Transmission by a Magnetic Current Loop around a Long Cylinder Piercing a Circular Aperture

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Abstract

Electromagnetic power transmission of a magnetic current loop into a wire-penetrated aperture is investigated. We use the superposition principle, the associated Weber transform, and the mode-matching method to constitute a set of simultaneous equations. Our theoretical model for a wire-penetration is useful for the analysis of EMI and its applications.

Keywords : wire-penetrating aperture, associated Weber transform, mode-matching

1. Introduction

A wire-penetrating aperture is a canonical model that can be used to represent locally a via hole in a printed circuit board (PCB), an antenna feed or cable systems linking different equipments. While there have been many works done in the area of a wire-penetrating aperture [1] – [4], they all assumed a small aperture, a thin wire or an infinitesimally thin screen and their analysis methods are inherently based on the numerically intensive integral equation. Recently, we have obtained a numerically efficient series solution for the field penetration into a wire-penetrated aperture when excited by an electrical current loop [5]. The present paper is a continuation of [5] to obtain a fast convergent series solution for the transmission by a magnetic current loop. We will use the mode-matching, the superposition principle, as used in [5], and the associated Weber transform [6], which is somewhat different from the original one. A brief theoretical summary is given and the power transmission characteristics of a wire-penetrating aperture are investigated.

2. Theory

Consider an infinitely long conducting cylinder piercing a circular aperture in a thick conducting screen. The geometry of the problem is shown in Fig. 1. The cylindrical coordinates (ρ,ϕ,z) are used for analysis and a time-harmonic factor $e^{-i\omega t}$ is suppressed throughout. Region (I) is the upper half-space ($z>0,\rho>a$), region (II) is an annular aperture (radii: a and b , depth: d), and region (III) is the lower half-space ($z<0,\rho>a$). All regions are assumed to be air with the wavenumber $k=\omega\sqrt{\mu_0\varepsilon_0}$. A magnetic ring current source $\bar{M}(\bar{r})=\hat{\phi}M_0\delta(\rho-\rho')\delta(z-z')$ is placed in region (I), where $\delta(\cdot)$ is the Dirac delta function. Due to the azimuthal symmetry, only the TM^z modes are excited [7] for this case. The total field in region (I) consists of the primary and secondary components based on the superposition principle [5]. The primary magnetic field H_ϕ^p resulting from $\bar{M}$ is

$$H_\phi^p(\rho,z)=-i\omega\varepsilon_0M_0\rho'\int_0^\infty\frac{Z_1(\kappa\rho_<)H_1^{(1)}(\kappa\rho_<)}{H_0^{(1)}(\kappa a)}\cos\zeta z\cos\zeta z'd\zeta \tag{1}$$

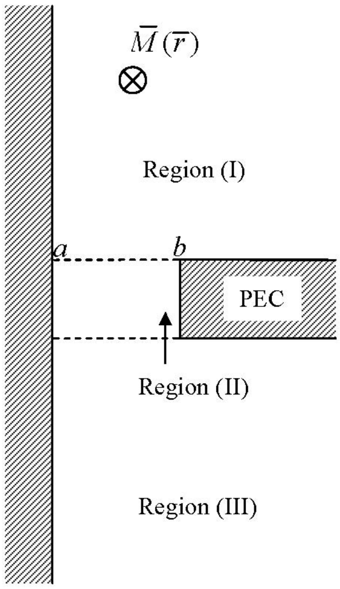

Figure 1: Magnetic ring current source around a long cylinder penetrating a circular aperture.

where $Z_1(\kappa\rho) = J_1(\kappa\rho)N_0(\kappa a) - N_1(\kappa\rho)J_0(\kappa a)$ and $\kappa = \sqrt{k^2 - \zeta^2}$. The functions $J_p(\cdot)$ and $N_p(\cdot)$ represent the Bessel functions of the first and second kinds of order p, respectively. The function $H_p^{(1)}(\cdot)$ is the Hankel function of the first kind of order p. In (1), the notation $\rho_<$ ($\rho_>$) designates the smaller (larger) of ρ and ρ'. Based on the associated Weber transform [6], the secondary magnetic fields in region (I) and (III) take the form of

$$H_\phi^I(\rho, z) = \int_0^\infty \tilde{H}_\phi^I(\zeta) e^{i\kappa z} \frac{Z_1(\zeta\rho)}{J_0^2(\zeta a) + N_0^2(\zeta a)} e^{i\kappa z} \zeta d\zeta \tag{2}$$

$$H_\phi^{III}(\rho, z) = \int_0^\infty \tilde{H}_\phi^{III}(\zeta) e^{-i\kappa(z+d)} \frac{Z_1(\zeta\rho)}{J_0^2(\zeta a) + N_0^2(\zeta a)} e^{i\kappa z} \zeta d\zeta \tag{3}$$

Note that $\tilde{H}_\phi^{I,III}(\zeta)$ is the associated Weber transform defined by $\tilde{H}_\phi^{I,III}(\zeta) = \int_a^\infty H_\phi^{I,III}(\rho, 0) Z_1(\zeta\rho)\rho d\rho$. In region (II), the magnetic field can be represented in terms of the discrete mode summation

$$H_\phi^{II}(\rho, z) = \left[a_0 \frac{e^{ik\left(z+\frac{d}{2}\right)}}{\rho} + b_0 \frac{e^{-ik\left(z+\frac{d}{2}\right)}}{\rho} \right] + \sum_{m=1}^\infty \left[a_m e^{i\kappa_m\left(z+\frac{d}{2}\right)} + b_m e^{-i\kappa_m\left(z+\frac{d}{2}\right)} \right] R_1(\gamma_m \rho) \tag{4}$$

where $\kappa_m = \sqrt{k^2 - \gamma_m^2}$ and

$$R_1(\gamma_m \rho) = J_1(\gamma_m \rho) - \frac{J_0(\gamma_m a)}{N_0(\gamma_m a)} N_1(\gamma_m \rho) \tag{5}$$

and the eigenvalue γ_m is determined by $R_0(\gamma_m b) = J_0(\gamma_m b) - \dfrac{J_0(\gamma_m a)}{N_0(\gamma_m a)} N_0(\gamma_m b) = 0$. In order to obtain the unknown modal coefficients a_α and b_α ($\alpha = 0, 1, 2, \cdots$), we enforce the boundary conditions on the field continuities from regions (I) through (III). The tangential E_ρ component continuity at $z=0$ gives

$$\tilde{H}_\phi^I(\zeta)=\frac{1}{\kappa}\left\{k\Xi_0(\zeta)\left[a_0 e^{ik\frac{d}{2}}-b_0 e^{-ik\frac{d}{2}}\right]+\sum_{m=1}^{\infty}\kappa_m \Xi_m(\zeta)\left[a_m e^{i\kappa_m\frac{d}{2}}-b_m e^{-i\kappa_m\frac{d}{2}}\right]\right\} \tag{6}$$

where

$$\Xi_0(\zeta)=-\frac{1}{\zeta}Z_0(\zeta b)=-\frac{1}{\zeta}\left[J_0(\zeta b)N_0(\zeta a)-N_0(\zeta b)J_0(\zeta a)\right] \tag{7}$$

$$\Xi_m(\zeta)=-\frac{2\zeta Z_0(\zeta b)}{\pi\gamma_m N_0(\gamma_m b)(\zeta^2-\gamma_m^2)} \tag{8}$$

Then the tangential H_ϕ continuity at $z=0$ yields a set of simultaneous equation for a_α and b_α. It is necessary to obtain another set of simultaneous equation for a_α and b_α by using the boundary condition at $z=-d$. After truncating an infinite series up to N terms, it is trivial to solve the simultaneous equations for the unknown modal coefficients by matrix inversion.

3. Calculations

Fig. 2 displays the two-dimensional curves of the magnitude of H_ϕ field with several cuts. In order to check the validity of our theory, the numerical results are compared with the COMSOL Multiphysics results. Our computation results are in good agreement with the data of COMSOL. Using $N=5$, we performed computations throughout this section to assure reasonable accuracy. Fig. 3. shows the dependence of the power transmission coefficients ($T=P_{tran}/P_{inc}$) on the plane thickness d/λ. It clearly reveals undulating characteristics with the wavelength λ. Unlike the electric current source excitation [5], T does not approach 0 as the aperture size becomes smaller since the coaxial region (II) can support the TEM mode.

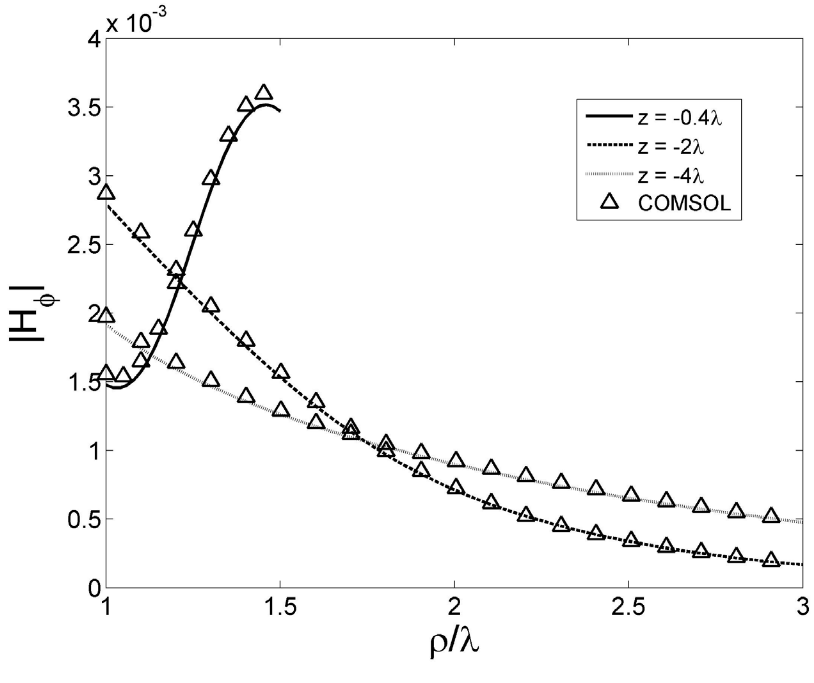

Figure 2: Magnitude of total magnetic field H_ϕ for a magnetic loop ($M_0=1$) located at $\rho'=1.25\lambda$, $z'=0.5\lambda$ with $a=1\lambda$, $b=1.5\lambda$, $d=0.8\lambda$, and $N=5$.

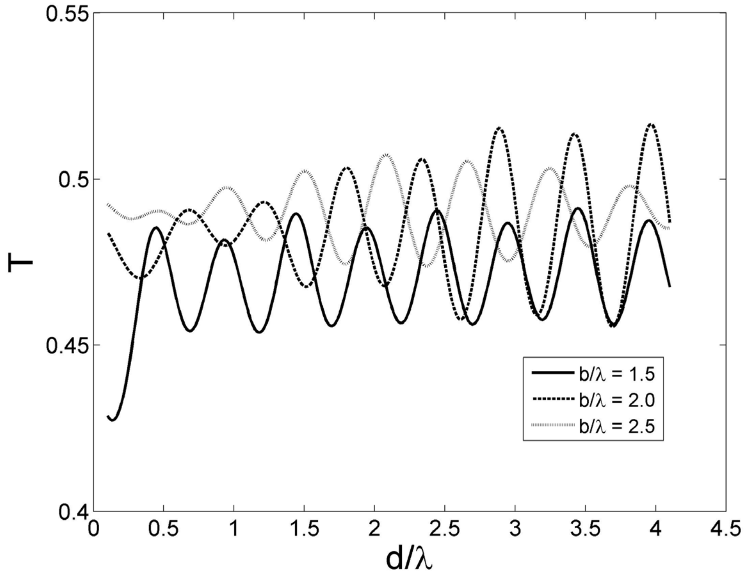

Figure 3: Transmission coefficients T versus d/λ when $a = 1\lambda$, $\rho' = 1.25\lambda$, $z' = 0.5\lambda$ and $M_0 = 1$

4. Conclusion

Electromagnetic power penetration into a wire-penetrating aperture has been discussed by using the associated Weber transform, the superposition principle and the mode-matching method. A magnetic current loop was assumed as an excitation and the effects of the plane thickness and aperture size on the transmission coefficients were also investigated. The solution was formulated in terms of the fast convergent series form. Our formulation can be further applied to the practical situations such as the EMI calculation associated with via holes on a PCB.

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