Scattering of Electromagnetic Waves by Inhomogeneous Dielectric Gratings Loaded with Parallel Perfectly Conducting strips

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Abstract

We have proposed a new method for the scattering of electromagnetic waves b y inhomogeneous dielectric gratings loaded with parallel perfectly conducting strips using the combination of improved Fourier series expansion method and point matching method. Numerical results are given for the transmitted scattered characteristics for TE and TM cases.

Keywords : Inhomogeneous Dielectric Gratings, Perfectly Conducting Strips

1. Introduction

Recently, the refractive index can easily be controlled to make the periodic structures such as optoelectronic devices, photonic bandgap crystals, frequency selective devices, and other applications by the development of manufacturing technology of optical devices. Thus, the scattering and guiding problems of the inhomogeneous gratings have been considerable interest, and many analytical and numerical methods which are applicable to the dielectric gratings having an arbitrarily periodic structures combination of dielectric and metallic materials[1]-[4] .

In this paper, we proposed a new method for the scattering of electromagnetic waves by inhomogeneous dielectric gratings with parallel perfectly conducting strips[10] using the combination of improved Fourier series expansion method[5]-[7] and point matching method[8]-[9].

Numerical results are given for the transmitted scattered characteristics for the case of frequency loaded with parallel perfectly conducting strips for TE and TM cases. The effects of the inhomogeneous dielectric gratings comparison with that of the slanted angle of the perfectly conducting strips on the transmitted power are discussed. Our approach also can treat periodic configurations having arbitrary combinations of dielectric, metallic, and perfectly conducting components.

2. Method of Analysis

We consider inhomogeneous dielectric gratings loaded with parallel perfectly conducting strips shown in Fig.1. The grating is uniform in the *y*-direction and the permittivity $\varepsilon(x,z)$ is an arbitrary periodic function of z with period p. The permeability is assumed to be μ_0 . The time dependence is $\exp(-i\omega t)$ and suppressed throughout.

In the formulation, the TE wave is discussed. When the TE wave (the electric field has only the *y*-component) is assumed to be incident from $x>0$ at the angle θ_0 ,

$$E_y^{(i)} = e^{ik_1(z\sin\theta_0 - x\cos\theta_0)}, \quad k_1 \triangleq \omega\sqrt{\varepsilon_1\mu_0} \qquad (1)$$

the electric fields in the regions $S_1\,(x \le 0)$, and $S_3\,(x \ge D)$ are expressed[10] as

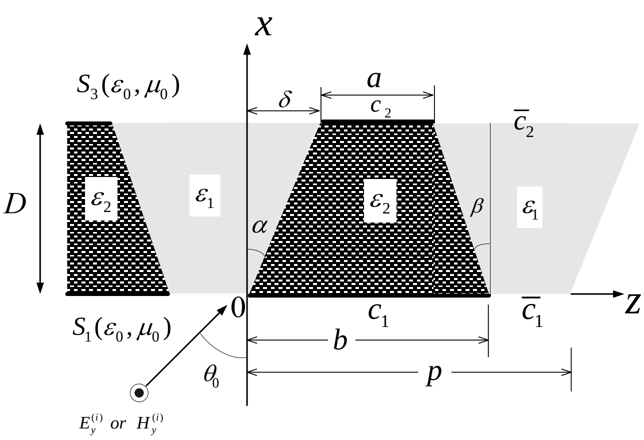


Fig.1 Structure of inhomogeneous dielectric gratings loaded with three perfectly conducting strips

$\underline{S_1(x \le 0)\ :}$

$$E_y^{(1)} = E_y^{(i)} + e^{ik_1 z\sin\theta_0} \sum_{n=-N}^{N} b_n^{(1)} e^{i\left(-k_n^{(1)}x + 2\pi nz/p\right)} \tag{2}$$

$\underline{S_3(x \ge D)\ :}$

$$E_y^{(3)} = e^{ik_1 z\sin\theta_0} \sum_{n=-N}^{N} c_n^{(3)} e^{i\left\{ k_n^{(3)}(x-D) + 2\pi nz/p\right\}} \tag{3}$$

$$H_z^{(j)} = \left\{ i\omega\mu_0 \right\}^{-1} \partial E_y^{(j)} \big/ \partial x\ ,\ (j = 1, 3)$$

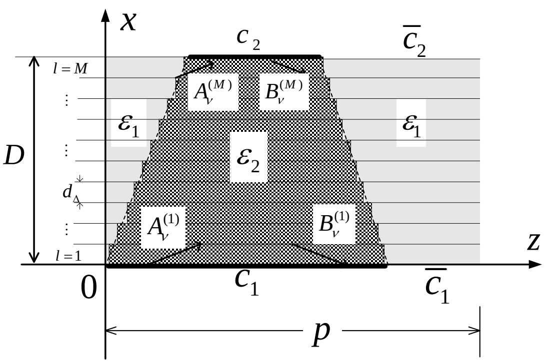


Fig.2 Approximated inhomogeneous layer

The inhomogeneous layer is approximated by a stratified layers of modulated index profile with d_Δ shown in Fig.2.and taking each layer as a modulated dielectric grating, the electromagnetic fields are expanded appropriately by a finite Fourier series as follows:

$\underline{S_2(0 < x < D)\ :}$

$$E_y^{(2,l)} = \sum_{\nu=1}^{2N+1} \left[A_\nu^{(l)} e^{ih_\nu^{(l)} x} + B_\nu^{(1)} e^{-ih_\nu^{(l)}(x - d_\Delta)} \right] f_\nu^{(l)}(z) \tag{4}$$

$$f_\nu^{(l)}(z) \triangleq e^{ik_1 \sin\theta_0 z} \sum_{m=-N}^{N} u_m^{(\nu,l)} e^{i2\pi mz/p}\quad ,l = 1 \sim \mathrm{M}$$

where $b_n^{(1)}, A_\nu^{(1\sim \mathrm{M})}, B_\nu^{(1\sim \mathrm{M})}, and\ c_n^{(3)}$, and $C_n^{(3)}$ are unknown coefficients to be determined from boundary conditions. $k_n^{(j)}$ $(j = 1, 3)$ is propagation constants in the x direction, and , $h_\nu^{(k)}$ $,u_n^{(\nu,l)}(l = 1 \sim \mathrm{M})$,the propagation constant and eigenvectors, are satisfy the following eigenvalue equation in regard to h [5]

$$\mathbf{\Lambda U} = h^2 \mathbf{U} \tag{5}$$

where,

$$\mathbf{U}^{(\nu,l)} \triangleq \left[u_{-N}^{(\nu,l)}, \cdots, u_0^{(\nu,l)}, \cdots u_N^{(\nu,l)} \right]^T, T\ :\ \text{transpose},\quad \mathbf{\Lambda} \triangleq \left[a_{m,n}^{(l)} \right],\quad a_{n,m}^{(l)} \triangleq k_1{}^2 \xi_{n,m}^{(l)} - (2\pi n/p + k_1 \sin\theta_0)^2\ ,$$

$$\xi_{n,m}^{(l)} \triangleq \frac{1}{p}\int_0^p \left\{ \frac{\varepsilon_2^{(l)}(z)}{\varepsilon_0} \right\} e^{i2\pi(n-m)z/p} dz\ , m,n = (-N, \cdots, 0, \cdots, N)$$

We obtain the matrix form combination of metallic region C and the dielectric region $\overline{C}$ using boundary condition $Z_j = (j-1)p/[(2N+1)]$; $j = 1 \sim (2N+1)$ at the matching points on $x =$ 0, *and* D. Boundary condition using Point Matching are as follows:

$$Z_j \in C_1\ ;\left[E_z^{(1)} = 0,\ E_z^{(2,1)} = 0 \right]_{x=0}\quad ,\quad Z_j \in \overline{C_1}\ ;\left[E_y^{(1)} = E_y^{(2,1)} \right]_{x=0}\ ,\ \left[H_z^{(1)} = H_z^{(2,1)} \right]_{x=0} \tag{6}$$

$$Z_j \in C_3\ ;\left[E_z^{(2,M)} = 0,\ E_z^{(3)} = 0 \right]_{x=D}\quad ,\quad Z_j \in \overline{C_3}\ ;\left[E_y^{(2,M)} = E_y^{(3)} \right]_{x=D}\ ,\ \left[H_z^{(2,M)} = H_z^{(3)} \right]_{x=D} \tag{7}$$

In the boundary condition at Eq.(6), and Eq.(7), it is satisfied in all matching points by using the orthogonality properties of $\{e^{i2\pi nz/p}\}$,we get following equation in regard to $A_\nu^{(1)}, B_\nu^{(1)}, A_\nu^{(M)}, and\ B_\nu^{(M)}$

$$\mathbf{Q}_1\mathbf{A}^{(1)} + \mathbf{Q}_2\mathbf{B}^{(1)} = \mathbf{F}\quad ,\quad \mathbf{Q}_3\mathbf{A}^{(M)} + \mathbf{Q}_4\mathbf{B}^{(M)} = \mathbf{0} \tag{8}$$

where $\mathbf{F} \triangleq \left[0(Z_k \in C_1)\ , 2k_0^{(1)}(Z_k \in \overline{C_1}) \right]^T$

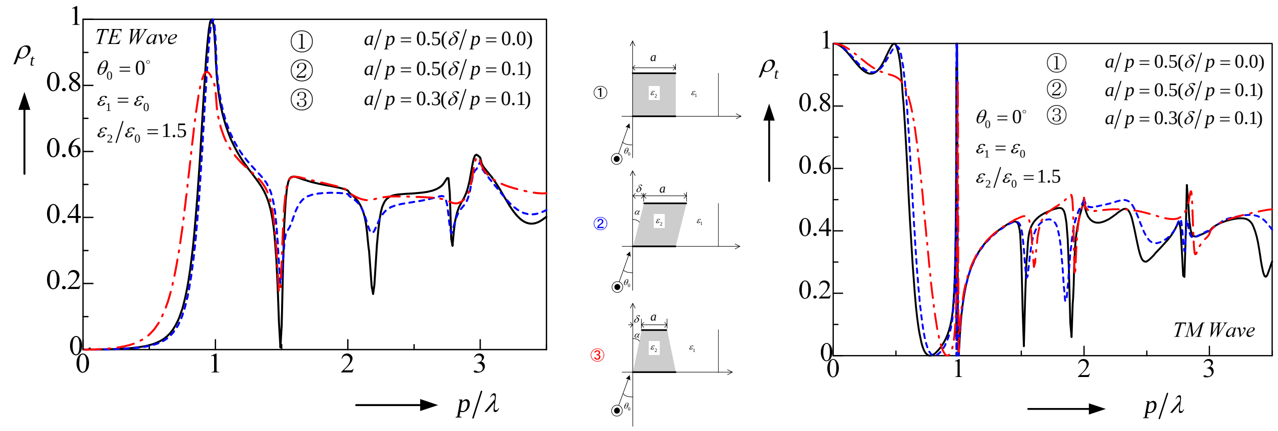
$$\mathbf{A}^{(k)} \triangleq \left[A_1^{(k)}, A_2^{(k)}, \cdots, A_{2N+1}^{(k)} \right]^T, k = 1,2\quad \mathbf{B}^{(k)} \triangleq \left[B_1^{(k)}, B_2^{(k)}, \cdots, B_{2N+1}^{(k)} \right]^T, k = 1, M$$

$$\mathbf{Q}_1 \triangleq \mathbf{X}_{(1)}^C + \mathbf{K}^{(1)}\mathbf{X}_{(1)}^G + \mathbf{X}_{(1)}^G\mathbf{H}^{(1)},\quad \mathbf{Q}_2 \triangleq \mathbf{X}_{(1)}^C + (\mathbf{K}^{(1)}\mathbf{X}_{(1)}^G + \mathbf{X}_{(1)}^G\mathbf{H}^{(1)})\mathbf{D}^{(1)},$$

$$\mathbf{Q}_3 \triangleq \mathbf{X}_{(M)}^C\mathbf{D}^{(M)} + (\mathbf{K}^{(M)}\mathbf{X}_{(M)}^G - \mathbf{X}_{(M)}^G\mathbf{H}^{(M)})\mathbf{D}^{(M)},\quad \mathbf{Q}_4 \triangleq \mathbf{X}_{(M)}^C + \mathbf{K}^{(M)}\mathbf{X}_{(M)}^G + \mathbf{X}_{(M)}^G\mathbf{H}^{(M)}$$

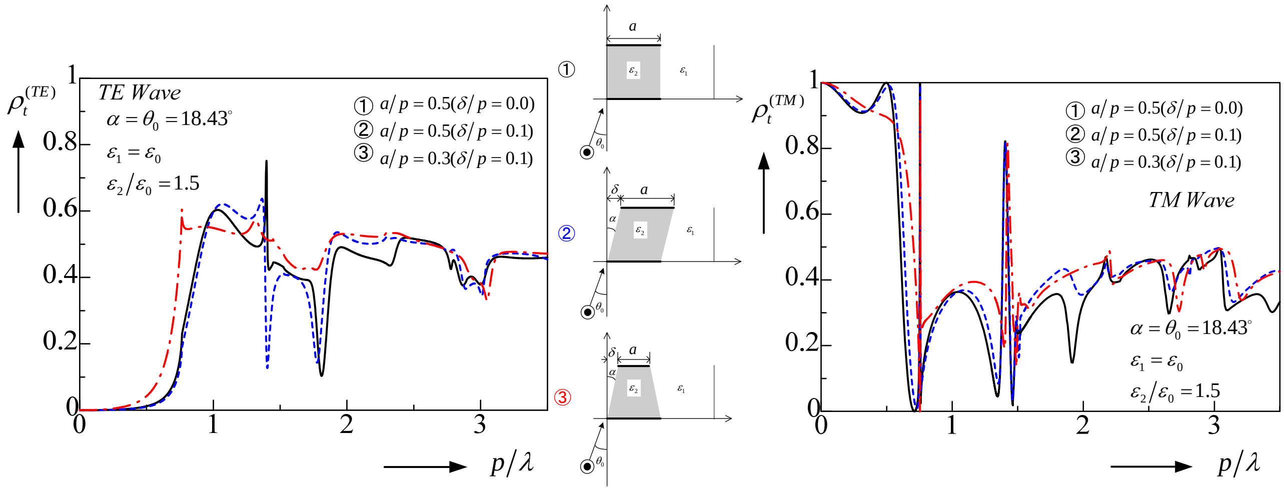
$$n = (-N, \cdots, 0\ , \cdots, N)\ , \nu = 1 \sim (2N_f + 1)$$

$$\mathbf{X}_{(l)}^C = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots \\ e^{-iNz_j} & \cdots & e^{i0z_j} & \cdots & e^{iNz_j} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} Z_j \in C_1 \bullet \mathbf{U}^{(l)} \\ \left.\right\} Z_j \in \overline{C}_1 \end{matrix}\quad ,\mathbf{X}_{(l)}^G = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & & 0 \\ \vdots & \vdots & \vdots & \vdots \\ k_{-N}^{(l)} e^{-iNz_j} & k_0^{(l)} e^{i0z_j} & & k_N^{(l)} e^{iNz_j} \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} Z_j \in C_1 \bullet \mathbf{U}^{(l)} \\ \left.\right\} Z_j \in \overline{C}_1 \end{matrix}$$



(a) TE Wave (b) TM wave

Fig.3. Power transmission coefficients ρ_t vs. normalized frequency p/λ



(a) TE Wave (b) TM wave

Fig.4. Power transmission coefficients ρ_t vs. normalized frequency p/λ

$$\mathbf{U}^{(l)} \triangleq \left[{}^{(l)}u_n^{(\nu)} \right],\ \mathbf{D}^{(l)} \triangleq \left[e^{ih_\nu^{(l)}d_\Delta} \cdot \delta_{\nu,\nu} \right],\quad \mathbf{H}^{(l)} \triangleq \left[h_\nu{}^{(l)} \cdot \delta_{\nu,\nu} \right],\quad \mathbf{K}^{(l)} \triangleq \left[k_n^{(l)} \cdot \delta_{\nu,\nu} \right],\ l = 1,\ M\ ,,\ \nu = 1 \sim (2N_f + 1)$$

We obtain the relationship between $\mathbf{A}^{(1)}, \mathbf{B}^{(1)}$ in the first layer and, $\mathbf{A}^{(M)}, \mathbf{B}^{(M)}$ in the end of unit layer using boundary condition at $x = -ld_\Delta$ $(l = 1 \sim M - 1)$.

$$\left[E_y^{(2,l)} = E_y^{(2,l+1)} \right]_{x=-ld_\Delta},\ \left[H_z^{(2,l)} = H_z^{(2,l+1)} \right]_{x=-ld_\Delta} \tag{9}$$

$$\begin{pmatrix} \mathbf{A}^{(1)} \\ \mathbf{B}^{(1)} \end{pmatrix} = \begin{pmatrix} \mathbf{S}_1^{(1)} & \mathbf{S}_2^{(1)} \\ \mathbf{S}_3^{(1)} & \mathbf{S}_4^{(1)} \end{pmatrix} \begin{pmatrix} \mathbf{S}_1^{(2)} & \mathbf{S}_2^{(2)} \\ \mathbf{S}_3^{(2)} & \mathbf{S}_4^{(2)} \end{pmatrix} \cdots \begin{pmatrix} \mathbf{S}_1^{(M)} & \mathbf{S}_2^{(M)} \\ \mathbf{S}_3^{(M)} & \mathbf{S}_4^{(M)} \end{pmatrix} \begin{pmatrix} \mathbf{A}^{(M)} \\ \mathbf{B}^{(M)} \end{pmatrix} = \begin{pmatrix} \mathbf{S}_1 & \mathbf{S}_2 \\ \mathbf{S}_3 & \mathbf{S}_4 \end{pmatrix} \begin{pmatrix} \mathbf{A}^{(M)} \\ \mathbf{B}^{(M)} \end{pmatrix}, \tag{10}$$

$$where\ \ \mathbf{S}_k^{(l)} \triangleq \left[{}^{(l)}s_{n,\nu}^{(k)} \right],\ k = 1 \sim 4,\ l = 1 \sim M$$

$${}^{(l)}s_{n,\nu}^{(1)} \triangleq \frac{1}{2}\left[1 + h_{n+N+1}^{(l+1)} / h_\nu^{(l)} \right] e^{-ih_\nu^{(l)}d_\Delta},\quad {}^{(l)}s_{n,\nu}^{(2)} \triangleq {}^{(l)}s_{n,\nu}^{(3)} \cdot e^{i\left\{ h_{n+N+1}^{(l)} - h_\nu^{(l)} \right\} d_\Delta},$$

$${}^{(l)}s_{n,\nu}^{(3)} \triangleq \frac{1}{2}\left[1 - h_{n+N+1}^{(l+1)} / h_\nu^{(l)} \right],\quad {}^{(l)}s_{n,\nu}^{(4)} \triangleq \frac{1}{2}\left[1 + h_{n+N+1}^{(l+1)} / h_\nu^{(l)} \right] e^{ih_{n+N+1}^{(l)}d_\Delta},$$

$$v_{n,\nu}^{(l)} = \left[u_{\nu,n}^{(l)} \right]^{-1} \left[u_{\nu,n}^{(l+1)} \right].$$

By using matrix relationship between Eq.(8), and Eq.(10), we get the following homogeneous matrix equation in regard to $A_\nu^{(M)}$ $(\nu = 1 \sim 2N + 1)$.

$$\mathbf{W}\cdot\mathbf{A}^{(M)}=\mathbf{F}\ , \tag{11}$$

where $\mathbf{W}\triangleq\left[\mathbf{Q}_1\mathbf{S}_1+\mathbf{Q}_2\mathbf{S}_3-\left(\mathbf{Q}_1\mathbf{S}_2+\mathbf{Q}_2\mathbf{S}_4\right)\mathbf{Q}_4^{-1}\mathbf{Q}_3\right]$.

The mode power transmission coefficients ρ_t is given by

$$\rho_t\triangleq\sum_{n=-N}^{N}\mathrm{Re}\left[k_n^{(3)}\right]|\,c_n^{(3)}\,|^2\ , \tag{12}$$

where、 $C_n^{(3)}\triangleq\sum_{n=-N}^{N}\left[A_\nu^{(M)}e^{ih_\nu^{(M)}d_\Delta}+B_\nu^{(M)}\right]U_n^{(\nu)}$.

3. Numerical Analysis

We consider the following profiles of inhomogeneous dielectric gratings:

$$\varepsilon_2\,(x,z)\triangleq\begin{cases}\varepsilon_1:(0\sim\alpha,\,0\sim\beta)\\ \varepsilon_2:(\pi/2\text{-}\alpha,\ \pi/2\text{-}\beta)\end{cases} \tag{13}$$

The values of parameters chosen are $\varepsilon_1=\varepsilon_3=\varepsilon_0$, $(a=b)/p=0.5$, $\varepsilon_1/\varepsilon_0=1.0$, $\theta_0=\alpha=\beta=18.43^\circ$, $\varepsilon_2/\varepsilon_0=1.5$ and $D/p=0.3$ .The relative error are less than about 0.1% and the energy error is less than about 10^{-3} for TE and TMwaves when we computed with $N=15$ at $a/b=1$ and $p/\lambda=1.5$ .Figures 2 shows ρ_t for various values of normalized frequency (p/λ) for $\varepsilon_1/\varepsilon_0=1.0$ and 2.0 at ①$\alpha(=\beta)=0^\circ$,②$\alpha=(-\beta)=18.43^\circ$, and ③$\alpha=(\beta)=18.43^\circ$.

We note that the characteristic tendencies of coupling resonance cases of ①～③are approximately same for the TE and TM at $1.4<p/\lambda>1.5$, but for $0.8<p/\lambda>1.3$ at ③ is more significant for the power transmission coefficients with constants.Figures 4 shows ρ_t for the various values of normalized frequency (p/λ) with the case of $\varepsilon_1/\varepsilon_0=\varepsilon_d/\varepsilon_0=1.5$ for the same parameters Fig.3. We note that the characteristic tendencies for the effect of the equivalent permittivity are approximately same.

Next, we consider the three strip gratings for the case of $d/p=0.15$ .Figures 5 shows ρ_t for various values of normalized frequency (p/λ) with the same parameters as in Fig.4.

4. Conclusion

In this paper, we have proposed a new method for the scattering of electromagnetic waves by inhomogeneous dielectric gratings loaded with perfectly conducting strips using the combination of improved Fourier series expansion method and point matching method.

Numerical results are given for the transmitted scattered characteristics for the case of frequency loaded with the parallel perfectly conducting strips for TE and TM cases. The effects of the inhomogeneous dielectric gratings comparison with that of the slant angle on the transmitted power are discussed.

This method also can be applied to the inhomogeneous dielectric gratings having an arbitrarily periodic structures combination of dielectric and metallic materials.

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