A Perturbative Representation of Depolarized Electromagnetic Waves Propagated Through Random Medium Screen

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Abstract

We have derived an integral equation using the dyadic Green's function on the assumption that there exists a random medium screen of which the dielectric constant is fluctuating randomly. From the integral equation, an analytic expression of the depolarized EM wave has been given by using the perturbation method.

Keywords: random medium, depolarized electromagnetic wave, perturbation method,

1. Introduction

Studies of the electromagnetic(EM) wave propagation through random media have been continued into the important development of various measurements and communications. It is mainly assumed in the studies that the fluctuating intensity of continuous random medium is so weak and the fluctuating scale-size is much larger than the wave length of EM wave[1, 2]. Therefore the scalar approximation has been used. When the optical path-length becomes large in long propagation through random medium, however, then we have to consider the effect of depolarization of EM waves. In previous studies, quantitative analysis of the depolarization has not been investigated sufficiently.

In this paper, to analyze the depolarized EM waves, we first have derived an integral equation using the dyadic Green's function on the assumption that there exists a random medium screen of which the dielectric constant is fluctuating randomly. Next we have modified the integral equation on the assumption that the observation point is very far from the screen. From this modified integral equation, an analytic expression of the depolarized EM wave has been given by using the perturbation method. Finally we have shown the representation of the first order perturbation of the depolarized EM wave; and will be able to discuss quantitatively the depolarization of EM wave propagated through the random medium screen.

2. Formulation

Let us consider the problem of EM wave scattering by a random medium with volume V. When we designate an incident EM wave by $\mathbf{F}_{in} = [\mathbf{E}_{in}, \mathbf{H}_{in}]_{\mathrm{t}}$, the scattered EM wave by $\mathbf{F}_s = [\mathbf{E}_s, \mathbf{H}_s]_{\mathrm{t}}$, and the total EM wave by $\mathbf{F} = [\mathbf{E}, \mathbf{H}]_{\mathrm{t}}$, then $\mathbf{F}$ satisfies the Maxwell's equations in overall region $(V + \bar{V})$. Therefore we obtain

$$\overline{\overline{L}}\mathbf{F} \;=\; -\overline{\overline{T}}\,\overline{\overline{T}}\,\overline{\overline{M}}\,\mathbf{F}\;;\;\; \overline{\overline{L}} = \left[\begin{array}{c|c} \nabla\times & -j\omega\mu_0\overline{\overline{1}} \\ \hline j\omega\varepsilon_0\overline{\overline{1}} & \nabla\times \end{array}\right],\;\; \overline{\overline{M}}(\mathbf{r}) = j\omega\left[\begin{array}{c|c} \overline{\overline{0}} & -\delta\mu(\mathbf{r})\mu_0\overline{\overline{1}} \\ \hline \delta\varepsilon(\mathbf{r})\varepsilon_0\overline{\overline{1}} & \overline{\overline{0}} \end{array}\right] \tag{1}$$

where $[1 + \delta\varepsilon(\mathbf{r})]\varepsilon_0$ and $[1 + \delta\mu(\mathbf{r})]\mu_0$ are the dielectric constant and magnetic permeability of the random medium, respectively, $\delta\varepsilon(\mathbf{r})$ and $\delta\mu(\mathbf{r})$ are random function, and

$$[\,\nabla\times\,] \equiv \begin{bmatrix} 0 & -\partial/\partial z & \partial/\partial y \\ \partial/\partial z & 0 & -\partial/\partial x \\ -\partial/\partial y & \partial/\partial x & 0 \end{bmatrix},\; \overline{\overline{T}} = \left[\begin{array}{c|c} \overline{\overline{0}} & -\overline{\overline{1}} \\ \hline \overline{\overline{1}} & \overline{\overline{0}} \end{array}\right],\; \overline{\overline{1}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\; \overline{\overline{0}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{2}$$

Using the dyadic Green's function $\mathbf{G}$, we can express solutions of Eq.(1) with $\mathbf{F}_{in}$ as these of the following integral equation:

$$\mathbf{F} = \mathbf{F}_{in} + \int_V \mathbf{G}(\mathbf{r}-\mathbf{r}') \odot \overline{\overline{T}}\,\overline{\overline{M}}(\mathbf{r}')\,\mathbf{F}(\mathbf{r}')\,\mathrm{d}\mathbf{r}' \tag{3}$$

Let us define a vector $\mathbf{A}$ as $\mathbf{A} = [\mathbf{A}_e,\ \mathbf{A}_m]_\mathrm{t}$ where $\mathbf{A}_e = [A_{ex}\ A_{ey}\ A_{ez}]_\mathrm{t}$ and $\mathbf{A}_m = [A_{mx}\ A_{my}\ A_{mz}]_\mathrm{t}$. Then, using $\mathbf{G} = \mathbf{G}_e + \mathbf{G}_m$ where $\mathbf{G}_e$, $\mathbf{G}_m$, respectively, are due to electric and magnetic point sources, we define the operator $\odot$ as follows:

$$\mathbf{G} \odot \mathbf{A} = \sum_{\alpha=x,y,z} (\mathbf{G}_{e\alpha} A_{e\alpha} + \mathbf{G}_{m\alpha} A_{m\alpha}) \tag{4}$$

where $\mathbf{G}_e$ is expressed as

$$\mathbf{G}_e = \sum_\alpha \mathbf{G}_{e\alpha} = \sum_\alpha \begin{bmatrix} \mathbf{G}^e_{e\alpha} \\ \mathbf{G}^m_{e\alpha} \end{bmatrix} \quad ; \quad \mathbf{G}^e_{e\alpha} = \sum_\alpha \begin{bmatrix} G^{ex}_{e\alpha} \\ G^{ey}_{e\alpha} \\ G^{ez}_{e\alpha} \end{bmatrix} \quad , \quad \mathbf{G}^m_{e\alpha} = \sum_\alpha \begin{bmatrix} G^{mx}_{e\alpha} \\ G^{my}_{e\alpha} \\ G^{mz}_{e\alpha} \end{bmatrix} \tag{5}$$

and $\mathbf{G}_m$ is expressed by changing the subscript of $\mathbf{G}_e$ from e to m. Here it should be noted that $G^{e\beta}_{e\alpha}$, $G^{m\beta}_{e\alpha}$, respectively, are the β-directional electric and magnetic wave fields radiated from the α-directional electric point source.

3. Analysis of Dyadic Green's Function

Figure 1 shows geometry of the problem. Assuming TE-wave incidence shown in Fig.1 and applying Eq.(4) on $\mathbf{A}$ with $A_{m\alpha} = 0$ because of $\delta\mu(\mathbf{r}) = 0$, we have

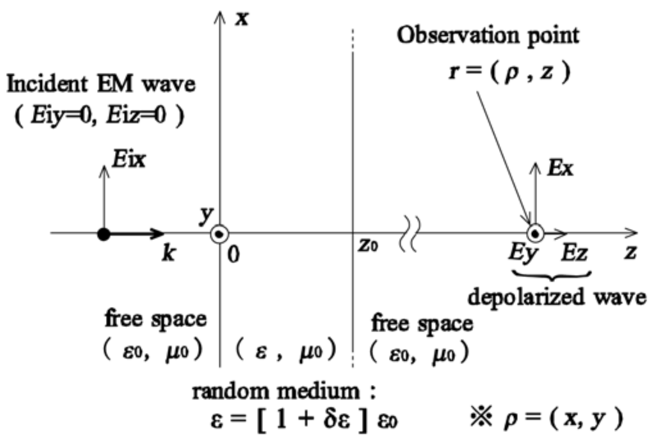

Figure 1: Geometry of the problem

$$\begin{bmatrix} E_x(\boldsymbol{\rho},z) \\ E_y(\boldsymbol{\rho},z) \\ E_z(\boldsymbol{\rho},z) \end{bmatrix} = \begin{bmatrix} E_{ix}(\boldsymbol{\rho},z) \\ 0 \\ 0 \end{bmatrix} - j\omega \int_V \sum_\alpha \begin{bmatrix} G^{ex}_{e\alpha}(\boldsymbol{\rho}-\boldsymbol{\rho}', z-z')\delta\varepsilon(\boldsymbol{\rho}',z')\varepsilon_0 E_\alpha(\boldsymbol{\rho}',z') \\ G^{ey}_{e\alpha}(\boldsymbol{\rho}-\boldsymbol{\rho}', z-z')\delta\varepsilon(\boldsymbol{\rho}',z')\varepsilon_0 E_\alpha(\boldsymbol{\rho}',z') \\ G^{ez}_{e\alpha}(\boldsymbol{\rho}-\boldsymbol{\rho}', z-z')\delta\varepsilon(\boldsymbol{\rho}',z')\varepsilon_0 E_\alpha(\boldsymbol{\rho}',z') \end{bmatrix} \mathrm{d}\mathbf{r}' \tag{6}$$

where $\boldsymbol{\rho} = (x,y)$ and $\int_V \mathrm{d}\mathbf{r}' = \int_0^{z_0} \mathrm{d}z' \int_{S_\infty} \mathrm{d}\boldsymbol{\rho}'$. The dyadic Green's function $\mathbf{G}^e_{e\alpha}$ is expressed as

$$\begin{aligned} \mathbf{G}^e_{e\alpha}(\mathbf{r},z) &= j\omega\mu_0 \left[\mathbf{i}_\alpha + (\mathbf{i}_\alpha\cdot\nabla)\nabla\right] G(\mathbf{r},z) \\ &= j\frac{1}{\omega\varepsilon_0}\left[\left(-k^2 + \frac{jk}{r} + \frac{1}{r^2}\right)(\mathbf{i}_\alpha\times\mathbf{i}_r)\times\mathbf{i}_r + 2\mathbf{i}_r(\mathbf{i}_\alpha\cdot\mathbf{i}_r)\left(\frac{jk}{r}+\frac{1}{r^2}\right)\right] G(\mathbf{r},z) \end{aligned} \tag{7}$$

where $G(\mathbf{r}, z)$ is the scalar Green's function in free space, $k = \omega\sqrt{\varepsilon_0\mu_0}$ is the wavenumber in free space and $\mathbf{i}_\alpha$ $(\alpha = x, y, z)$, $\mathbf{i}_r$ are the $\alpha-$ and $r-$ directional unit vector, respectively.

When $kr \gg 1$, then $\mathbf{G}^e_{e\alpha}$ is approximated by

$$\mathbf{G}^e_{e\alpha}(\mathbf{r}, z) = j\frac{1}{\omega\varepsilon_0}\left[-k^2(\mathbf{i}_\alpha \times \mathbf{i}_r) \times \mathbf{i}_r\right] G(\mathbf{r}, z) \tag{8}$$

Here we assume a far field of which the observation point is in the neighborhood of the z-axis; i.e., $z \gg \rho, x, y$, and $r \approx z \to \infty$. Then we obtain

$$\mathbf{G}^e_e = \begin{bmatrix} G^{ex}_{ex} & G^{ex}_{ey} & G^{ex}_{ez} \\ G^{ey}_{ex} & G^{ey}_{ey} & G^{ey}_{ez} \\ G^{ez}_{ex} & G^{ez}_{ey} & G^{ez}_{ez} \end{bmatrix} = \frac{k^2 G(\mathbf{r}, z)}{j\omega\varepsilon_0} \begin{bmatrix} R^{ex}_{ex} & R^{ex}_{ey} & R^{ex}_{ez} \\ R^{ey}_{ex} & R^{ey}_{ey} & R^{ey}_{ez} \\ R^{ez}_{ex} & R^{ez}_{ey} & R^{ez}_{ez} \end{bmatrix} = \frac{k^2 G(\mathbf{r}, z)}{j\omega\varepsilon_0}\mathbf{R}\,; \tag{9}$$

$$\mathbf{R} = \frac{1}{r^2}\begin{bmatrix} x^2 - r^2 & yx & zx \\ xy & y^2 - r^2 & zy \\ xz & yz & z^2 - r^2 \end{bmatrix} = \frac{1}{r^2}\begin{bmatrix} -(y^2 + z^2) & yx & zx \\ xy & -(x^2 + z^2) & zy \\ xz & yz & -(x^2 + y^2) \end{bmatrix} \tag{10}$$

4. Representation of EM wave using perturbation method

Here we use a perturbation method to estimate effects of depolarization due to the propagation in the random medium. Equation(6) can be expressed by the following formal equation.

$$\mathbf{E} = \mathbf{E}_{in} + \mathbf{L}\mathbf{E} \tag{11}$$

where $\mathbf{E} = [E_x, E_y, E_z]_{\mathrm{t}}$, $\mathbf{E}_{in} = [E_{ix}, E_{iy}, E_{iz}]_{\mathrm{t}}$, and $\mathbf{L} = -j\omega\int_V d\mathbf{r}' \cdot [\sum_\alpha \mathbf{G}^e_{e\alpha}(\mathbf{r} - \mathbf{r}')\varepsilon_d(\mathbf{r}')]$. Here, $\mathbf{L}$ is divide into $\mathbf{L} = \mathbf{L}_0 + \Delta\mathbf{L}$, where $\mathbf{L}_0$ and $\Delta\mathbf{L}$ are the unperturbed and perturbed operators, respectively. In this case, $\mathbf{E}$ can also be defined as $\mathbf{E} = \sum_{n=0}^{\infty}\Delta^n\mathbf{E}^{(n)}$. According as the power of Δ, the field equations may be expressed by

$$\Delta^0 : \mathbf{E}^{(0)} = \mathbf{E}_{in} + \mathbf{L}_0\mathbf{E}^{(0)}\,, \quad \cdots \quad , \; \Delta^{(n)} : \mathbf{E}^{(n)} = \Delta\mathbf{L}\mathbf{E}^{(n-1)} + \mathbf{L}_0\mathbf{E}^{(n)}. \tag{12}$$

In above equations, $\mathbf{E}^{(0)}$ is the unperturbed field which is not depolarized through propagation in the random medium. By putting $\Delta^{(n)} = 1$, then $\mathbf{E}$ can be expressed as $\mathbf{E} = \sum_{n=0}^{\infty}\mathbf{E}^{(n)} = \mathbf{E}^{(0)} + \mathbf{E}^{(1)} + \mathbf{E}^{(2)} + \cdots + \mathbf{E}^{(n)} + \cdots$. On the other hand, $\mathbf{E}$ can be expressed by

$$\begin{aligned} \mathbf{E} &= (\,\mathbf{E}_{in} + \mathbf{L}_0\mathbf{E}^{(0)}\,) + (\,\Delta\mathbf{L}\mathbf{E}^{(0)} + \mathbf{L}_0\mathbf{E}^{(1)}\,) + (\,\Delta\mathbf{L}\mathbf{E}^{(1)} + \mathbf{L}_0\mathbf{E}^{(2)}\,) + \cdots \\ &= \mathbf{E}_{in} + (\mathbf{L}_0 + \Delta\mathbf{L})(\mathbf{E}^{(0)} + \mathbf{E}^{(1)} + \mathbf{E}^{(2)} + \cdots) = \mathbf{E}_{in} + \mathbf{L}\mathbf{E} \end{aligned} \tag{13}$$

Because $\mathbf{E}_{in} = [E_{ix}, 0, 0]_{\mathrm{t}}$ as shown in Fig.1, the unperturbed wave is given as the solution of the following equation.

$$\begin{aligned} E^{(0)}_x(\boldsymbol{\rho}, z) &= E^{(0)}_{ix}(\boldsymbol{\rho}, z) - k^2\int_0^{z_0} dz' \int_{S_\infty} d\boldsymbol{\rho}' \\ &\quad \cdot \left[G(\boldsymbol{\rho} - \boldsymbol{\rho}', z - z')R^{ex}_{ex}(\boldsymbol{\rho} - \boldsymbol{\rho}', z - z')\delta\varepsilon(\boldsymbol{\rho}', z')E^{(0)}_x(\boldsymbol{\rho}', z')\right] \end{aligned} \tag{14}$$

Here $E^{(0)}_y(\boldsymbol{\rho}, z) = 0$ and $E^{(0)}_z(\boldsymbol{\rho}, z) = 0$. From Eq.(22), we obtain the unperturbed operator $\mathbf{L}_0$.

$$\mathbf{L}_0 = L_0\mathbf{D}_{L_0}\,;\quad L_0 = -k^2\int_0^{z_0} dz' \int_{S_\infty} d\boldsymbol{\rho}' \cdot [G(\boldsymbol{\rho} - \boldsymbol{\rho}', z - z')\delta\varepsilon(\boldsymbol{\rho}', z')]\,, \tag{15}$$

$$\mathbf{D}_{L_0} = \frac{-1}{|\mathbf{r} - \mathbf{r}'|^2}\begin{bmatrix} (y - y')^2 + (z - z')^2 & 0 & 0 \\ 0 & (x - x')^2 + (z - z')^2 & 0 \\ 0 & 0 & (x - x')^2 + (y - y')^2 \end{bmatrix} \tag{16}$$

On the other hand, using Eqs.(9) and (10), we can express Eq.(11) as

$$\begin{aligned}\mathbf{E}(\boldsymbol{\rho}, z) &= \mathbf{E}_{in}(\boldsymbol{\rho}, z) - k^2 \int_0^{z_0} dz' \int_{S_\infty} d\boldsymbol{\rho}' \\ &\quad \cdot [G(\boldsymbol{\rho}-\boldsymbol{\rho}', z-z')\mathbf{R}(\boldsymbol{\rho}-\boldsymbol{\rho}', z-z')\delta\varepsilon(\boldsymbol{\rho}', z')\mathbf{E}(\boldsymbol{\rho}', z')] \end{aligned} \tag{17}$$

From above equation, we obtain the operator $\mathbf{L}$.

$$\mathbf{L} = L_0 \mathbf{D}_L \ ; \tag{18}$$

$$\begin{aligned}\mathbf{D}_L &= \mathbf{R}(\boldsymbol{\rho}-\boldsymbol{\rho}', z-z') \\ &= \frac{1}{|\mathbf{r}-\mathbf{r}'|^2}\begin{bmatrix} -[(y-y')^2+(z-z')^2] & (x-x')(y-y') & (x-x')(z-z') \\ (x-x')(y-y') & -[(x-x')^2+(z-z')^2] & (y-y')(z-z') \\ (x-x')(z-z') & (y-y')(z-z') & -[(x-x')^2+(y-y')^2] \end{bmatrix}\end{aligned} \tag{19}$$

Because $\Delta\mathbf{L} = \mathbf{L} - \mathbf{L}_0$, we can obtain the perturbed operator $\Delta\mathbf{L}$.

$$\Delta\mathbf{L} = L_0 \Delta\mathbf{D} \ ; \quad \Delta\mathbf{D} = \mathbf{D}_L - \mathbf{D}_{L_0} \tag{20}$$

From Eqs.(9),(10),(12) and (20), we have

$$\begin{aligned}\Delta\mathbf{L}\mathbf{E}^{(0)} &= L_0\Delta\mathbf{D}\mathbf{E}^{(0)} = -k^2 \int_0^{z_0} dz' \int_{S_\infty} d\boldsymbol{\rho}' \\ &\cdot G(\boldsymbol{\rho}-\boldsymbol{\rho}', z-z') \begin{bmatrix} 0 \\ R_{ex}^{ey}(\boldsymbol{\rho}-\boldsymbol{\rho}', z-z')\delta\varepsilon(\boldsymbol{\rho}', z')E_x^{(0)}(\boldsymbol{\rho}', z') \\ R_{ex}^{ez}(\boldsymbol{\rho}-\boldsymbol{\rho}', z-z')\delta\varepsilon(\boldsymbol{\rho}', z')E_x^{(0)}(\boldsymbol{\rho}', z') \end{bmatrix} \equiv \mathbf{E}_{in}^{(1)}(\boldsymbol{\rho}, z) = \begin{bmatrix} 0 \\ E_{iy}^{(1)}(\boldsymbol{\rho}, z) \\ E_{iz}^{(1)}(\boldsymbol{\rho}, z) \end{bmatrix}\end{aligned} \tag{21}$$

Therefore, $E_x^{(1)}(\boldsymbol{\rho}, z) = 0$, and $E_y^{(1)}(\boldsymbol{\rho}, z)$, $E_z^{(1)}(\boldsymbol{\rho}, z)$ satisfy the following equation.

$$\begin{aligned}E_{\alpha'}^{(1)}(\boldsymbol{\rho}, z) &= E_{i\alpha'}^{(1)}(\boldsymbol{\rho}, z) - k^2 \int_0^{z_0} dz' \int_{S_\infty} d\boldsymbol{\rho}' \\ &\cdot \left[G(\boldsymbol{\rho}-\boldsymbol{\rho}', z-z')R_{e\alpha'}^{e\alpha'}(\boldsymbol{\rho}-\boldsymbol{\rho}', z-z')\delta\varepsilon(\boldsymbol{\rho}', z')E_{\alpha'}^{(1)}(\boldsymbol{\rho}', z')\right] \quad (\alpha' = y, z)\end{aligned} \tag{22}$$

If $z_0 \ll z$, then $R_{ex}^{ex} \approx -1$, $R_{ey}^{ey} \approx -1$ and $R_{ez}^{ez} \approx 0$ in Eq.(22). In this case, we have obtained $\mathbf{E}^{(0)}$: the solution of Eq.(22) approximately in a compact form[2]. Therefore we can discuss quantitatively the depolarization through the analysis of Eq.(12).

5. Conclusion

We derived an integral equation using the dyadic Green's function on the assumption that there exists a random medium screen of which the dielectric constant is fluctuating randomly. To solve the depolarization problem, we modified the integral equation under the condition that the observation point is very far from the screen. From this modified integral equation, an analytic expression of the depolarized EM wave has been given by using the perturbation method. Finally we have shown the representation of the first order perturbation of the depolarized EM wave. Because the representation is written in a compact form using an ordered exponential function[2], this result is useful for the analysis of the depolarization of EM wave propagated through a random medium screen.

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