# Single Mode Analysis of Periodic Narrow Slit with a finite thickness

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## Abstract

A transmitted wave through a periodic narrow slit is analyzed numerically by a moment method limited to a single mode. Normal and oblique incidence cases are analyzed and compared to 3D simulation results. This analysis is very fast compared to 3D simulators and gives equivalent results.
**Keywords :** periodic narrow slit, moment method, aperture admittance, extraordinary transmission

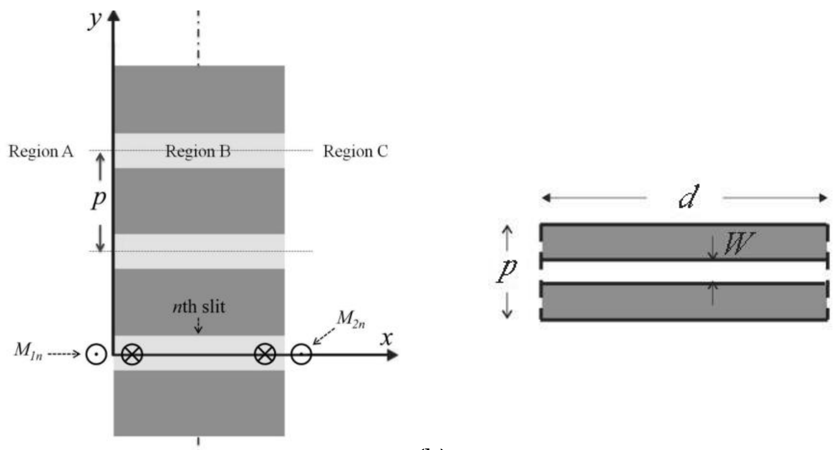
## 1. Introduction

Electromagnetic wave transmission through a single narrow slit has been studied in many fields. Here, the single narrow slit means that it has a very small aperture compared to the electrical length of the incident wave. The transmission characteristics of a single slit using a moment method [1], and the single mode transmission characteristics of a narrow single slit have been studied using equivalent circuit [2]. Along with these studies, the transmission characteristics of periodic slit array have been studied by various means, where an important problem is on extraordinary transmission peak and Fabry-Pérot resonances phenomena [3].

Recently, single mode transmission characteristics through periodic narrow slits were analyzed using a Fabry-Pérot regime and equivalent circuit [4]. This research provided clear physical insights about the problem, but is limited to the normal incident case.

In this paper, we will treat both a normal incident case and oblique incident case using the applied moment method. This analysis converges very fast compared to 3D simulators and gives equivalent results.

## 2. Admittance Matrix of unit cell




(a) (b)

Figure 1: (a) Periodic narrow slit array, (b) Unit cell slit structure

Slits are infinitely positioned along the y-direction with period *p* assuming also that they are infinite along the z-direction. The thickness of a slit is *d* and the width of each slit is *w*. $M_{1n}$ and $M_{2n}$

are equivalent magnetic sources of the *n*th slit aperture at each side. In this paper, three assumptions are needed to simplify the problem. First, a slit is very narrow. This means that the frequency of incident wave must be in $0 \le f_a \le f_{TM_1}$ , where the $f_{TM1}$ is a first cut off frequency of higher order modes. Second, we regard the incident wave as a plane wave. Third, region A and region C are the same media. So, we can extract the matrix based on the moment method.

$$\begin{bmatrix} [Y_a] & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & [Y_c] \end{bmatrix} \cdot \begin{bmatrix} V_{11} \\ \vdots \\ \underline{V_{1N}} \\ V_{2N} \\ \vdots \\ V_{2N} \end{bmatrix} + \begin{bmatrix} [Y_{11}^b] & [Y_{12}^b] \\ [Y_{21}^b] & [Y_{22}^b] \end{bmatrix} \cdot \begin{bmatrix} V_{11} \\ \vdots \\ \underline{V_{1N}} \\ V_{2N} \\ \vdots \\ V_{2N} \end{bmatrix} = \begin{bmatrix} I_1 \\ \vdots \\ \underline{I_N} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{1}$$

Because all slits have the same width, $V_{a1} \cdots V_{aN}$ are equal to the same constant. So, (1) can be expressed as,

$$\left. \begin{aligned} Y'_{an}V_{1n} + Y_{11}^b V_{1n} + Y_{12}^b V_{2n} &= I_n \\ Y_{21}^b V_{1n} + Y_{22}^b V_{2n} + Y'_{cn} V_{2n} &= 0 \end{aligned} \right) ;$$

$$\text{here,} \quad \begin{bmatrix} Y'_{an} = Y_{n1}^a + Y_{n2}^a \cdots + Y_{nn}^a + \cdots Y_{nN}^a \\ Y'_{cn} = Y_{n1}^c + Y_{n2}^c \cdots + Y_{nn}^c + \cdots Y_{nN}^c \end{bmatrix} \tag{2}$$

In (2), admittances $Y_a$ and $Y_c$ can be easily extracted from the equations of the equivalent magnetic field by finite current source [2] and $Y^b$s' are the components of the admittance matrix of a two wire transmission line.

$$Y_{nm} = \frac{k_a}{2\eta_a} \int_{\Delta y_m} \int_{\Delta y'_n} M_{1n} W_{1m} \cdot H_0^{(2)}(k_a |y - y'|) dy dy' \tag{3}$$

$$Y'_{an} = \sum_{m=-\infty}^{\infty} \frac{k_a}{2\eta_a} \int_{\Delta y_m} \int_{\Delta y'_n} M_{1n} \cdot W_{1m} \cdot H_0^{(2)}(k_a |y - y'|) dy dy' \tag{4}$$

$$\begin{bmatrix} Y_{11}^{n\ b} & Y_{12}^{n\ b} \\ Y_{21}^{n\ b} & Y_{22}^{n\ b} \end{bmatrix} = \begin{bmatrix} -jY_0 \cot k_b d & -jY_0 \csc k_b d \\ -jY_0 \csc k_b d & -jY_0 \cot k_b d \end{bmatrix} \tag{5}$$

In (3) and (4), we need to substitute a spatial integral of the second kind Hankel function to a spectral integral of it to be able to get exact solutions. So, (4) becomes as,

$$Y'_{an} = \frac{k_a}{\pi\eta_a} \cdot \sum_{m=-\infty}^{\infty} \int_{\Delta y_m} \int_{\Delta y_{n'}} M_{1n} \cdot W_{1m} \cdot \left\{ \left( \int_0^{k_a} \frac{1}{\sqrt{{k_a}^2 - {k_y}^2}} \cdot e^{-jk_y y} dk_y \right) dy dy' + j \left( \int_{k_a}^{\infty} \frac{1}{\sqrt{{k_y}^2 - {k_a}^2}} \cdot e^{-jk_y y} dk_y \right) dy dy' \right\} \tag{6}$$

Calculating the spectral integral of (6) using a Fourier series formation gives $Y'_{an}$ as,

$$Y'_{an} = \left( \frac{\pi}{\lambda_a \eta_a} + \frac{2}{\eta_a p} \right) + j \left( \frac{2}{\lambda_a \eta_a} \cdot \ln\left( \frac{1}{C k_a w} \right) + \frac{2}{\eta_a p} \cdot \left\{ \frac{\sin(\pi\delta)}{\pi\delta} \right\}^2 \cdot \frac{1}{\sqrt{\left( \frac{f_{TM_{02}}}{f_a} \right)^2 - 1}} \right) \tag{7}$$

Here, we can let $M_{1n}$ and $W_{1m}$ be $1/w$ and $\delta$ is the ratio $w$ and $p$. This means that the Galerkin Method is used and that their integrated values are equal to one because of the assumption of single mode transmission [2]. In (7), the real and imaginary terms represent the impedance of a single slit aperture and the coupled impedance of the slit array structure. These represent a Floquet mode of single mode. In the case of oblique incident, $M_{1n}$ has a specific phase term according to the position of the *n*th slit and the incident angle of the wave.

# 3. Transmittance and Extraordinary Transmission

Transmittance is defined as the ratio of incident power and transmitted power. From [2], we can also use the same admittance of $Y_{12}$ of the *n*th slit.

$$Y_{12} = (Y^a + Y^c)\cos(k_b d) + j\left( Y_0 + \frac{Y^a Y^c}{Y_0} \right)\sin(k_b d) \tag{8}$$

Because of the assumption of a plane wave, the incident power and transmitted power are

$$P_{inc} = \frac{|H_0|^2 w}{\mathrm{Re}\{Y^a\}} \quad \text{and} \tag{9}$$

$$P_{trans} = \left| \frac{I_i}{Y_{12}} \right|^2 \mathrm{Re}\{Y_c\} \tag{10}$$

So, the transmittance can be expressed as,

$$\text{Transmittance} = \frac{P_{trans}}{P_{inc}} = \frac{4\,\mathrm{Re}\{Y^c\}^2}{w|Y_{12}|^2} \tag{11}$$

# 4. Results of Numerical analysis

Because of the limited space, we show results about the normal incident case.

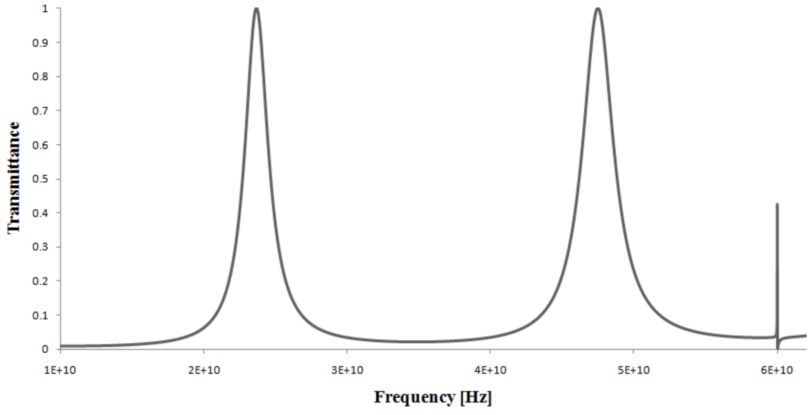



Figure 2: Transmittance vs. frequency using proposed analysis.

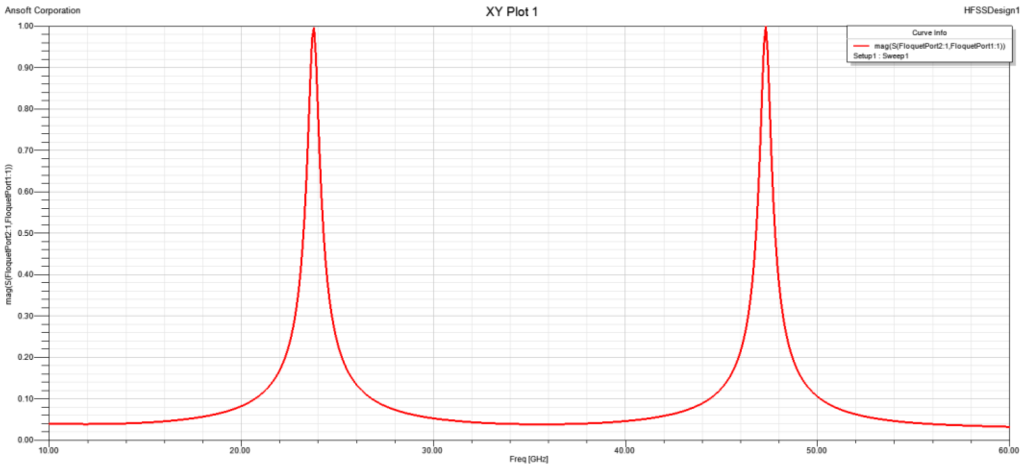



Figure 3: Transmittance using CST simulator.

3D simulation takes about one hour but we needed several seconds to get a desired result. Only things we need are the dimensions of the structure and the incident angle of the wave.

## 5. Conclusions

This paper introduced a new analysis of single mode transmission through a periodic narrow slit array. This analysis expresses the aperture admittance as three different parts: Floquet mode, dominant mode and higher order mode. The transmittance coefficient results matched those of 3D simulator normal and oblique incident cases. At 60 GHz, the extraordinary transmission peak appears very sharply. This analysis reduces the time to get to desired results and produces good physical insight.

## References

[1] R.F.Harrington, "Electromagnetic transmission through a filled slit in a conducting plane of finite thickness, TE case," *IEEE Trans.Microwave Theory Tech.*, vol.MTT-26, NO.7, July 1978

[2] R.F.Harrington, "Electromagnetic transmission through narrow slots in thick conducting screen -s," *IEEE Trans. Antennas Propagat.*, vol.AP-28, No.5, September 1980

[3] Yunping Qi, Jungang Miao, Sheng Hong." Characterization of Extraordinary Transmission for a Single Subwavelength Slit: A Fabry-Pérot-Like Formula Model," *IEEE Trans.Microwave Theory Tech.*, vol.58, NO.12, December 2010

[4] F.Medina, F.Mesa, "Extraordinary Transmission Through Arrays of Slits: A Circuit Theory Model," *IEEE Trans.Microwave Theory Tech.*, vol.58, NO.1, January 2010

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