Surface and Leaky-Wave Modes in a Grounded Dielectric Slab Covered With Graphene

[#]S. Couture, D. L. Sounas, and C. Caloz
Department of Electrical Engineering, École Polytechnique de Montréal
QC, H3T 1J4, Canada, [#]simon-2.couture@polymtl.ca

Abstract

The surface and leaky-wave modes supported by a grounded dielectric slab covered with graphene are studied. The dispersion curves of these modes are obtained by using an equivalent transmission line model. It is found that the surface and leaky-wave modes are profoundly influenced by the presence of conductive graphene. The tunability of the leaky modes through the chemical potential is explored and suggested as a possibility for a steerable leaky-wave antenna.

Keywords : Surface waves Leaky-waves Graphene Dielectric slab

1. Introduction

Recently, many research efforts have been dedicated to graphene, a 2D material with unique physical properties. While the electronic properties of graphene have received a great deal of attention towards the development of nanoscale devices, little research has been done regarding its electromagnetic properties at the macroscopic scale. The most important of studies in the area include the analysis of surface waves guided along a graphene sheet [1] and the recent demonstration of non-reciprocal gyrotropy in graphene [2]. This paper investigates the surface and leaky-wave modes supported by a grounded dielectric slab covered with graphene.

2. Physical Structure and Field Solution

The structure studied is shown in Fig. 1. It consists of a grounded dielectric slab of thickness d covered with a sheet of graphene. Graphene is modeled as a zero-thickness sheet with surface conductivity [3]

$$\sigma = -j\frac{e^2 k_B T}{\pi\hbar^2(\omega - j2\Gamma)}\left[\frac{\mu_c}{k_B T} + 2\ln\left(1 + e^{-\frac{\mu_c}{k_B T}}\right)\right], \tag{1}$$

where e is the electron charge, k_B is the Boltzmann's constant, $\hbar$ is the Planck's constant, T is the temperature, ω is the angular frequency, Γ is the scattering rate and μ_c is the chemical potential.

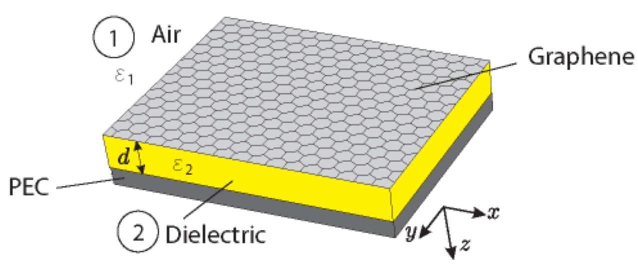

Figure 1: Graphene sheet covering a grounded dielectric slab of thickness d. Region 1 corresponds to the air while region 2 corresponds to the dielectric slab.

The field solution to the problem of Fig. 1 is separated into TE_z and TM_z modes, which have zero electric and magnetic field components along the z-axis, respectively. The development for the TE_z modes is presented below. The TM_z modes are obtained by a similar procedure. Consider a TE_z mode propagating along the x-axis with wavenumber k_x. Taking into account that

E_z=0 and k_y=0, the Maxwell Gauss equation, $\nabla\cdot\mathbf{E}=j(k_xE_x+k_yE_y+k_zE_z)=0$, yields E_x=0. Therefore, the electric field has only a y component and the Maxwell curl equations take the form

$$\frac{\partial}{\partial z}E_{y1,2}=j\omega\mu_{1,2}H_{x1,2}, \tag{2-a}$$

$$\frac{\partial}{\partial z}H_{x1,2}=j\omega\varepsilon_{1,2}\left(\frac{k_{z1,2}^2}{k_{1,2}^2}\right)E_{y1,2}, \tag{2-b}$$

where $k_{1,2}^2=\omega^2\mu_{1,2}\varepsilon_{1,2}$, $k_{z1,2}^2=k_{1,2}^2-k_x^2$ and the subscript 1 or 2 denotes the corresponding region.

Equations (2) are equivalent to the telegrapher's equations in transmission line theory if we define the voltage as $V_{1,2}=E_{y1,2}$ and the current as $I_{1,2}=-H_{x1,2}$. Then, the wavenumber for propagation along the z-axis is $k_{z1,2}$ and the characteristic impedance $Z_{01,2}=\omega\mu_{1,2}/k_{z1,2}$. The structure of Fig. 1 can therefore be modeled by the transmission line equivalent circuit shown in Fig. 2. To model the graphene sheet, notice that it supports a surface current $J_y=\sigma E_y$. By inserting this current into the boundary condition $\hat{\mathbf{n}}\times(\mathbf{H}_1-\mathbf{H}_2)=\mathbf{J}$ at the air-dielectric interface, where $\hat{\mathbf{n}}=-\hat{\mathbf{z}}$, we get $\sigma E_y=H_{x2}-H_{x1}$, or in terms of the equivalent voltages and currents, $V=(I_1-I_2)/\sigma$. This allows the graphene sheet to be modeled by a simple shunt impedance $1/\sigma$.

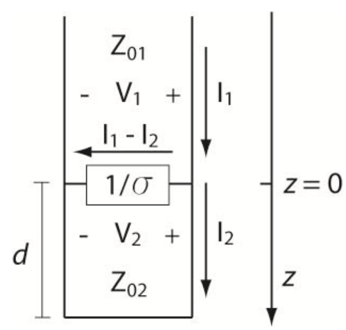

Figure 2 : Transmission line equivalent model for the structure of Fig. 1. The grounded dielectric slab is modeled by a short-circuited transmission line of length d while the graphene sheet is modeled by a simple shunt impedance.

The field in any of the regions 1 and 2 in Fig. 1 may be expressed as the superposition of two waves travelling along $\pm z$ directions. However, the surface waves and the leaky waves exist without any incident wave (a wave propagating along the $+z$ direction) and, therefore, the field in region 1 consists of only a reflected wave, propagating along the $-z$ direction. The existence of a reflected wave without an incident one is only possible when the reflection coefficient is infinite. Thus, the surface and leaky waves are found from the poles of the reflection coefficient. The latter is found from the transmission line model of Fig. 2 as

$$R=\frac{j(1+\sigma Z_{01})Z_{02}\tan(k_{z2}d)-Z_{01}}{j(1+\sigma Z_{01})Z_{02}\tan(k_{z2}d)+Z_{01}}, \tag{3}$$

and its poles are the solutions to the equation

$$Z_{01}+j(1+\sigma Z_{01})Z_{02}\tan(k_{z2}d)=0. \tag{4}$$

3. Results and Discussion

Dispersion curves for the modes supported by the structure of Fig. 1 are shown in Fig. 3 for both the TE$_z$ and TM$_z$ cases. Because of $k_{z1}=\sqrt{k_1^2-k_x^2}$ and the ambiguity in the sign of the square root, two sets of modes exist: proper modes with Im$\{k_{z1}\}$<0, which decrease exponentially as we move away from the structure along the z-axis, and improper modes with Im$\{k_{z1}\}$>0, which increase exponentially as we move away from the structure along the z-axis. Notice that there is no such ambiguity for k_{z2} since (4) is an even function of k_{z2}.

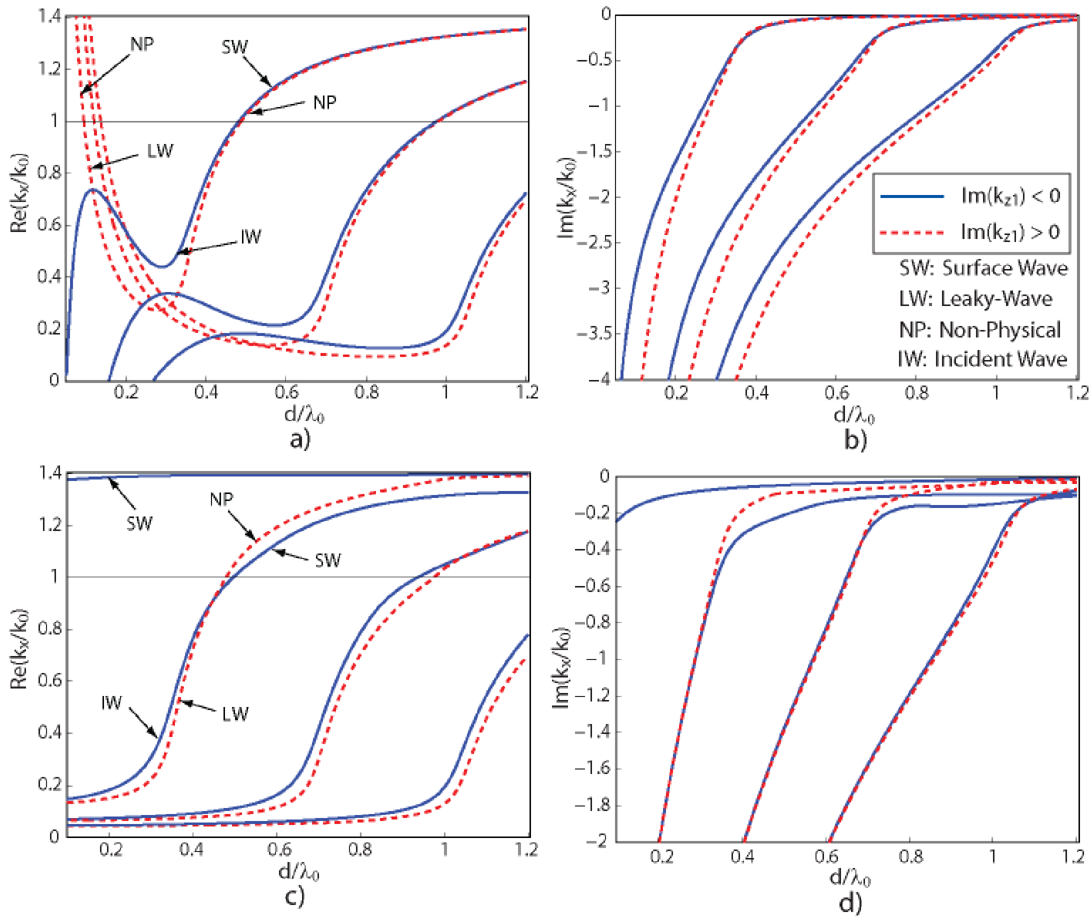

Figure 3: Dispersion curves for the surface and leaky wave modes supported by the structure of Fig.1, for ε_2=2, d=10 mm, μ_c=0 eV and T=300 K. (a) Re$\{k_x/k_0\}$ for the TE$_z$ modes. (b) Im$\{k_x/k_0\}$ for the TE$_z$ modes. (c) Re$\{k_x/k_0\}$ for the TM$_z$ modes. (d) Im$\{k_x/k_0\}$ for the TM$_z$ modes.

The dispersion diagrams shown in Fig. 3 differ in several ways from the dispersion diagrams for the problem with no graphene sheet (e.g. [4]). In the TE$_z$ case presented in Fig. 3(a), the conductive layer of graphene is responsible for the appearance of a 1st order leaky-wave (characterized by only one maximum of the electric field in the substrate, not shown here) which is not present in the case without graphene. It is also noted that below the cutoff of the standard proper surface wave, corresponding to Re$\{k_x\}$=k_0, the dispersion curve is continuous and extends in the Re$\{k_x\}$<k_0 region, seeming to indicate the existence of a fast, proper wave. However, it is postulated that these solutions correspond to a wave incident on the grounded dielectric slab satisfying the Salisbury screen condition, hence for which there is no reflected wave. It is noted that the improper modes in the slow wave region Re$\{k_x\}$>k_0 are unphysical, because they are characterized by an exponential increase of the fields in the air region away from the dielectric slab while not providing radiation toward a real physical angle in space [5].

The fast and proper wave found in the TE$_z$ case is also present in the TM$_z$ case. While TM$_z$ leaky-wave modes are also present in the case without graphene, here these leaky-waves do not have a lower cutoff frequency. Additionally, the spectral gap between the upper cutoff frequency of these modes and the cutoff frequency of the surface modes is greatly reduced by the presence of graphene, a phenomenon similar to what is reported in [6].

An important characteristic of graphene is tunable conductivity through μ_c, which provides control over the characteristics of the surface and leaky modes in the structure of Fig. 1. Figure 4 depicts the effect of varying μ_c on k_x for the left-most TM_z leaky wave shown in figures 3(c) and 3(d) for three different d/λ_0 values. At low frequencies (d=0.33λ_0), the tuning range of k_x is large, but accompanied with high losses, as seen from Fig. 3(d). As the frequency increases, losses decrease, however with a simultaneous decrease in the tunability range on k_x. This control over k_x could be potentially used in the steering of the radiation angle of a leaky-wave antenna based on this structure. The tuning of μ_c may be realized by applying a DC voltage between graphene and the ground plane.

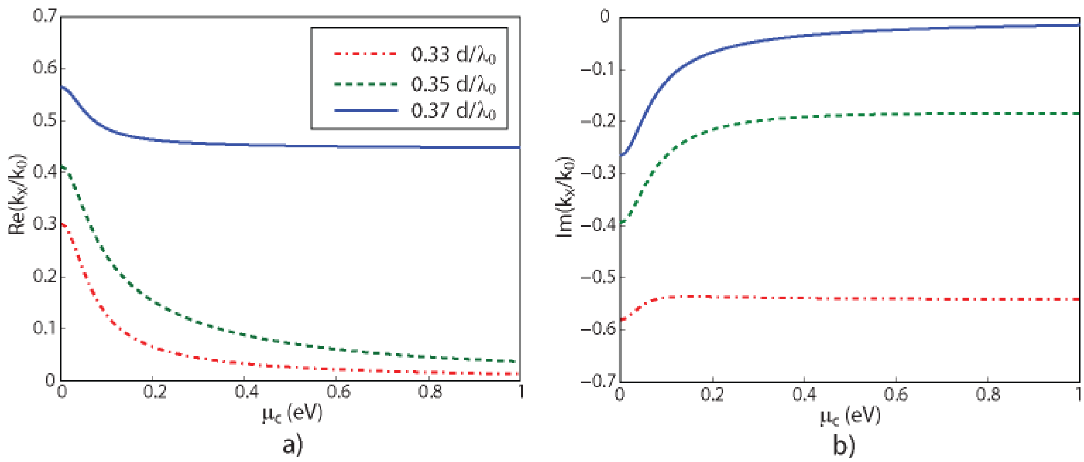

Figure 4 : Value of k_x versus the chemical potential μ_c for the left-most TM_z leaky-wave shown in Figs. 3(c) and 3(d), for different values of d/λ_0. (a) Re$\{k_x/k_0\}$. (b) Im$\{k_x/k_0\}$.

4. Conclusion

The surface and leaky-wave modes in a grounded dielectric slab covered with graphene have been investigated. The presence of the conductive graphene sheet changes considerably the modes dispersion characteristics compared to the case without graphene. Furthermore, the possibility of tuning the leaky-mode characteristics through the chemical potential has been explored. The potential application of this phenomenon in a steerable leaky-wave antenna has been suggested.

References

[1] A. H. C. Neto, F. Guinea, N. M. R. Peres, K. S. Novoselov, and A. K. Geim, "The electronic properties of graphene," *Rev. Mod. Phys.*, vol. 81, pp. 109–162, 2009.

[2] D. L. Sounas and C. Caloz, "Electromagnetic nonreciprocity and gyrotropy of graphene," *Appl. Phys. Lett.*, vol. 98, 021911, 2011.

[3] G. W. Hanson, "Dyadic Green's functions and guided surface waves for a surface conductivity model of graphene," *J. Appl. Phys.*, vol. 103, 064302, 2008.

[4] A. Shahvarpour, A. Alvarez-Melcon, and C. Caloz, "Broadband and low beam squint 2D leaky-wave antenna constituted by a uniaxially anisotropic grounded slab," *Radio Sci.*, To be published.

[5] D. R. Jackson and A. A. Oliner, "Leaky-Waves Antennas," in *Modern Antenna Handbook*, C. A. Balanis, Ed., John Wiley & Sons, 2008, ch. 7.

[6] H. Shigesawa, M. Tsuji, and A. A. Oliner, "The nature of the spectral gap between bound and leaky solutions when dielectric loss is present in printed-circuit lines," *Radio Sci.*, 28, pp. 1235-1243, 1993.