Time-Domain Asymptotic Solution for Edge-Surface Diffracted Ray

#Keiji Goto, Toru Kawano, and Toyohiko Ishihara
Department of Communication Engineering, National Defense Academy
Hashirimizu, Yokosuka, 239-8686 Japan, e-mail: keigoto@nda.ac.jp

Abstract

We derive a time-domain asymptotic solution for the edge-surface diffracted ray excited by the edge of a thin cylindrically curved conducting surface from the corresponding frequency-domain asymptotic solution. The validity of the asymptotic solution is confirmed by comparing with the reference solution calculated numerically.

Keywords : Time-domain Asymptotic solution Edge-surface diffracted ray

1. Introduction

By recent technological advances in the area of radar cross section and target identification, it becomes important to study the asymptotic analysis methods for the frequency-domain (FD) and the time-domain (TD) scattered fields by curved objects with edges or wedges [1]-[3].

In the previous study [4], we have derived the TD asymptotic solution for the transient whispering gallery (WG) mode radiation field radiated from the *concave* side of a thin cylindrically curved conducting open surface. It has been clarified that the instantaneous angular frequency of the transient WG mode radiation field *increases* as a function of a time.

In this study, we derive the TD asymptotic solution for the surface diffracted ray (SD ray) excited by the edge and shed from the *convex* portion of a thin cylindrically curved conducting open surface. It is assumed that the transient SD ray is excited by a high-frequency pulse source with a truncated Gaussian time variation [4]. The validity of the TD asymptotic solution derived here is confirmed by comparing with the reference solution calculated numerically. We clarify that the instantaneous angular frequency of the transient SD ray *decreases* as a function of a time.

2. Formulation and Time-Domain Asymptotic Solution

2.1 Formulation and Integral Representation for Transient SD Ray

We have shown in Fig.1 the thin cylindrically curved conducting open surface $(a, \phi_{\rm AB})$ defined by $\rho = a, 0 \le \phi \le \phi_{\rm AB}$, the two-dimensional coordinate systems (ρ, ϕ), (x, y), and (r, ψ), and the SD ray $\rm A \curvearrowright A_1 \to P$ excited by the edge A. The TM-type plane wave (magnetic field is directed normal to the plane of incidence) is incident on the cylindrically curved conducting surface from the $-x$ direction. The creeping wave (CW) excited by the edge A propagates along the convex surface and is diffracted at the surface diffraction point $\rm A_1$ $(\rm A \curvearrowright A_1 (= L_2))$. Then at the point $\rm A_1$, the cylindrical wave leaves the convex surface tangentially and reaches the observation point P $(\rm A_1 \to P (= L_3))$.

The ℓ th order FD−SD ray $(\text{FD} - \text{SD}_\ell$ ray) $u_{\text{SD},\ell}(\boldsymbol{r}, \omega) (\equiv u_{\text{SD},\ell}(\omega)$, hereafter the position vector $\boldsymbol{r} = (r, \psi)$ is dropped) with total propagation distance $L = L_1 + L_2 + L_3$ may be expressed by [5]:

$$u_{\text{SD},\ell}(\omega) = A_{\text{SD},\ell}(\omega) \exp[-\alpha_\ell(\omega) L_2] \exp[i\omega(L_1/c + L_2/c_{\text{CW},\ell}(\omega) + L_3/c)], \quad (1)$$

$$c_{\text{CW},\ell}(\omega) = c/c_\ell(\omega) < c, \quad c_\ell(\omega) = 1 + c\,\beta_\ell(\omega)/\omega, \quad (2)$$

$$\alpha_\ell(\omega) = \frac{\sqrt{3}}{2}\frac{\sigma_\ell}{a}[\omega a/(2c)]^{1/3} \ (>0), \quad \beta_\ell(\omega) = \frac{1}{2}\frac{\sigma_\ell}{a}[\omega a/(2c)]^{1/3} \ (>0), \quad (3)$$

where $L_1 (= \rm Q \to A)$ denotes the distance from the reference point Q to the edge A. The term

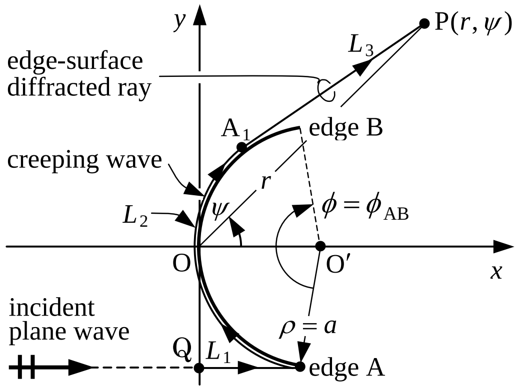
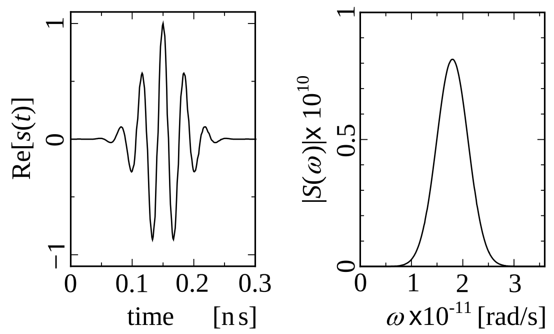


Fig.1 Edge-surface diffracted ray excited by edge A of a thin cylindrically curved conducting open surface and two-dimensional coordinate systems (ρ,ϕ), (x,y), and (r,ψ).



(a) $s(t)$ in Eq. (4) (b) $S(\omega)$ in Eq. (5)

Fig.2 Gaussian-type modulated UWB pulse source. Numerical parameters: $\omega_0 = 1.799\times 10^{11}$ rad/s, $t_0 = 1.5\times 10^{-10}$ s, $d = 2.3\times 10^{-11}$ s, $\bar{d} = 6.75\times 10^{-11}$ s.

$\alpha_\ell(\omega)$ denotes the ℓ th order attenuation constant, and c and $c_{\mathrm{CW},\,\ell}(\omega)\,(<c)$ denote, respectively, the speed of light and the phase velocity of $\mathrm{FD-CW}_\ell$. Notation σ_ℓ in Eq. (3) is the ℓ th zero of the Airy function derivative (i.e., $Ai'(-\sigma_\ell)=0$). The time factor $\exp(-i\omega t)$ is suppressed.

We assume that the transient SD_ℓ ray is excited by the truncated Gaussian-type modulated pulse source [4] defined by

$$s(t) = \exp[-i\omega_0(t-t_0) - (t-t_0)^2/(4d^2)] \ \text{ for } 0 \le t \le 2t_0\,, \quad s(t)=0:\text{elsewhere}\,, \tag{4}$$

where ω_0 denotes the central angular frequency, and t_0 and d are constant parameters. The frequency spectrum $S(\omega)$ of $s(t)$ in Eq. (4) is given by

$$S(\omega) = 2d\sqrt{\pi}\,\mathrm{Re}[\mathrm{erf}\beta(\omega)]\exp[i\omega t_0 - d^2(\omega-\omega_0)^2]\,, \quad \beta(\omega) = t_0/(2d) - i\,d(\omega-\omega_0)\,. \tag{5}$$

Figs. 2(a) and 2(b) show respectively the real part of pulse source $s(t)$ in Eq. (4) and the frequency spectrum $S(\omega)$ in Eq. (5). The pulse source $s(t)$ defined in Eq. (4) becomes an ultra-wideband (UWB) pulse source when d satisfies $d \le \bar{d}$ where $\bar{d}$ is the value of d such that the fractional bandwidth of $S(\omega)$ is 25.0 % [6].

The TM-type transient SD_ℓ ray $y_{\mathrm{SD},\,\ell}(\boldsymbol{r},t)\,(\equiv y_{\mathrm{SD},\,\ell}(t))$ excited by a Gaussian-type modulated pulse source $s(t)$ in Eq. (4) can be derived from the following inverse Fourier transform [4]:

$$y_{\mathrm{SD},\,\ell}(t) = \frac{d}{\sqrt{\pi}}\int_{-\infty}^{\infty} F_\ell(\omega)\exp[q_\ell(\omega)]\,d\omega\,, \tag{6}$$

$$F_\ell(\omega) = A_\ell(\omega)\,\mathrm{Re}[\mathrm{erf}\beta(\omega)]\exp[-\alpha_\ell(\omega)L_2]\,, \tag{7}$$

$$q_\ell(\omega) = -d^2(\omega-\omega_0)^2 - i\omega T_\ell(\omega)\,, \quad T_\ell(\omega) = t - t_0 - L_1/c - L_2/c_{\mathrm{CW},\,\ell}(\omega) - L_3/c\,. \tag{8}$$

2.2 Time-Domain Asymptotic Solution for Edge-Surface Diffracted Ray

In the integral $y_{\mathrm{SD},\,\ell}(t)$ in Eq. (6), we assume that the functions $\alpha_\ell(\omega)$ and $T_\ell(\omega)$ in the integrand may be approximated by

$$h_\ell(\omega) \sim h_\ell(\omega_0) + (\omega-\omega_0)\,h'_\ell(\omega_0) + \frac{(\omega-\omega_0)^2}{2}h''_\ell(\omega_0)\,, \quad \text{where } h_\ell(\omega) = \alpha_\ell(\omega) \text{ or } T_\ell(\omega)\,. \tag{9}$$

Substituting the 2nd order approximation in Eq. (9) into Eq. (6) and applying the saddle point technique [4], one may obtain the following TD asymptotic solution for the SD_ℓ ray:

$$y_{\mathrm{SD},\,\ell}(t) = \hat{F}_\ell(\omega_{\mathrm{s},\,\ell})\exp[-ig_\ell(t)]\exp[f_\ell(t)]\,, \tag{10}$$

$$\hat{F}_\ell(\omega_{\mathrm{s},\,\ell}) = d\sqrt{\gamma_\ell + i\eta_\ell}\;\{1+\xi_1(t,\ell,\omega_0)\}\;A_\ell(\omega_{\mathrm{s},\,\ell})\,\mathrm{Re}[\mathrm{erf}\beta(\omega_{\mathrm{s},\,\ell})]\,, \tag{11}$$

$$g_\ell(t) = \omega_0 T_\ell(\omega_0) + \frac{\eta_\ell}{4}\left[1 - \frac{\hat{a}_\ell}{2}(\eta_\ell^2 - 3\gamma_\ell^2)(t-t_{\mathrm{P},\,\ell})\right](t-t_{\mathrm{P},\,\ell})^2 - \xi_2(t,\ell,\omega_0)\,, \tag{12}$$

$$f_\ell(t) = -\alpha_\ell(\omega_0) L_2 - \frac{\gamma_\ell}{4}\left[1 + \frac{\hat{a}_\ell}{2}(\gamma_\ell^2 - 3\eta_\ell^2)(t - t_{\mathrm{P},\ell})\right](t - t_{\mathrm{P},\ell})^2 + \xi_3(t, \ell, \omega_0)\,, \tag{13}$$

where $\hat{F}_\ell(\omega_{s,\ell})$, $g_\ell(t)$, and $\exp[f_\ell(t)]$ denote respectively the amplitude term, the phase term, and the envelope shape of the $\mathrm{TD}-\mathrm{SD}_\ell$ ray and $\xi_n(t, \ell, \omega_0)$, $n = 1, 2, 3,$ etc. in the above equations are defined as follows:

$$\xi_1(t, \ell, \omega_0) = \frac{3}{4}\hat{a}_\ell \left[\{(\gamma_\ell^2 - \eta_\ell^2)(t - t_{\mathrm{P},\ell}) + 2\gamma_\ell \eta_\ell e_\ell\} + i\{2\gamma_\ell \eta_\ell (t - t_{\mathrm{P},\ell}) - (\gamma_\ell^2 - \eta_\ell^2) e_\ell\}\right], \tag{14}$$

$$\begin{aligned}\xi_2(t, \ell, \omega_0) = {} & \frac{3}{8}\hat{a}_\ell \gamma_\ell (\gamma_\ell^2 - 3\eta_\ell^2) e_\ell (t - t_{\mathrm{P},\ell})^2 + \frac{e_\ell}{2}\left[\gamma_\ell - \frac{3}{4}\hat{a}_\ell\, \eta_\ell\, (\eta_\ell^2 - 3\gamma_\ell^2) e_\ell\right](t - t_{\mathrm{P},\ell}) \\ & + \frac{{e_\ell}^2}{4}\left[\eta_\ell - \frac{1}{2}\hat{a}_\ell\, \gamma_\ell\, (\gamma_\ell^2 - 3\eta_\ell^2) e_\ell\right],\end{aligned} \tag{15}$$

$$\begin{aligned}\xi_3(t, \ell, \omega_0) = {} & \frac{3}{8}\hat{a}_\ell \eta_\ell (\eta_\ell^2 - 3\gamma_\ell^2) e_\ell (t - t_{\mathrm{P},\ell})^2 - \frac{e_\ell}{2}\left[\eta_\ell - \frac{3}{4}\hat{a}_\ell \gamma_\ell (\gamma_\ell^2 - 3\eta_\ell^2) e_\ell\right](t - t_{\mathrm{P},\ell}) \\ & + \frac{{e_\ell}^2}{4}\left[\gamma_\ell - \frac{1}{2}\hat{a}_\ell\, \eta_\ell\, (\eta_\ell^2 - 3\gamma_\ell^2) e_\ell\right],\end{aligned} \tag{16}$$

$$\gamma_\ell = a_\ell \big/ (a_\ell^2 + b_\ell^2)\,, \quad \eta_\ell = b_\ell \big/ (a_\ell^2 + b_\ell^2)\,, \tag{17}$$

$$a_\ell = d^2 + \frac{L_2 c_\ell'(\omega_0)}{2\sqrt{3}\,c}\,, \quad b_\ell = \frac{L_2 c_\ell'(\omega_0)}{c} + \omega_0\, \hat{a}_\ell\,, \quad \hat{a}_\ell = \frac{L_2 c_\ell''(\omega_0)}{2c}\,, \quad e_\ell = L_2 \alpha_\ell'(\omega_0)\,, \tag{18}$$

$$t_{\mathrm{P},\ell} = t_0 + L_1 \big/ c + L_2 \big/ v_{g,\ell} + L_3 \big/ c\,, \quad v_{g,\ell} = \frac{c_{\mathrm{CW},\ell}(\omega_0)}{1 - \omega_0\, c'_{\mathrm{CW},\ell}(\omega_0) \big/ c_{\mathrm{CW},\ell}(\omega_0)} > c_{\mathrm{CW},\ell}(\omega_0)\,. \tag{19}$$

Here, $\omega_{s,\ell}$ is the saddle point of the integrand in Eq. (6) given by

$$\omega_{s,\ell} = \omega_0 + \frac{1}{2}\{\eta_\ell (t - t_{\mathrm{P},\ell}) - \gamma_\ell e_\ell\} - i\frac{1}{2}\{\gamma_\ell (t - t_{\mathrm{P},\ell}) - \eta_\ell e_\ell\}\,. \tag{20}$$

It is clarified from Eq. (19) that the transient CW_ℓ $(\mathrm{A} \curvearrowright \mathrm{A}_1)$ constructing the transient SD_ℓ ray $y_{\mathrm{SD},\ell}(t)$ propagates with the group velocity $v_{g,\ell}$ in Eq. (19) *faster* than the phase velocity $c_{\mathrm{CW},\ell}(\omega_0)$ (see Eq. (2)) and that the transient incident wave $(\mathrm{Q} \to \mathrm{A})$ and the transient cylindrical wave $(\mathrm{A}_1 \to \mathrm{P})$ constructing $y_{\mathrm{SD},\ell}(t)$ propagate with the group velocity coincident with the speed of light *c*. It is also apparent from Eq. (10) that the transient SD_ℓ ray takes the maximum value $\gamma_{\max,\ell}$ $(= \max[y_{\mathrm{SD},\ell}(t_{\mathrm{P},\ell})])$ at the observation time $t = t_{\mathrm{P},\ell}$ (see Eq. (19)).

The instantaneous angular frequency $\omega_{\mathrm{SD},\ell}(t)$ of the $\mathrm{TD}\text{-}\mathrm{SD}_\ell$ ray may be derived analytically from (see Eq. (12))

$$\omega_{\mathrm{SD},\ell}(t) = \frac{d}{dt} g_\ell(t)\,. \tag{21}$$

3. Numerical Results and Discussions

In order to confirm the validity of the TD asymptotic solution derived in Section **2**, we calculate the TM-type lowest order transient SD_1 ray (i.e., $\ell = 1$) excited by the truncated Gaussian-type modulated UWB pulse source $s(t)$ given Fig.2(a).

In Fig.3, we have shown the instantaneous angular frequency $\omega_{\mathrm{SD},1}(t)$ of $y_{\mathrm{SD},1}(t)$ in Eq. (6) vs. time *t* curves. It is confirmed that the asymptotic solution (——) in Eq. (21) decreases as the function of time *t* and agrees well with the reference solution (•••) calculated numerically from Eq. (6) in the period $24.32\ \mathrm{ns} \le t \le 24.44\ \mathrm{ns}$ including $t = t_{\mathrm{p},1} = 24.387$ ns. We have also shown the instantaneous angular frequency of the pulse source $s(t - t_{\mathrm{P},1} + t_0)$ (– – –·) in Eq. (4). The instantaneous angular frequency (– – –·) of $s(t - t_{\mathrm{P},1} + t_0)$ is kept constant at the central angular frequency ω_0.

In Fig.4, we have shown the response waveform of $y_{\mathrm{SD},1}(t)$ in Eq. (6). It is observed that the asymptotic solution (——) calculated from Eq. (10) agrees excellently with the reference solution

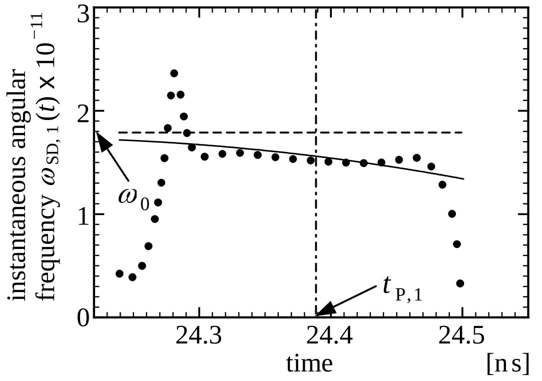


Fig.3 Instantaneous angular frequency $\omega_{SD,1}(t)$ vs. time t curves. ——: asymptotic solution in Eq. (21), ●●●: reference solution obtained numerically from Eq. (6), - - - ·: pulse source $s(t - t_{P,1} + t_0)$. Numerical parameters: $(a, \phi_{AB}) = (0.5\,\mathrm{m}, 160\,\mathrm{deg.})$, observation point $P(r, \psi) = (6.0\,\mathrm{m}, 20\,\mathrm{deg.})$.

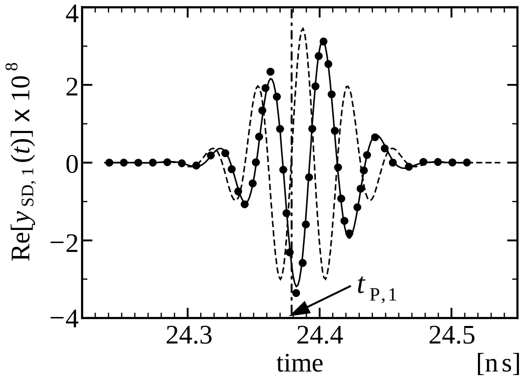


Fig.4 Response waveform of $y_{SD,1}(t)$. ——: asymptotic solution in Eq. (10), ●●●: reference solution calculated by numerical integration in Eq. (6), and - - - ·: normalized pulse source $\mathrm{Re}[\gamma_{max,1}\, s(t - t_{P,1} + t_0)]$. Numerical parameters: same as those given in Fig.3.

(●●●) obtained by applying the numerical integration in Eq. (6) in the period $24.24\,\mathrm{ns} \le t \le 24.51$ ns. We have also shown in Fig.4 the pulse source $\mathrm{Re}[\gamma_{max,1}\, s(t - t_{P,1} + t_0)]$. (- - - ·) normalized by the maximum value $\gamma_{max,1}$. It is confirmed that the instantaneous angular frequencies $\omega_{SD,1}(t)$ of $y_{SD,1}(t)$ (——) and the reference solution (●●●) decrease continuously as the function of time t in the period $24.32\,\mathrm{ns} \le t \le 24.44\,\mathrm{ns}$. This phenomenon agrees with the result shown in Fig.3.

4. Conclusion

In this study, we have derived the time-domain asymptotic solution for the edge-surface diffracted ray excited by the high-frequency pulse source. The validity of the asymptotic solution derived here has been confirmed by comparing with the numerical reference solution.

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