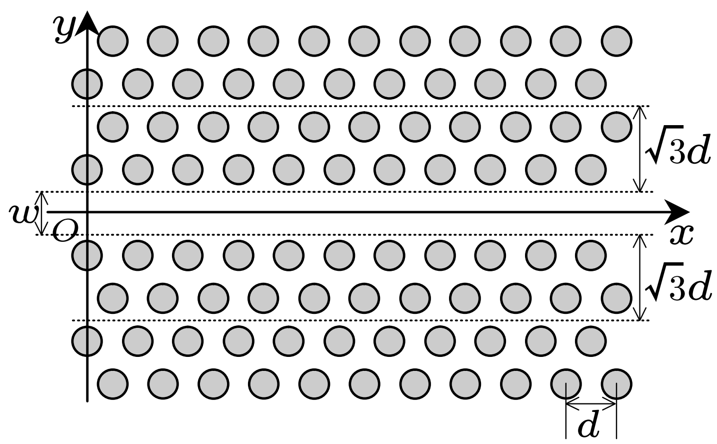

Figure 1: Geometry under consideration

where the scattering-matrices are given by

$$\boldsymbol{S}_{c,11} = \boldsymbol{B}^{(+)} \left(\boldsymbol{T}^{-1} - \boldsymbol{L}\right)^{-1} \boldsymbol{C}^{(-)} \tag{5}$$

$$\boldsymbol{S}_{c,12} = \boldsymbol{I} + \boldsymbol{B}^{(+)} \left(\boldsymbol{T}^{-1} - \boldsymbol{L}\right)^{-1} \boldsymbol{C}^{(+)} \tag{6}$$

$$\boldsymbol{S}_{c,21} = \boldsymbol{I} + \boldsymbol{B}^{(-)} \left(\boldsymbol{T}^{-1} - \boldsymbol{L}\right)^{-1} \boldsymbol{C}^{(-)} \tag{7}$$

$$\boldsymbol{S}_{c,22} = \boldsymbol{B}^{(-)} \left(\boldsymbol{T}^{-1} - \boldsymbol{L}\right)^{-1} \boldsymbol{C}^{(+)} \tag{8}$$

with

$$\left(\boldsymbol{B}^{(\pm)}\right)_{n,m} = \frac{2}{d\beta_n}\left(\frac{-i\alpha_n \pm \beta_n}{k_s}\right)^m, \qquad \left(\boldsymbol{C}^{(\pm)}\right)_{n,m} = \left(\frac{i\alpha_m \pm \beta_m}{k_s}\right)^n \tag{9}$$

$$(\boldsymbol{L})_{n,m} = \sum_{l=1}^{\infty} H^{(1)}_{m-n}(l\, k_s\, d)\left[(-1)^{m-n}\, e^{ild\xi} + e^{-ild\xi}\right] \tag{10}$$

$$(\boldsymbol{T}_m)_{n,m} = \delta_{n,m}\begin{cases} \frac{\zeta_s J_n(k_s a) J_n'(k_c a) - \zeta_c J_n'(k_s a) J_n(k_c a)}{\zeta_c H_n^{(1)\prime}(k_s a) J_n(k_c a) - \zeta_s H_n^{(1)}(k_s a) J_n'(k_c a)} & \text{for TM-polarization} \\ \frac{\zeta_c J_n(k_s a) J_n'(k_c a) - \zeta_s J_n'(k_s a) J_n(k_c a)}{\zeta_s H_n^{(1)\prime}(k_s a) J_n(k_c a) - \zeta_c H_n^{(1)}(k_s a) J_n'(k_c a)} & \text{for TE-polarization} \end{cases}. \tag{11}$$

ζ_c and ζ_s denote the characteristic impedances of the cylinder and the surrounding media, respectively, and k_c denotes the wavenumber in the cylinder medium. The relation (4) can be rewritten in the following form:

$$\begin{pmatrix} \boldsymbol{\psi}^{(+)}(y_c+0) \\ \boldsymbol{\psi}^{(-)}(y_c+0) \end{pmatrix} = \boldsymbol{K} \begin{pmatrix} \boldsymbol{\psi}^{(+)}(y_c-0) \\ \boldsymbol{\psi}^{(-)}(y_c-0) \end{pmatrix} \tag{12}$$

where

$$\boldsymbol{K} = \begin{pmatrix} \boldsymbol{S}_{c,12} - \boldsymbol{S}_{c,11}\boldsymbol{S}_{c,21}^{-1}\boldsymbol{S}_{c,22} & \boldsymbol{S}_{c,11}\boldsymbol{S}_{c,21}^{-1} \\ -\boldsymbol{S}_{c,21}^{-1}\boldsymbol{S}_{c,22} & \boldsymbol{S}_{c,21}^{-1} \end{pmatrix}. \tag{13}$$

Then, the transfer relation between the plane-wave amplitudes for the structure $w/2 < y < w/2 + \sqrt{3}d$, which is a periodicity unit for the upper PC wall, is derived as

$$\begin{pmatrix} \boldsymbol{\psi}^{(+)}(w/2+\sqrt{3}d) \\ \boldsymbol{\psi}^{(-)}(w/2+\sqrt{3}d) \end{pmatrix} = \boldsymbol{F}^{(u)} \begin{pmatrix} \boldsymbol{\psi}^{(+)}(w/2) \\ \boldsymbol{\psi}^{(-)}(w/2) \end{pmatrix} \tag{14}$$

where

$$\boldsymbol{F}^{(u)} = \left(\boldsymbol{U}(\tfrac{d}{2}, \tfrac{\sqrt{3}}{4}d)\boldsymbol{K}\boldsymbol{U}(0, \tfrac{\sqrt{3}}{4}d)\right)^2 e^{-id\xi} \tag{15}$$

$$\boldsymbol{U}(x,y) = \begin{pmatrix} \boldsymbol{V}(x,y) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{V}(x,-y) \end{pmatrix}, \qquad (\boldsymbol{V}(x,y))_{n,m} = \delta_{n,m} e^{i(\alpha_n x + \beta_n y)}. \tag{16}$$

Here, we use a matrix $\boldsymbol{R}^{(u)}$ given by

$$\boldsymbol{R}^{(u)} = \begin{pmatrix} \boldsymbol{R}_{11}^{(u)} & \boldsymbol{R}_{12}^{(u)} \\ \boldsymbol{R}_{21}^{(u)} & \boldsymbol{R}_{22}^{(u)} \end{pmatrix} = \left(\cdots\ \boldsymbol{r}_n^{(u)}\ \cdots\right). \tag{17}$$

with $\boldsymbol{r}_n^{(u)}$ are the nth-eigenvectors of $\boldsymbol{F}^{(u)}$, and define column matrices $\boldsymbol{b}^{(u,\pm)}(y)$ as

$$\begin{pmatrix} \boldsymbol{b}^{(u,+)}(w/2) \\ \boldsymbol{b}^{(u,-)}(w/2) \end{pmatrix} = \boldsymbol{R}^{(u)^{-1}} \begin{pmatrix} \boldsymbol{\psi}^{(+)}(w/2) \\ \boldsymbol{\psi}^{(-)}(w/2) \end{pmatrix} \tag{18}$$

The elements of the column matrices $\boldsymbol{b}^{(u,\pm)}(w/2)$ give the amplitudes of the Floquet modes propagating in the y-directions in the upper PC wall at $y = w/2$. Here, the eigenvectors $\boldsymbol{r}_n^{(u)}$ are arranged in such a way that the column matrices $\boldsymbol{b}^{(u,+)}(w/2)$ and $\boldsymbol{b}^{(u,-)}(w/2)$ correspond respectively to the Floquet-modes propagating in the $+y$- and $-y$-directions. Since $\boldsymbol{b}^{(u,-)}(w/2) = \boldsymbol{0}$ for the guided Floquet-modes of the considering PCW, we have the following relation:

$$\boldsymbol{\psi}^{(-)}(w/2) = \boldsymbol{R}_{21}^{(u)} \boldsymbol{R}_{11}^{(u)^{-1}} \boldsymbol{\psi}^{(+)}(w/2) \tag{19}$$

and the reflection matrix of the upper PC wall is given by $\boldsymbol{R}_{21}^{(u)} \boldsymbol{R}_{11}^{(u)^{-1}}$. Following the similar manipulation, the reflection matrix of the lower PC wall can be derived as $\boldsymbol{R}_{12}^{(l)} \boldsymbol{R}_{22}^{(l)^{-1}}$ where the superscript (l) indicates that the matrices are associated to the lower PC wall.

Considering plane-wave propagation in the gap between PC walls, we obtain an equation:

$$\left(\boldsymbol{R}_{21}^{(u)} \boldsymbol{R}_{11}^{(u)^{-1}} \boldsymbol{V}(0, w) \boldsymbol{R}_{12}^{(l)} \boldsymbol{R}_{22}^{(l)^{-1}} \boldsymbol{V}(0, w) - \boldsymbol{I} \right) \boldsymbol{\psi}^{(-)}(w/2) = \boldsymbol{0}. \tag{20}$$

Equation (20) has a nontrivial solution only when

$$\det \left(\boldsymbol{R}_{21}^{(u)} \boldsymbol{R}_{11}^{(u)^{-1}} \boldsymbol{V}(0, w) \boldsymbol{R}_{12}^{(l)} \boldsymbol{R}_{22}^{(l)^{-1}} \boldsymbol{V}(0, w) - \boldsymbol{I} \right) = 0. \tag{21}$$

This equation gives the dispersion equation for the Floquet-modes, and we obtain the propagation constants by solving Eq.(21) in terms of ξ.

3. Conclusion

This paper has presented a formulation of two-dimensional PCW consisting of circular cylinder array in triangular lattice. The fields in PCW can be expressed in the plane-wave expansions and the dispersion equation is obtained using the reflection matrices of PC walls. The present formulation derived the reflection matrix by the eigenvalue/eigenvector analysis of the transfer matrix for the periodicity unit of PC wall. This kind of approach has been found that there are limit in accuracy. To avoid this difficulty, we have also applied RTMA that uses the cylindrical-wave expansions to treat the boundary conditions adequately.

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