EBG periodic structures analysis with Parallel FETI Domain Decomposition Techniques

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Abstract

This paper presents the implementation of Finite Element Tearing and Interconnecting (FETI) methods for solving electromagnetic frequency domain problems encountered in the design of electromagnetic band gap materials (EBG).

Keywords: EBG materials, Domain Decomposition Techniques, FETI, Finite Element Method

1. Introduction

Domain Decomposition Methods have demonstrated efficiency and accuracy during the resolution of Maxwell Equations in the frequency domain for both RCS applications and Antenna structures interactions [1]. In the domain of Finite Element Methods, and for the resolution of acoustic Helmoltz equations, efficient sub-domain connecting techniques have been applied and called « Dual-Primal Finite Element Tearing and Interconnecting » [2]. These techniques are known under the acronym FETI-DP and have been adapted to electromagnetic (FETI-DPEM) for the calculation of antenna arrays and metamaterial periodic structures [3].

In this paper we describe the implementation of an algebraic parallel FETI solver developed at ONERA for both acoustic and electromagnetic applications. It has been called as a library in the FEM module of the FACTOPO tool developed for both Antenna and RCS applications. The efficiency of the proposed method will be assessed throughout the computation of the electromagnetic field transmission in an X band EBG material constituted by an array of dielectric rods.

2. FEM domain decomposition method

The general principle of the FETI methods for Maxwell Equations is to decompose the global computational domain in non overlapping sub-domains in which local solution fields are calculated by solving the Finite Element System with a direct method. We then impose the tangent field continuity on the interfaces by using Lagrange multiplier. It results a reduced problem on interfaces which would be solved by an iterative method. The solution of the interface problem would be used as a boundary condition for evaluating the field in each sub-domain. We denote $\Omega = \Omega_1 \cup \Omega_2 .... \Omega_N$ a partition of the initial computation domain. In each sub-domain Ω_i (Figure 1) we are calculating in each sub-domain the diffracted fields $\vec{E}^{\,i}$ verifying:

$$\nabla \times (\mu_{r,i}^{-1} . \nabla \times \vec{E}^i) - k_0^2 \varepsilon_{r,i} \vec{E}^i = k_0{}^2 (\varepsilon_{r,i} - \mu_{r,i}^{-1}) \vec{E}_{incident} \quad \text{in } \Omega_i \quad (1)$$

$$\vec{n}_{ext} \times \nabla \times \vec{E}^i + j k_0 \vec{n}_{ext} \times (\vec{n}_{ext} \times \vec{E}^i) = 0 \quad \text{in } \Gamma_{ABC} \; (2)$$

The vector $\vec{E}_{incident}$ is representing the electric incident field in the volume Ω_i. Γ_{ABC} represents the boundary of the volume Ω_i where the field is verifying Absorbing Boundary Conditions (ABC).

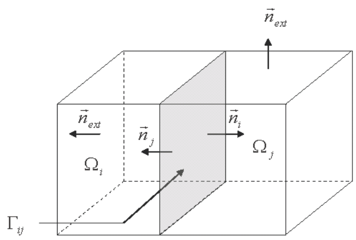


Figure 1 : interface problem

In the following we will denote $\vec{E}^i_j$ the electric field on the interface of the sub-domain Ω_i adjacent to the sub domain Ω_j . On the interfaces Γ^{robin}_{ij} separating two sub-domains Ω_i and Ω_j , we impose Robin type boundary conditions by using Lagrange multipliers $\vec{\Lambda}^i_j$ and $\vec{\Lambda}^j_i$ which will be new unknowns:

$$\vec{n}_i \times (\mu^{-1}_{r,i}.\nabla \times \vec{E}^i_j) + jk_0 \vec{n}_i \times (\vec{n}_i \times \vec{E}^i_j) = \vec{\Lambda}^i_j \qquad (3)$$

$$\vec{n}_j \times (\mu^{-1}_{r,j}.\nabla \times \vec{E}^j_i) + jk_0 \vec{n}_j \times (\vec{n}_j \times \vec{E}^j_i) = \vec{\Lambda}^j_i \qquad (4)$$

The tangential electric and magnetic field continuity on the interfaces Γ^{ij}_{robin} separating the two sub domains Ω_i and Ω_j leads to the following relations that should be verified by the multipliers $\vec{\Lambda}^i_j$ and $\vec{\Lambda}^j_i$:

$$\left.\begin{array}{l} \Lambda^i_j + \Lambda^j_i - 2jk_0 \vec{n}_i \times (\vec{n}_i \times \vec{E}^i_j) = 0 \\ \Lambda^i_j + \Lambda^j_i - 2jk_0 \vec{n}_j \times (\vec{n}_j \times \vec{E}^j_i) = 0 \end{array}\right. \quad \text{on } \Gamma^{ij}_{robin} \qquad (5)$$

The weak formulation used for the computation of the fields $\vec{E}^i$ in each volume Ω_i is:

Find $\vec{E}^i \in H(Rot^0, \Omega_i)$ such that, $\forall \vec{W} \in H(Rot^0, \Omega_i)$:

$$\begin{aligned} &\int_{\Omega_i} \left[\mu^{-1}_{r,i}.(\nabla \times \vec{E}^i).(\nabla \times \vec{W}) - k_0^2 \varepsilon_{r,i}.\vec{E}^i.\vec{W}\right] d\Omega + jk_0 \int_{\Gamma_{ext}} (\vec{n} \times \vec{E}^i).(n \times \vec{W}) d\Gamma \\ &+ jk_0 \int_{\Gamma^{ij}_{robin}} (\vec{n} \times \vec{E}^i_j).(n \times \vec{W}) d\Gamma = k_0{}^2 \int_{\Omega_i} (\varepsilon_{r,i} - \mu^{-1}_{r,i}) \vec{E}^i.\vec{W} d\Omega \end{aligned} \qquad (6)$$

The iterative resolution of the interface problem (5) is based on a Krylov sub-space. We write equivalently:

$$\lambda^i_j + \lambda^j_i - (M^i_j + M^j_i) E^i_j = 0 \qquad \forall \quad i = 1,2,N \text{ , } j \text{ neighbor of } i \qquad (7)$$

with

$$M^i_j = jk_0 \int_{\Gamma_{ij}} (\vec{n}_i \times \vec{W}_i).(\vec{n}_i \times \vec{W}_i) d\Gamma \qquad (8)$$

The iterative method is using for steps:

1. Calculation of local solutions in each sub domain with the use of Robin type conditions by solving the problem (6)
2. Exchange fields $\vec{E}$ and Lagrange multipliers $\vec{\Lambda}$ on each interface
3. Computation of $g^i_j = \lambda^i_j + \lambda^j_i - (M^i_j + M^j_i) E^i_j$ on each interface
4. Implementation of GMRES iterations with a stop criterion $\|g\| < \varepsilon$

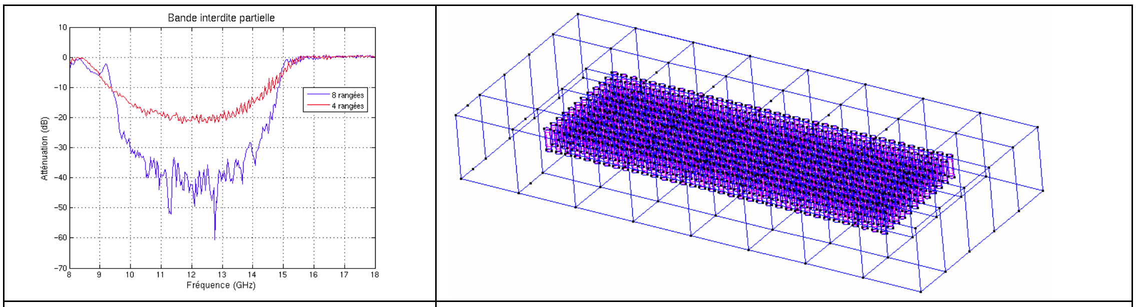
3. Implementation and prediction of EBG materials

In this paper the method is optimized for large periodic and finite arrays. The implementation is using MPI libraries and we target the exploitation of the methodology on massively parallel computers. Although the method is general and adapted for any 3D problem, only one unit cell of the periodic structure is considered and coincident edges on master and slave interface are considered. So as to solve large scale problems, the unit cell could be a periodic structure itself and the calculation of the local solution of step 1 would be done with the sparse direct PARDISO solver using the Intel "mkl" library on each processor. The implementation strategy was also to separate the electromagnetic part of the code to the FETI solver part. Then the volume and surface elementary matrices coming from the electromagnetic weak formulation (6) are formed in the FEM module of FACTOPO while the linear algebraic resolution of the local problem and the interface problem are driven with FORTRAN calls to the FETI library.

During the presentation, the accuracy and efficiency of the proposed method will be discussed for Electric Band Gap prediction. We consider here the calculation of the diffracted field by an array of alumina dielectric rods (Figure 2b). The array is exited by a unitary plane wave whose electric field is polarized parallel to the axis of the rods and whose incidence is perpendicular to the longer side of the array ($\theta = 90, \varphi = 0$). The calculation frequency is 12 GHz and is located in the forbidden frequency band as shown in the measured transmission diagram (Figure 2a) obtained by considering 4 rows (red curve) and 8 rows (blue curve). The leading dimensions and radio electric properties are summarized in the Table 1. The convergence of the FETI method is analyzed by considering array whose size increases (24, 432 and 864 rods). For all the arrays the unit cell is meshed with 326,650 edges and zero order Nedelec edge functions are used. The evolution of the CPU time and the number of iterations required for a convergence lower than 10-6 is indicated in Table 2 and Figure 4. The calculation of the electric field in the EBG structure shows a strong attenuation of the field as indicated by the measured data (Figure 2a).

Table 1: geometry of EBG rods

Rods diameter (mm)	4
Rod length (mm)	15
Array step (mm)	7
Alumina permittivity	$\varepsilon_r = 9.4$



a) Transmission coefficients | b) EBG array

Figure 2: EBG material investigated

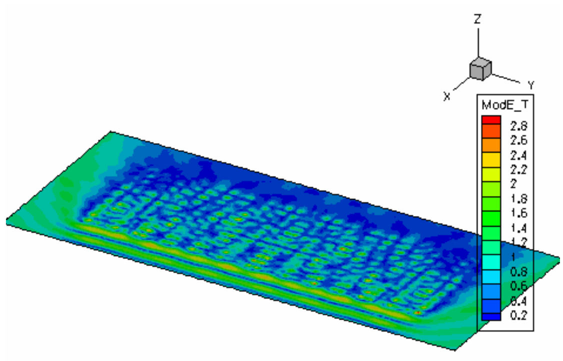
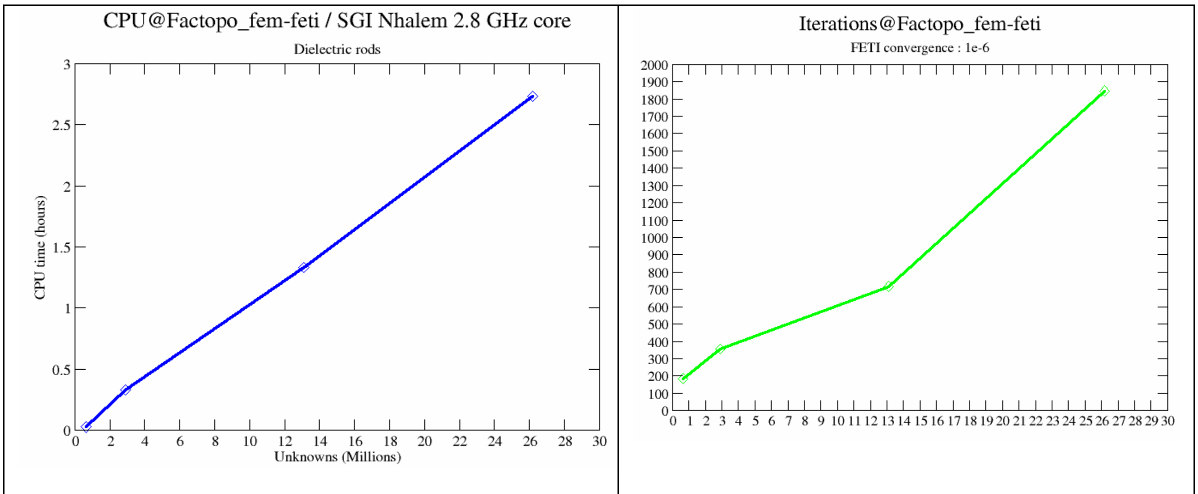


Figure 3: E fields at 12 GHz in EBG material

Table 2: CPU times for growing arrays

Total unknowns	2.9 M	13.1 M	26.2 M
Dielectric rods	24	432	864
Cores (SGI Nhalem, 2.8 GHz)	9	40	80
FETI Iterations	357	717	1847
convergence	10^{-6}	10^{-6}	10^{-6}
Factorisation of local problem	89 s	89 s	89 s
LU resolution	0.9 s	0.9 s	0.9 s
Total CPU time	20 minutes	1h20 minutes	2h44 minutes



a)CPU time	b) Iterations ; 10^{-6} convergence

Figure 4: CPU time and number of iteration for array growing from 4 rods to 864 rods

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