An Application of Particle Swarm Optimization to Reconstruction of a Homogeneous Dielectric Circular Cylinder

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Abstract

Particle swarm optimization (PSO) is applied to an inverse scattering problem that estimates the dielectric constant and the radius of a circular cylinder from scattered waves. A modification of PSO is introduced in order to reduce computational complexity when multiple-frequency data is available.
Keywords: inverse scattering problem, homogeneous dielectric circular cylinder, electromagnetic imaging, particle swarm optimization

1. Introduction

Various techniques have been studied for an inverse scattering problem to reconstruct an object from the scattered waves. Main difficulties of the problems are in the ill-posed and the nonlinear relations between the scattered waves and the object. An approach to reduce the ill-posed property has been studied by the author[1]; while much careful handling should be needed for the nonlinear property. For a strongly inhomogeneous object the problem reduces an optimization problem to minimize a cost function that relates differences between the measured and calculated scattered-waves. It is difficult in general to search for the minimum point of the cost function in terms of a gradient based optimization method because of existence of local minimum points.

Particle swarm optimization (PSO) is a stochastic nonlinear optimization algorithm, which is inspired by the social behavior of a bird flock or fish school. It has been applied to electromagnetic problems[2, 3], and is attractive because of its simplicity.

In this paper, we will apply PSO to a simple inverse scattering problem that estimate the dielectric constant and the radius of a homogeneous circular cylinder from scattered waves[4]. In inverse scattering problems, it is likely that data of different measurements is utilized in order to obtain detailed information of the object. In those cases it is natural that we should introduce a cost function as the sum of squared errors of the different measurement. It takes increased time to evaluate the summed cost function. In order to save the time a modification of PSO is also introduced.

2. Formulation

2.1. Scattering by a homogeneous dielectric circular cylinder

Let us begin with review of scattering problem of a homogeneous circular cylinder located at the origin in free space under E-wave time-harmonic excitations of the time factor $\exp(\mathrm{j}\omega t)$. The geometry is shown in Fig. 1. The relative dielectric constant and the radius of the cylinder are denoted by ε_r and r, respectively. When a plane wave propagating in the θ direction is incident to the cylinder, the scattered wave in the far-field is expressed as

$$u_\mathrm{s}(\rho,\phi) \simeq \sqrt{\frac{2}{\pi k\rho}}\mathrm{e}^{-\mathrm{j}k\rho+\frac{\mathrm{j}\pi}{4}}\bar{u}_\mathrm{s}(\phi) \tag{1}$$

where k is the wavenumber and $\bar{u}_\mathrm{s}(\phi)$ is the far-field (complex) pattern of scattered wave, which is represented as

$$\bar{u}_\mathrm{s}(\phi) = \sum_{m=-\infty}^{\infty}\beta_m\mathrm{j}^m\mathrm{e}^{\mathrm{j}m\phi}. \tag{2}$$

The coefficient β_m is calculated from ε_r and r according to

$$\beta_m = -\frac{\sqrt{\varepsilon_\mathrm{r}}\mathrm{J}_m(kr)\mathrm{J}_{m+1}(\sqrt{\varepsilon_\mathrm{r}}kr) - \mathrm{J}_m(\sqrt{\varepsilon_\mathrm{r}}ka)\mathrm{J}_{m+1}(kr)}{\sqrt{\varepsilon_\mathrm{r}}\mathrm{H}_m^{(2)}(kr)\mathrm{J}_{m+1}(\sqrt{\varepsilon_\mathrm{r}}kr) - \mathrm{J}_m(\sqrt{\varepsilon_\mathrm{r}}kr)\mathrm{H}_{m+1}^{(2)}(kr)} \times \mathrm{j}^{-m}\mathrm{e}^{-\mathrm{j}m\theta} \tag{3}$$

where J_m and $\mathrm{H}_m^{(2)}$ are the Bessel function and the Hankel function of the second kind, respectively.

2.2. Inverse Scattering Problem

Let us consider reconstruction of a homogeneous dielectric circular cylinder from the scattered waves. The center of the circular cylinder is fixed to the origin for simplicity; i.e., $\theta = 0$ is used because of the symmetry and only two parameter, ε_r and r, shall be recovered.

In order to recast the inverse problem to an optimization problem let us introduce the mean square error as

$$\Omega^{(i)}(\varepsilon_\mathrm{r}, r) = \frac{1}{2\pi}\int_0^{2\pi} \left|\bar{u}_\mathrm{s}^{(i)}(\phi; \varepsilon_\mathrm{r}, r) - \tilde{\bar{u}}_\mathrm{s}^{(i)}(\phi)\right|^2 d\phi \tag{4}$$

where $\bar{u}_\mathrm{s}$ and $\tilde{\bar{u}}_\mathrm{s}$ denote the measured and the calculated far-field patterns, respectively, and the superscript (i) means that the wavenumber $k = k_i$ is used. Substituting Eq. (2) into Eq. (4), we can obtain another form as

$$\Omega^{(i)}(\varepsilon_\mathrm{r}, r) \simeq \sum_{m=-M}^{M} \left|\beta_m^{(i)}(\varepsilon_\mathrm{r}, r) - \tilde{\beta}_m^{(i)}\right|^2, \qquad \tilde{\beta}_m^{(i)} = \frac{\mathrm{j}^{-m}}{2\pi}\int_0^{2\pi} \tilde{\bar{u}}_\mathrm{s}^{(i)}(\phi)\mathrm{e}^{-\mathrm{j}m\phi} d\phi \tag{5}$$

where M is a large number and β_m and $\tilde{\beta}_m$ denote the measured and the calculated coefficients of far-field patterns, respectively. Equation (5) is suitable for computation and is used as the cost function in the following numerical analysis.

2.3. Application of Particle Swarm Optimization

In PSO, multiple particles(candidate points) change their positions(the coordinates) in solution space according to their own experience and experience of the swarm. In order to apply PSO to the inverse problem that minimizes the cost function indicated in Eq. (5) let us set a swarm of P particles whose position is characterized as $\boldsymbol{x} = (\varepsilon_\mathrm{r}, r)$.

In the basic PSO the velocity of pth particle is updated according to

$$\boldsymbol{v}_p^{(t+1)} = w\boldsymbol{v}_p^{(t)} + c_1 r_1(\boldsymbol{p}_p^{(t)} - \boldsymbol{x}_p^{(t)}) + c_2 r_2(\boldsymbol{g}^{(t)} - \boldsymbol{x}_p^{(t)}) \tag{6}$$

where t is the iteration number, $\boldsymbol{p}_p^{(t)}$, which is called *pbest*, is the best solution which pth particle personally encountered, $\boldsymbol{g}^{(t)}$, which is called *gbest*, is the best solution which the entire swarm encountered, w is called the inertial weight, c_1, c_2 are scaling factors, and r_1, r_2 are random numbers uniformly distributed in $[0, 1]$. The position of pth particle $\boldsymbol{x}_p$ is updated (assuming the unit time is elapsed per iteration) according to

$$\boldsymbol{x}_p^{(t+1)} = \boldsymbol{x}_p^{(t)} + \boldsymbol{v}_p^{(t+1)}. \tag{7}$$

Each particle moves along its original course to some extent and is stochastically pulled to *pbest* and *gbest*.

Let us consider the case that scattering data of two different frequencies is available. A method in this case is to minimize the cost function $\Omega^{(1)} + \Omega^{(2)}$. The summed cost function would reduce rises and falls, emphasize its global minimum point, and facilitate the search for the global minimum point. There probably remain local minimum points, however, and time to evaluate the cost function is increased. As another method we introduce a novel PSO, which we call a split PSO in this paper, where the particles are split into two groups. The particles of the first group evaluate $\Omega^{(1)}$ and those of the other group do $\Omega^{(2)}$, moving according to

$$\boldsymbol{v}_p^{(k+1)} = w\boldsymbol{v}_p^{(t)} + c_1 r_1(\boldsymbol{p}_p^{(t)} - \boldsymbol{x}_p^{(t)}) + c_{21} r_{21}(\boldsymbol{g}_1^{(t)} - \boldsymbol{x}_p^{(t)}) + c_{22} r_{22}(\boldsymbol{g}_2^{(t)} - \boldsymbol{x}_p^{(t)}) \tag{8}$$

instead of Eq. (6) where c_{21}, c_{22} are scaling factors, r_{21}, r_{22} are random numbers uniformly distributed in $[0, 1]$, and $\boldsymbol{g}_i^{(t)}$ is the best solution of $\Omega^{(i)}$ that the ith group encountered.

The inverse algorithm is summarized as follows:

Step 1: Set the initial position of each particles by randomly selecting a value with uniform probability over the solution space. Similarly, set each dimension of the initial velocity of each particles by a random value in the range $[-V, V]$. Evaluate the cost function according to the position of particles and set *pbest* and *gbest*.

Step 2: Update the velocity of each particle by Eq.(6) or (8) and update position by Eq.(7). Evaluate the cost function at the updated positions and update the *pbest* and the *gbest*. The particles are allowed to move out of the solution space, however the particle outside the solution space are not evaluated for cost function; namely, invisible walls[2] are assumed as the boundary condition. This step is repeated until a termination criterion is satisfied.

3. Numerical Analysis

Let us examine a reconstruction of a lossless circular cylinder of $\varepsilon_{\rm r} = 3$ and $r/\lambda = 2$ where λ is a wavelength. The solution space is set to $1 \leq \varepsilon_{\rm r} \leq 5,\ 0.1 \leq r/\lambda \leq 4.1$ and V is set to be equal to the dynamic range of the solution space, i.e., 4. The parameters are set to $P = 10$, $w = 0.4, c_1 = c_2 = 2$ and $c_{21} = c_{22} = 1$. If all the dimensions of the velocity of particles and of difference between the positions of particles and *gbest* are less than $\epsilon = 0.01$ or the iteration number becomes larger than $I_{\max} = 200$, the minimization process is terminated. The scattering data of $k_1 = 2\pi/\lambda$ and/or $k_2 = \pi/\lambda$ is assumed to be available.

At first let us analysis the case that one frequency of k_1 or k_2 is available. Figures 2 and 3 show the contours of $\Omega^{(1)}$ and $\Omega^{(2)}$ and behaviors of particles searching for the minimum point according to the basic PSO, respectively. We see that the search for the minimum point fails in $\Omega^{(1)}$ and succeeds in $\Omega^{(2)}$.

Next let us examine the case that both frequencies are available. Figure 4 shows a movement of ten particles which have successfully searched for the minimum point of $\Omega^{(1)} + \Omega^{(2)}$ according to the basic PSO. The rises and falls of $\Omega^{(1)} + \Omega^{(2)}$ seem to be dull compared with $\Omega^{(1)}$ but there remain local minimum points. Figure 5 shows a movement of particles according to the split PSO, where each half of particles has searched for the minimum point of $\Omega^{(1)}$ or $\Omega^{(2)}$, respectively. Five different runs are executed for the same cylinder. Figures 6 and 7 respectively show that the estimated parameters and the value of the cost functions change similarly as the number of function evaluations are increased.

As the last analysis a hundred runs are executed for each of three different cylinders in the case that two frequencies is available. Table 1 shows the number of successfully recovering the parameters for each cylinder. We see from the table that the split PSO is slightly more successful than the basic one in searching for the global minimum point. Note that the split PSO requires the only one cost function of $\Omega^{(1)}$ or $\Omega^{(2)}$ to be evaluated for each particle. If the number of particles is increased to $P = 20$ we can see a great improvement of the possibility of successful search.

4. Conclusion

In this paper, we have considered a simple inverse scattering problem using the split PSO which simultaneously minimize two cost functions. The split PSO saves computational cost and tends to reduce traps at local minimum points. Analysis of characteristics of the PSO in details and its application to more complex problems which have many unknown parameters are subjects for future studies.

References

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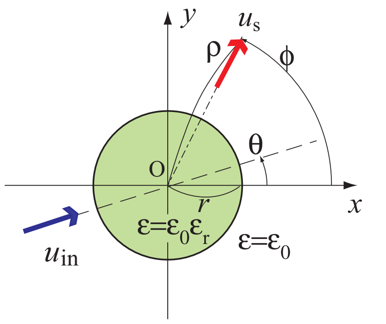

Figure 1: Geometry of the scattering problem

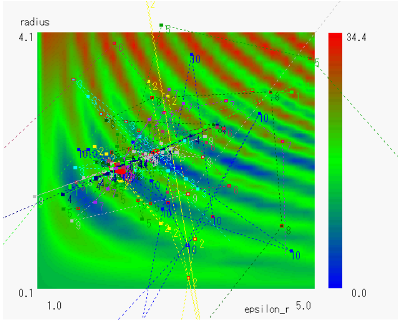

Figure 2: Cost function $\Omega^{(1)}$, whose minimum point is searched for by the basic PSO.

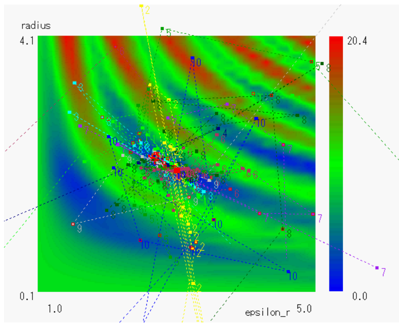

Figure 3: Cost function $\Omega^{(2)}$, whose minimum point is searched for by the basic PSO.

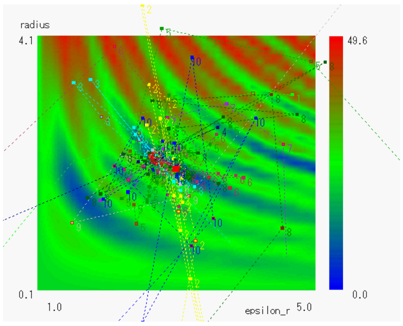

Figure 4: Cost function $\Omega^{(1)} + \Omega^{(2)}$, whose minimum point is searched for by the basic PSO.

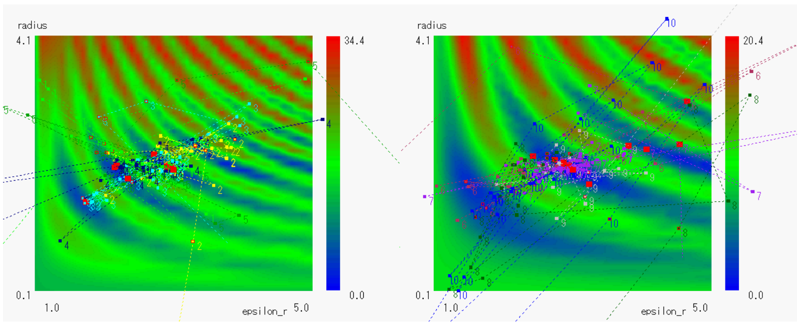

Figure 5: Cost functions $\Omega^{(1)}$ and $\Omega^{(2)}$, whose minimum points are simultaneously searched for by the split PSO.

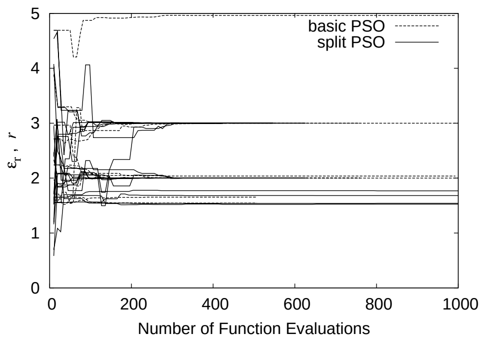

Figure 6: Estimated dielectric constant and radius of the cylinder versus the number of function evaluations. The results of five runs are superposed.

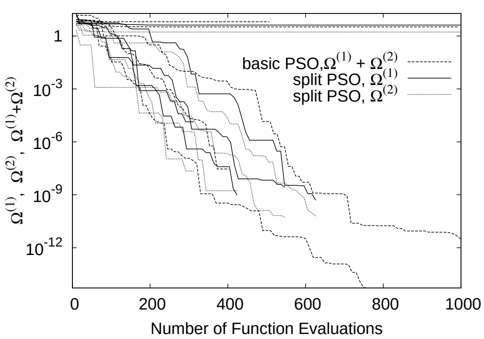

Figure 7: Value of the cost functions versus the number of function evaluations. The results of five runs are superposed.

Table 1: The number of successful estimation of 100 runs.

the number of particles	$P=10$	$P=10$	$P=20$
PSO type	basic	split	split
cylinder 1: $\varepsilon_r=3$, $r=2$	56	79	96
cylinder 2: $\varepsilon_r=2$, $r=1$	80	84	98
cylinder 3: $\varepsilon_r=4$, $r=3$	53	50	79