

# Field Distribution along Random Rough Surface in View of 1-Ray and 2-Rays Models

# Junichi Honda and Kazunori Uchida

Department of Information and Communication Engineering, Fukuoka Institute of Technology  
3-30-1 Wajiro-Higashi, Higashi-ku, Fukuoka, 811-0295 Japan  
Email: jhonda-fit@gmail.com, k-uchida@fit.ac.jp

## Abstract

This paper is concerned with an estimation of field distribution along random rough surfaces (RRSs) by using 1-ray and 2-rays models with two modification factors which are an amplitude modification  $\alpha$  and an order of propagation distance  $\beta$ . Comparing with the discrete ray tracing method (DRTM), we show that the 1-ray and 2-rays models can express approximately the field distribution along RRSs.

**Keyword:** 1-ray model, 2-rays model, DRTM, random rough surface, amplitude modification, order of propagation distance

## 1. Introduction

Recently, the information and communication technologies play an important role in many research fields, among them, the sensor network has attracted many researcher's interest [1]. Studies on the sensor network have been discussed mainly on network layer of OSI model. However, in order to develop the circuit such as antenna, radio propagation corresponding to the physical layer should be important factor.

When the sensor devices gather the physical information, the devices are distributed not only in the urban area with high rise buildings but also in natural environments such as dessert, hilly terrain and sea surfaces. Theses surfaces are assumed to be random rough surfaces (RRSs). The sensor devices placed just above RRSs are much influenced by scattered waves from RRSs. Therefore, it is important to investigate the propagation characteristics along RRSs [2].

We have proposed the discrete ray tracing method (DRTM) whose computation time is faster than the conventional ray tracing method (RTM) [2], [3]. In that method, we have discretized RRSs as well as ray searching between source and receiver. The DRTM is one of the suitable numerical technique for estimation of field intensity above RRSs. However, it needs a little computation time when we treat very complicated long RRSs or two dimensional (2D) RRSs. In this paper, we consider more simple numerical technique based on the 1-ray and 2-rays model. 1-ray model is the incident wave, and 2-rays model is the total field of incident and reflected waves. We use two modification factors ( $\alpha$ ,  $\beta$ ) in these method;  $\alpha$  is an amplitude modification, and  $\beta$  is an order of propagation distance. Comparing with the results of DRTM, we show that the field distribution along RRSs is expressed approximately by using 1-ray and 2-rays models with two parameters.

## 2. 1-Ray and 2-Ray Models

We introduce approximate solutions based on the 1-ray model and the 2-rays model. We first review those models. 1-ray model means incident wave, and 2-rays model means the total field which includes incident wave and reflected wave from the plane ground. The incident field emitted from antenna is expressed as follows [4]:

$$\mathbf{E}_i = \sqrt{30GP} \frac{e^{-jk_0 r}}{r} \mathbf{\Theta}^v(\mathbf{r}, \mathbf{p}_s) \quad (1)$$

where the time dependence  $e^{j\omega t}$  is assumed, and the wavenumber in the free space is given by  $\kappa_0 = \lambda/2\pi$ .  $P$  is input power of source, and we also assume the isotopic antenna as the absolute gain  $G = 1.0$ .  $\mathbf{p}_s$

is the unit vector of source, and  $\mathbf{r}$  is the position vector from source to observation point [4]. The unit vectors of electromagnetic polarization are given by

$$\Theta^v(\mathbf{r}, \mathbf{p}_s) = \frac{[(\mathbf{r} \times \mathbf{p}_s) \times \mathbf{r}]}{|(\mathbf{r} \times \mathbf{p}_s) \times \mathbf{r}|}, \quad \Theta^h(\mathbf{r}, \mathbf{p}_s) = \frac{(\mathbf{r} \times \mathbf{p}_s)}{|\mathbf{r} \times \mathbf{p}_s|}. \quad (2)$$

On the other hands, the reflected field is written as follows:

$$\mathbf{E}_r = \sqrt{30GP} \frac{e^{-jk_0 r_0}}{r_0} \mathbf{e}_0 \quad (3)$$

where  $r_0 = r_1 + r_2$ , and the following relation is given by

$$\mathbf{e}_0 = R^v(\theta_i)[\Theta^v(\mathbf{r}_1, \mathbf{p}) \cdot \Theta^v(\mathbf{r}_1, \mathbf{n})]\Theta^v(\mathbf{r}_2, \mathbf{n}) + R^h(\theta_i)[\Theta^h(\mathbf{r}_1, \mathbf{p}_s) \cdot \Theta^h(\mathbf{r}_1, \mathbf{n})]\Theta^h(\mathbf{r}_2, \mathbf{n}) \quad (4)$$

where  $\mathbf{r}_1$  is a distance vector from source to reflection point, and  $\mathbf{r}_2$  is a distance vector from reflection point to observation point.  $\mathbf{n}$  is a normal vector. The Fresnel reflection coefficients are given for vertical and horizontal electric components as follows:

$$R^h(\theta_i) = \frac{\cos \theta_i - \sqrt{\epsilon_c - \sin^2 \theta_i}}{\cos \theta_i + \sqrt{\epsilon_c - \sin^2 \theta_i}}, \quad R^v(\theta_i) = \frac{\epsilon_c \cos \theta_i - \sqrt{\epsilon_c - \sin^2 \theta_i}}{\epsilon_c \cos \theta_i + \sqrt{\epsilon_c - \sin^2 \theta_i}} \quad (5)$$

where  $\epsilon_c = \epsilon_r - j\sigma/\omega\epsilon_0$  is the complex permittivity of the ground plane with dielectric constant  $\epsilon_r$  and conductivity  $\sigma$ .

From the above discussion, we obtain the total field as follows:

$$\mathbf{E}_t = \mathbf{E}_i + \mathbf{E}_r. \quad (6)$$

Eq.(6) satisfies with electromagnetic theory. Considering the real situation, field intensity, however, is subject to a rapid disturbance due to reflections and diffraction by dense high rise building in urban area or complicated terrestrial surfaces. To deal with this problem, we introduce two modification factors ( $\alpha$ ,  $\beta$ ) into 1-ray and 2-rays models. Then, the incident and reflected fields are summarized as follows:

$$\mathbf{E}_i \simeq 10^{\alpha/20} \sqrt{30GP} \frac{e^{-jk_0 r}}{r^\beta} \Theta^v(\mathbf{r}, \mathbf{p}_s), \quad \mathbf{E}_r \simeq 10^{\alpha/20} \sqrt{30GP} \frac{e^{-jk_0 r_0}}{r_0^\beta} \mathbf{e}_0. \quad (7)$$

As a result, the total field is given by

$$\begin{aligned} \mathbf{E}_t &\simeq 10^{\alpha/20} \sqrt{30GP} \frac{e^{-jk_0 r}}{r^\beta} \mathbf{e}_t \\ \mathbf{e}_t &= \Theta^v(\mathbf{r}, \mathbf{p}) + \frac{r^\beta e^{-jk(r_0-r)}}{r_0^\beta} \mathbf{e}_0 \end{aligned} \quad (8)$$

where  $\alpha$  is the amplitude modification, and  $\beta$  is the order of propagation distance. Eq.(1) shows  $|E| \propto r^{-1}$ , that is,  $\beta = 1.0$  in the free space. In the complicated environment, however, we have  $\beta \neq 1.0$  and attenuation is enhanced as  $\beta$  is increased. Rewriting the total field in dB leads to the following equation:

$$|\mathbf{E}_t| \simeq A + \alpha - 20\beta \log_{10}(r) \quad [\text{dBV/m}] \quad (9)$$

$$A = 20 \log_{10}(\sqrt{30GW} |e_t|). \quad (10)$$

This equation is very simple, but it enables us to estimate easily the field distribution in complicated propagation environments by choosing two parameters.

Figure 1 shows the field distributions obtained by 1-ray and 2-rays models with  $\alpha = 0[\text{dB}]$ .  $\beta$  is changed in the range from 1.0 to 2.0. The source height and the receiver height are chosen as 30[m] and 1.5[m], respectively. We select the following parameters;  $f = 1.0[\text{GHz}]$ ,  $\epsilon_r = 5.0$  and  $\sigma = 0.0023[\text{S/m}]$ . Three curves are field distributions calculated by 1-ray model and the others are the results calculated by 2-rays model. It is demonstrated that the larger  $\beta$  is, the larger field attenuation becomes. And, it is shown that two curves are in good agreement with each other in the far field. It is also shown in the near field that 2-rays model is slightly different from 1-ray model due to the break point. As a result, we have found that the field decay only depends on  $\beta$ .

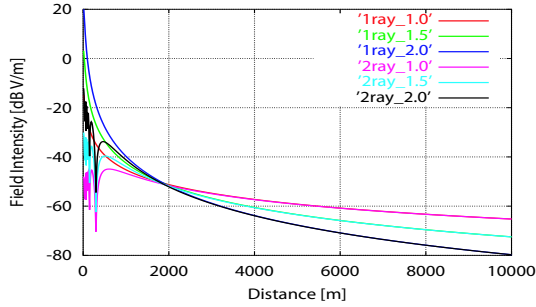


Figure 1: 1-ray model and 2-rays model.

### 3. Numerical Simulation

We compare the field distribution computed by DRTM with the field distribution estimated by 1-ray and 2-rays models. The DRTM has been proposed by authors, and the method carries out discretization of RRSs as well as discretization of searching rays from source to receiver. These operations save much computation time in comparison with the RTM. In this paper, we omit the detailed discussions on the DRTM [2], [3].

Figure 2 shows the geometry of the problem. Source point is placed on  $x = 0[m]$  and it is  $0.5[m]$  high above RRS. Observation points are  $0.5[m]$  high above RRS, and it is movable along RRS. Operating frequency is chosen as  $f = 1.0GHz$ . We assume here the RRS to be dry soil with material constants  $\epsilon_r = 5.0$  and  $\sigma = 0.0023[S/m]$ . The shapes of RRS are varied depending on the two RRS's parameters; correlation length ( $cl$ ) and deviation of its height ( $h$ ) [2]. Since a sensor device should be communicate with other sensor devices distributed on the RRSs, we assume the isotopic antenna as absolute gain  $G = 1.0$ . The input power is selected as  $P = 1[W]$ .

Figure 3 shows field distribution along RRSs with  $cl = 30[m]$  and  $h = 20[m]$ . Red line is a field distribution calculated by the DRTM. In this simulation we consider second order of scattered waves, and the curve is the averaged field distribution whose sampled number is 100. Green and blue lines are obtained by 1-ray and 2-rays models, respectively. We select  $\alpha = -15.0[dB]$  and  $\beta = 1.45$  for 1-ray and 2-rays models. It is shown that two curves obtained by 1-ray and 2-rays models agree with the result of DRTM in the far field. We can also see the little error between 1-ray and 2-rays models in the near field.

Figure 4 shows field distribution along RRSs with  $cl = 50[m]$  and  $h = 25[m]$ . Red line is a field distribution calculated by the DRTM, and the its ensemble average is 100 samples. Green and blue lines are obtained by 1-ray and 2-rays models, respectively. We select  $\alpha = -4.8[dB]$  and  $\beta = 1.5$  for 1-ray and 2-rays models. It is shown that two curves obtained by 1-ray and 2-rays models agree with the result of DRTM in the far field.

The results of the numerical simulations shown so far are summarized as follows. We could approximately estimate the field distribution along RRSs by using 1-ray and 2-rays models with two modification factors ( $\alpha, \beta$ ). In these numerical simulations, we could not see the break point because the source point is very close to RRSs. In the near field, the result of 1-ray model is slightly different from that of 2-rays model. However, two curves obtained by 1-ray and 2-rays models are in good agreement with the numerical result of DRTM in the far field. Since the 1-ray and 2-rays models are so simple, those calculation times are much faster than DRTM.

### 4. Conclusion

In this paper, we have introduced the field estimation method based on the 1-ray and 2-rays models. We have used two modification factors into 1-ray and 2-rays models: one is the amplitude modification ( $\alpha$ ), and the other is the order of propagation distance ( $\beta$ ). We have approximately estimated the field distribution along RRSs by using two modifications. It has been shown that the numerical results obtained by proposed 1-ray and 2-rays models are in agreement with that of DRTM. The present method is

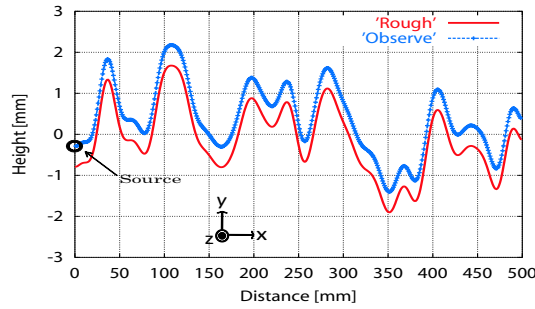


Figure 2: Source and observation points.

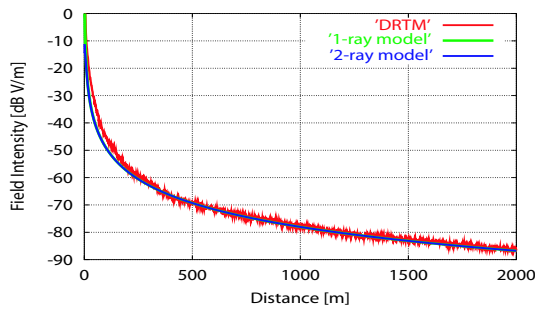


Figure 3: Field distribution along RRS  
( $cl = 30[m]$ ,  $h = 20[m]$ ).

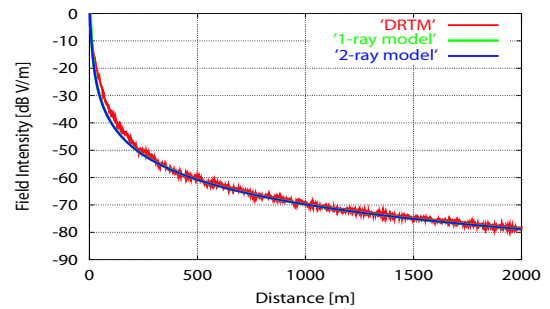


Figure 4: Field distribution along RRS  
( $cl = 50[m]$ ,  $h = 25[m]$ ).

a simple numerical method, and its computation time is faster than DRTM and other methods.

We have to investigate the relationship between two factors ( $\alpha, \beta$ ) and RRS's parameters ( $cl, h$ ). We also need to apply the proposed method to the allocation of sensor devices. These studies deserves as our near future work.

## Acknowledgments

The work was supported in part by a Grand-in Aid for Scientific Research (C) (21560421) from Japan Society for the Promotion of Science.

## References

- [1] Y. Tobe, "Trend of Technologies in Wireless Sensor Networks", *IEICE Trans. Commun.*, Vol. J90-B, No.8, pp. 711-719, Aug. 2007.
- [2] J. Honda, K. Uchida and K.Y. Yoon, "Estimation of Radio Communication Distance along Random Rough Surface", *IEICE Trans. Electron.*, Vol. E93-C, No. 1, pp. 39-45 Jan. 2010.
- [3] K. Uchida and J. Honda (2010) Estimation on Radio Communication Distance and Propagation Characteristics along Random Rough Surface for Sensor Networks, *Wireless Sensor Networks: Application-Centric Design*, Geoff V Merret and Yen Kheng Tan (Ed.), InTech, Chapter 13, pp. 231-248, Dec. 2010.
- [4] T. Tamaki, K. Uchida and J. Honda, "On Distance Characteristics Parameters Evaluated by a Two-Ray Ground Reflection Model", *Proceedings of ISAP 2010*, pp.73-76, Nov. 2010.