Effect of the Slit's Thickness to the Electromagnetic Plane Wave Scattering

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Abstract

Electromagnetic wave diffraction by a wide and thick slit has been formulated and analyzed the effect of the slit's thickness. From our numerical calculation, it has been found that for far field pattern, one can apply a rather simple infinitely thin slit formulation, when the thickness of the wide slit is less than one half wavelength.

keywords: Ray-mode coupling, Thick slit diffraction problem, Geometrical Theory of Diffraction

1. Introduction

Electromagenetic wave diffraction by a slit is rather classical canonical problem and many authors have been studied. For a slit on an infinitely thin screen, a solution by eigen function series expansion in terms of Mathieu functions has been reported by Morse and Rubinstein [1]. Also Nomura and Katsura applied a different series expansion using Weber Schafheitlin's discontinuous integrals [2], which may be called as Kobayashi Potential (KP) method [3]. These results can be good references for small aperture. Karp and Russek has applied the Geometrical Theory of Diffraction (GTD) proposed by Keller [4] to solve the diffraction by wide slits [5].

When one wants to apply the infinitely thin slit diffraction formulation to actual scattering prediction problems, thickness effect of the slit is always an arguing point. In order to answer this question, one need a reliable reference solution for thick slit diffraction.

Formulation of the diffraction by a slit perforated at a thick screen is rather involved. Hongo has extended the KP method to derive a solution, which is numerically effective for rather narrow slit aperture case [6, 7].

In order to analyze wide and thick slit diffraction problems, high frequency ray-mode coupling formulation [8, 9] would be effective, and we shall formulate to discuss about the thickness effect.

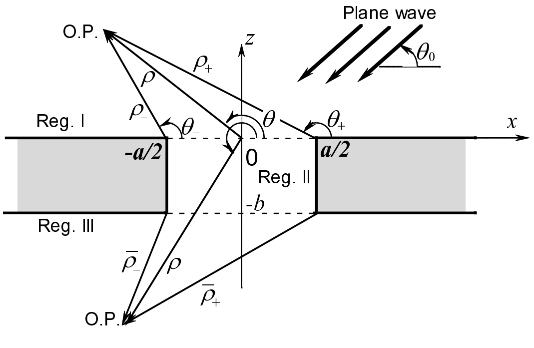


Figure 1: Geometry of the problem.

Time harmonic factor $e^{-i\omega t}$ is assumed and suppressed throughout the context.

2. Formulation

As depicted in Fig. 1, a H-polarized plane wave:

$$u^i(= H^i_y) = e^{-ikx\cos\theta_0 - ikz\sin\theta_0} \tag{1}$$

illuminates a slit on an infinitely long perfect conducting screen. Width and thickness of the slit are a and b, respectively, and k is free space wavenumber. For convenience, the entire region is divided into three regions: (I) semi-infinite upper half space ($z > 0$), (II) the slit region ($-b < z < 0$), and (III) the lower half space ($z < -b$).

In region I, one may observe the reflected plane wave and edge diffracted waves excited at the aperture edges. The incident wave impinges on the edges of the upper aperture($x = \pm a/2,\ z = 0$), and the edge diffracted waves are generated. These primary edge diffracted waves, which play an important role for modal excitation, may be calculated by GTD formulation as [8].

A part of the primary edge diffracted wave also propagates into the slit region II, and re-radiates after several internal reflections and diffraction. According to the ray-mode conversion [8, 9], the parallel plane waveguide modes propagating along negative z-direction are excited by the primary edge diffracted waves. These excited modes may be represented as

$$\dot{u} = \sum_{m=0}^{\infty} A_m U_m^-, \quad U_m^\pm = \cos\left\{\frac{m\pi}{a}\left(x + \frac{a}{2}\right)\right\}\exp(\pm i\zeta_m z), \tag{2}$$

where $U_m^\pm$ is the m-th parallel plate waveguide mode with $\zeta_m = \sqrt{k^2 - (m\pi/a)^2} = k\cos\theta_m$, $\theta_m = \sin^{-1}\{m\pi/(ka)\}$, and A_m denotes the modal excitation coefficient for mode U_m^-. For modes with $m > ka/\pi$, the propagation constant ζ_m along z-direction becomes pure imaginary, and the corresponding mode U_m^- is evanescent. Though one can derive the excitation coefficients A_m for these evanescent modes, these modes decay exponentially. Accordingly, modal sum in Eq.(2) can be truncated at the last propagating mode U_N.

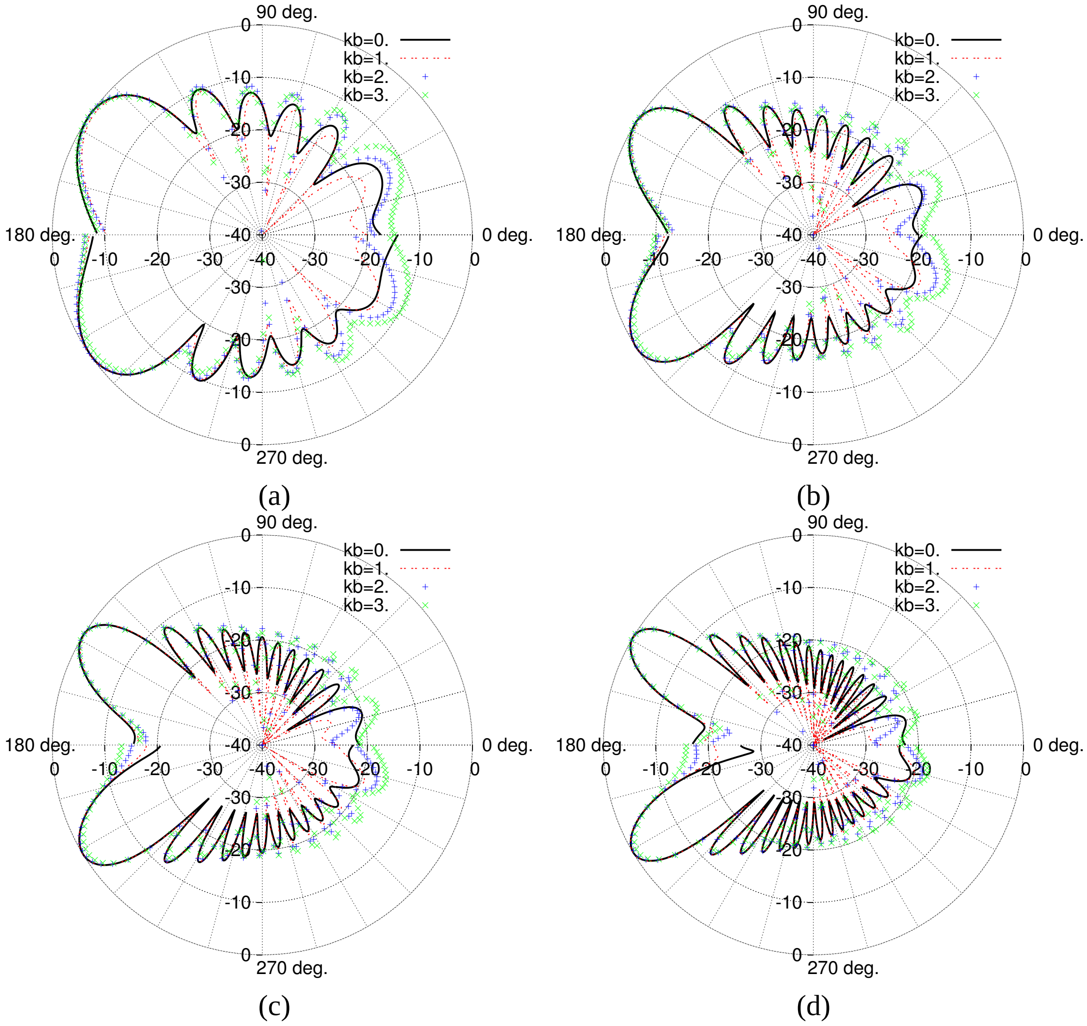
A mode U_m^- propagates toward to the lower open end $z = -b$, at which modal radiation and modal reflection occur. The modal radiation field u_1 in region III can be formed by collecting all radiation fields generated by the propagation modes U_m^- at the lower aperture. At the same time, modal coupling occurs to generate waveguide modes $U_n^+(n = 0, \cdots, N)$ propagating along z-positive direction with new excitation coefficients.

Consequently, this successive process of modal radiation and reflection/coupling continues to generate modal radiation fields u_{2n} for $z > 0$ and those u_{2n+1} for $z < -b$. This modal radiation continues until all the energy of bouncing waveguide modes is dissipated. Total radiation (diffraction) fields u_t may be written in a compact matrix form. Because of a limited space, the detail of the derivation is omitted here, and may be found elsewhere [10].

3. Numerical Results and Discussion

Formulation using ray-mode coupling has shown to be effective for open resonance structures such as a trough on the ground [8] and an open-ended finite parallel plane waveguide [9]. In the previous investigation [10], numerical results calculated from our formulation for a wide and thick slit have been compared with those obtained by KP method, and good agreement has been found to validate our formulation even for rather narrow aperture case.

Figure 2 shows the normalized far field diffraction pattern with incidence angle $\theta_0 = 30°$. Patterns for $kb = 0.,\ 1.,\ 2.,\ 3.$ are compared for $ka = 20.,\ 30.,\ 40.,\ 50$, respectively. In this figure, contribution from reflected wave has been omitted. For the case of infinitely thin slit ($kb = 0$), the diffraction pattern in regions I and III should be symmetric with respect to the screen. In our calculation, we took the limit for zero thickness in our formulation. One noticed slight differences at the screen boundary directions ($\theta = 0°, 180°, 360°$). This is due to the fact that our solution is composed by two right angle wedges, and cannot recover the correct edge behavior of half plane with zero thickness. Accordingly, our formulation inherits a different edge condition and cannot predict the correct field behavior. Beam amplitude and width in reflection direction $\theta = 150°$ does not change. This diffraction pattern is composed essentially from the primary edge diffracted waves at upper edges of the slit's aperture, and modal re-radiation wave is weak even for thin slit. The change of thickness causes level change in lower minor lobes whose



(a) (b) (c) (d)

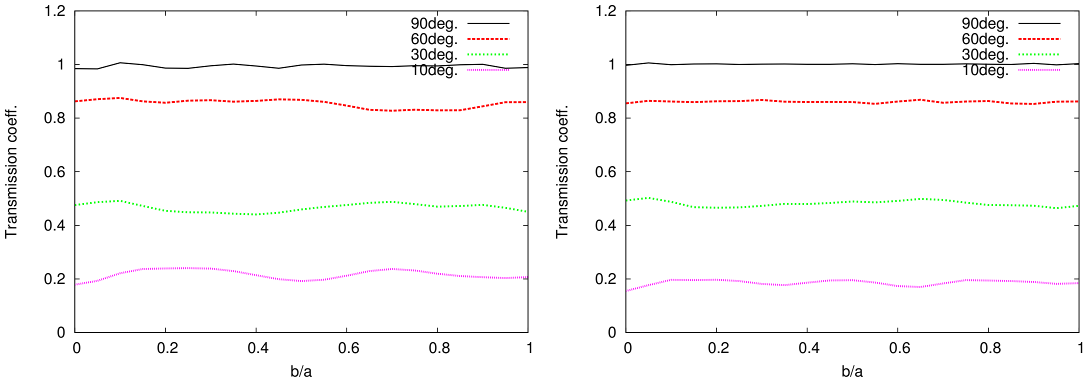
Figure 2: Far field patterns in dB. $\theta_0 = 30°$, $kb = 0., 1., 2., 3.$. (a) $ka = 20$. (b) $ka = 30$. (c) $ka = 40$. (d) $ka = 50$.

amplitudes are 10 dB lower than that of the main one. In the transmission region III, while the diffraction pattern does not change the main beam peak at the forward direction, symmetricity slowly deteriorates as the thickness of the slit gets thicker. As the aperture width gets wider, the far field radiation pattern has more oscillation with narrower lobe width.

In order to see the total diffraction effect by the thick slit, the transmission coefficients have been calculated. For the infinitely thin slit case, the transmission coefficient may be found from the far field directional pattern function $P(\theta)$ in the forward direction ($\theta = \pi + \theta_0$), but not for those with finite thickness. Transmission coefficient T of the slit may be expressed as [7]

$$T = \frac{\pi}{2ka \sin \theta_0} \int_{\pi}^{2\pi} |P(\theta)|^2 d\theta. \tag{3}$$

Figure 3 shows the transmission coefficients $T \sin \theta_0$ for various incidence angle θ_0. For thin slit case, transmission coefficient T becomes $1/\sin \theta_0$ as the aperture gets wider. As the slit gets thicker, the transmission coefficient becomes constant. This tendency is different from the narrow slit case [7]. While the transmission beam may change its direction due to the reflection at the internal waveguide walls, the slit's thickness causes little effect to the transmission coefficient, which is calculated from the radiation pattern $P(\theta)$ in entire region III. One may also conclude from Figs. 2 and 3 that the scattering pattern can be predicted by a simple infinitely thin slit solution, if the thickness of the slit is less than one half wavelength.



(a) (b)

Figure 3: Change of transmission coefficient $T \sin\theta_0$ in Eq.(3). $\theta_0 = 10^\circ,\ 30^\circ,\ 60^\circ,\ 90^\circ$. (a) $ka = 20$. (b) $ka = 30$.

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