Microwave Imaging of the Buried Dielectric Objects Based on Non-Radiating Subspace Reconstruction

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Abstract

Imaging of dielectric cylinders buried in a known inhomogeneous medium based on non-radiating (NR) current reconstruction is presented. For this, we use the finite element-boundary integral (FE-BI) method to formulate the scattering problem, then, we modify the subspace optimization method (SOM) by using sequential quadratic programming (SQP) for solving inverse scattering problem. Also the effect of NR current is determined

Keywords : Inverse scattering SQP-SOM NR current

1. Introduction

Microwave imaging of the buried objects has been found wide applications such as through-wall imaging, detection of bomb buried in the ground and visualization of breast cancer. Non-radiating (NR) currents which belong to the null space of the mapping operator [1] reveal important ambiguous characteristics in inverse source problems [2] and improve the resolution of imaging in inverse scattering problems [3].

In [4] radiating and NR current are expanded by singular value decomposition (SVD) of the scattering operator which map the induced current to the outside scattered field. Reference [5] consider the inverse scattering as an inverse source problem to obtain NR currents for imaging systems. In [6] dielectric objects buried in an inhomogeneous media are reconstructed by using subspace optimization method (SOM). For this the objective function consisting of the scattering data mismatch and induced current residue, is constructed and then minimized by contrast source inversion (CSI) algorithm.

In present work we use finite element-boundary integral (FE-BI) method to formulate the current state equation and scattering data equation based on nodes of the background subunits. Then we form the corresponding objective function by SOM, and minimize this equation by sequential quadratic programming (SQP) technique. Also we compare the imaging results of this objective function with the results of the radiating objective function which is constructed only with radiating induced current.

2. FE-BI Formulation of the Problem

FE-BI method uses finite element method (FEM) for the interior field and surface integral equation (SIE) for the exterior field [7]. Consider an inhomogeneous medium which buries some dielectric cylinders as illustrated in Fig. 1, and a equispaced array of antennas around the inhomogeneous background (domain D) which incident the TM^z waves and receive the scattered fields.

The background field $E_b(\boldsymbol{r})$, and the total field $E(\boldsymbol{r})$, satisfy the Helmholtz equation in D, and from combination of these Helmholtz equations one can obtain [6]:

$$[\nabla^2 + k_0^2 \varepsilon_{rb}(\boldsymbol{r})](E(\boldsymbol{r}) - E_b(\boldsymbol{r})\ ) = -k_0^2[\varepsilon_r(\boldsymbol{r}) - \varepsilon_{rb}(\boldsymbol{r})]E(\boldsymbol{r}) \quad (1)$$

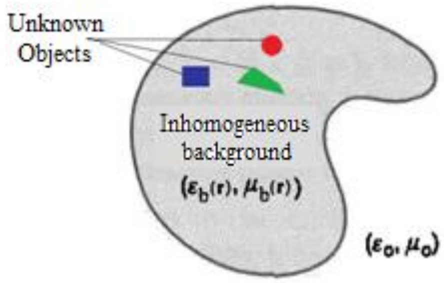

Figure 1: Unknown dielectric cylinders buried in an inhomogeneous background

where $\varepsilon_{rb}(\boldsymbol{r})$ and $\varepsilon_r(\boldsymbol{r})$ are the relative permittivity of the inhomogeneous background and buried objects, respectively, $\boldsymbol{r}$ denotes the position vector and $k_0 = \omega\sqrt{\varepsilon_0\mu}$ is the free space wave number with angular frequency ω, and free space permeability μ. We can write the right-hand side of relation (1) as $j\omega\mu I^{cur}(\boldsymbol{r})$, which $I^{cur}(\boldsymbol{r})$ is the induced current in the buried objects and reradiates the scattered field $E^{sca}(\boldsymbol{r}) = E(\boldsymbol{r}) - E_b(\boldsymbol{r})$ in the domain D. For exterior of the domain D one can use the surface integral equation [8] and obtain the scattered field as

$$E^{sca}(\boldsymbol{r}) = \oint_C \left\{-G(\boldsymbol{r},\boldsymbol{r}')I^{bou}(\boldsymbol{r}') + E^{sca}(\boldsymbol{r}')\frac{\partial G(\boldsymbol{r},\boldsymbol{r}')}{\partial n'}\right\} dC' \qquad (2)$$

where $I^{bou}(\boldsymbol{r}') = \frac{\partial E^{sca}(\boldsymbol{r}')}{\partial n'}$ represents the equivalent boundary current, $G(\boldsymbol{r},\boldsymbol{r}') = -\frac{j}{4}H_0^{(2)}(k_0|\boldsymbol{r} - \boldsymbol{r}'|)$ is two dimensional free space Green's function, $H_0^{(2)}(.)$ denotes the zeroth-order Hankel function of the second kind, C denotes the boundary of D, and $\frac{\partial}{\partial n'}$ represents the normal derivative along boundary C. relation (2) at the boundary $(\boldsymbol{r}\epsilon C)$ gives the boundary condition known as kirchhoff's boundary integral. For Discretization of kirchhoff's boundary integral we can use piecewise constant expansion for boundary field $E^{sca}(\boldsymbol{r}\epsilon C)$ and boundary current $I^{bou}(\boldsymbol{r}\epsilon C)$ with point matching approach [8], [9].

3. SQP-SOM for Inverse Scattering Problem

In conventional SOM [4], data and state equations are based on centers of the subunits of the background domain. Also, In [6] data equation for inhomogeneous background obtained based on centers of the subunits by using node to center operator, but because of the singularity errors corresponding to that operator, we did not use it in section 2. We obtained data and state equations based on nodes of subunits of the background. In this modified SOM, the objective function is evaluated at the nodes of the subunits. For optimization procedure, In [4] and [6] contrast source inversion (CSI) method is used, but in this modified SOM we use SQP algorithm which has fast internal linear algebra for solving quadratic programs [10]. Also, In [4] and [6] the vectors $\bar{\alpha}_p^n$ (coefficients of non-radiating singular vectors) are obtained as unknowns of the objective function, but in this modified SOM, we obtained the vectors $\bar{\alpha}_p^n$ from least squares procedure to decrease the unknowns of the objective function and increase the speed of the reconstruction procedure.

4. Numerical Results

Consider three dielectric cylinders buried in the inhomogeneous background as illustrated in Fig. 2. We use 10 numbers of transmitting and receiving antennas, and 124 subunits for grid mesh. In Fig. 2(b), the square with relative permittivity 1.5 represents the inhomogeneous medium, the bottom object with relative permittivity 2, represents object 1, and the two top objects in left and right sides with relative permittivity 1.8 and 1.3 represent objects 2 and 3, respectively.
The result of reconstructed objects based on NR current is shown in Fig. 3. The reconstructed result based on NR current is a successful and accurate solution. But, the reconstructed result without NR current, obtained from radiating objective function, is not acceptable as shown in Fig 4.

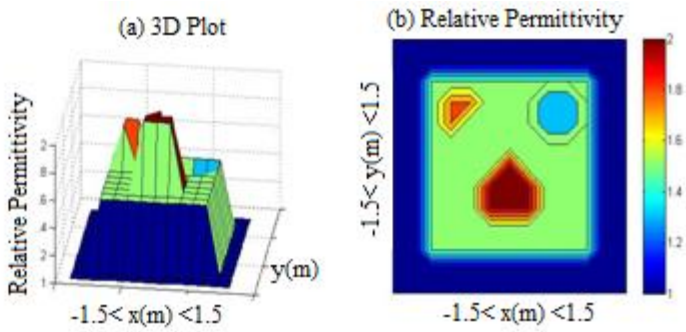

Figure 2: Exact relative permittivity of the buried bject

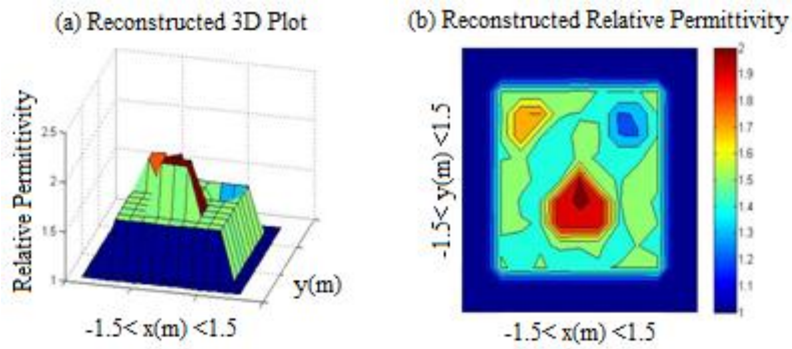

Figure 3: Objects buried in the inhomogeneous medium: (a) 3D plot of reconstructed relative permittivity, (b) 2D image of reconstructed relative permittivity.

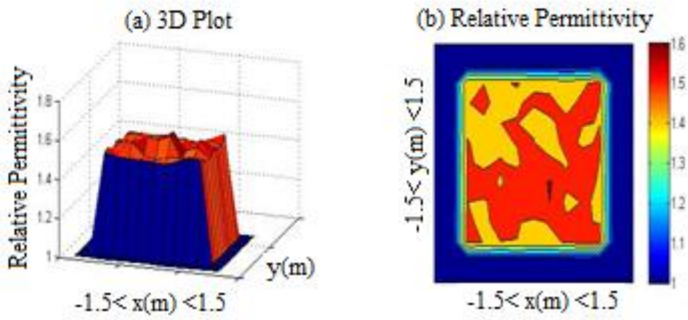

Figure 4: Reconstructed results with radiating objective function: (a) 3D plot of reconstructed relative permittivity, (b) 2D image of reconstructed relative permittivity.

For integrated circuit failure analysis [11] or detection of buried object, only the single-side view (semicircular array) for transmitting and receiving is possible as illustrated in Fig. 5.

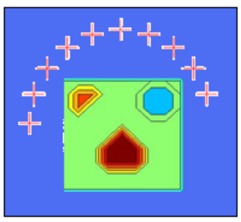

Figure 5: Semicircular array of antennas for single side incidence

To increase the resolution for this case, one can use frequency hopping procedure [12] which reconstructed permittivity of lower frequency is taken as the initial point for higher frequency. As shown in Fig. 6 the resolution of the results at the fourth step (600 MHz) becomes acceptable.

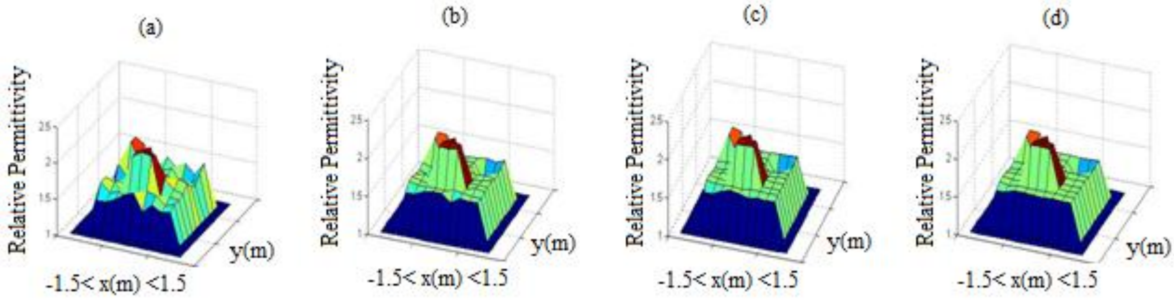

Figure 6: Reconstructed results using frequency hopping procedure: (a) $f = 300\ MHz$, (b) $f = 400\ MHz$; (c) $f = 500\ MHz$; (d) $f = 600\ MHz$.

5. Conclusions

In this paper we proposed a fast and accurate imaging of the dielectric objects buried in an inhomogeneous medium, based on NR current. For this we modified SOM for solving inverse problem and used SQP as a fast optimization algorithm to minimize the corresponding objective function. To discuss the role of NR current we defined a radiating objective function. The numerical results showed the buried dielectric objects could not reconstructed without NR subspace of the current. For single side view, the frequency hopping approach could improve the resolution of the reconstructed buried objects.

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