# Localization of Physical Optics Radiation Integrals Area in Scattering from a Rectangular Plate 

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#### Abstract

This paper presents an approximation method for 3-dimensional plate that localizes an integration area and reduces a computational load. The method is based upon the locality of scattering phenomena. The proposed method is the good approximation method and has the effect of computational load reduction at the high frequencies.


Keywords : Locality, Fresnel Zone, Scattering Problem, Physical Optics

## 1. Introduction

Procedure of calculating a scattering field is composed of two stages. First is the analysis of currents induced on the scatterer surface. The method of moments (MoM) is one of the method commonly used. Second is radiation surface integral. The induced currents are multiplied with the Green's function and are surface-integrated over whole the scatterer surface. At higher frequencies, computation loads in both the stages become heavier, especially in the former, while locality of scattering phenomena appears.

This phenomenon is visualized in Figure 1 [1]. The contribution from each part of the induced currents is localized on the plate; near the reflection point and edge diffraction points as is defined in the GTD. This property is called locality. The locality considered, the scattering field can be approximated by integrating the currents on these local areas and the computation loads can be decreased.

Figure 2 shows the steps in the approximation based upon the locality and the location of this paper. First step is that integration area is localized after analysis for full model. Next step is that current distribution on reduced scatterer leaving only the areas around the point of reflection and diffraction is analyzed by MoM. The approximation methods in both the steps are examined for 2-dimensional structure [2], [3]. This paper presents an approximation method in the former step.

## 2.Concept of Locality

In order to extract the locality, it is necessary to understand the method of stationary phase. Figure 3 shows a concept of stationary phase property. The integrand contains a oscillating function which comes from the phase change associated with the change in the path length from the source $(\mathrm{S})$ to the observer ( O ) via the integration point (I) on the surface. The vibration becomes more rapid at higher frequencies. Figure 3 depicts the behavior of the real (or imaginary) part of the integrand in the radiation integral. A result of integrating that function in the area where the phase changes is more rapid is approximately equal to zero due to the cancellation effect. The areas in which the cancellation effect is weak exist near the stationary phase point (SPP) and edge diffraction points, where the path length becomes an extreme value. The result of integrating in that area is not zero. Therefore, we use Fresnel zone, a concept based upon the path length, as a criterion for determining local areas.

## 3. Determination of Local Integration Areas and Weighting function

### 3.1 Stationary Phase Point (SPP)

The Fresnel zone number $n$ is defined as $n=L /(2 \lambda)$, where $L$ is the length of path from source via point on the scatterer to observer and $\lambda$ is the wavelength. The local areas are specified by the difference of the Fresnel zone number; in Figure 4, $\left|n_{I}-n_{P}\right|=\Delta n \leq n_{0}$, where $n_{P}$ and $n_{I}$ are
the Fresnel zone number at the SPP and the point of interest on the scattering surface, respectively, and $n_{0}$ is the parameter to determine the size of local areas. We put $n_{0}=3$ here. In other words, the area where difference in the path is within $3 / 2$ wavelength is defined as local area.

In extracting the locality, a weighting is also important. After localized, fictitious edges which do not exist in the original problem appear and would produce undesired edge contribution. A weighting is necessity to suppress these. EYE function is used as a weighting function in this paper. EYE function is defined by (1).

$$
\operatorname{EYE}(\Delta n)=\left\{\begin{array}{cc}
\frac{1}{2}\left\{\cos \left(\frac{\Delta n}{n_{0}} \pi\right)+1\right\} & \left(\Delta n \leq n_{0}\right)  \tag{1}\\
0 & \left(\Delta n>n_{0}\right)
\end{array}\right.
$$

To summarize the above discussion, a scattering field can be approximated by integrating the currents weighted by EYE function in the only localized area determined by the Fresnel zone.

### 3.2 Edge Diffraction Points

A model in Figure 5 is examined in this paper. The above approximation method can be applied for the SPP but the diffraction points. We propose a way to determine local areas around the diffraction points [4], [5]. The procedure is as follows and shown in Figure 6.

First, imaginary tangential infinite planes are set at edge and corner diffraction points. These planes are satisfied with a reflection law. Secondly, local areas are determined on these imaginary planes under the same condition for the SPP; $\left|n_{I}-n_{P}\right|=\Delta n \leq n_{0}$, and then, EYE function is applied to these local areas. Finally, the local areas and values of a weight on the imaginary planes are projected on the real scatterer surface. These projected areas are the real local integral areas.

## 4. Numerical Demonstration

Figure 7 shows a total field calculated by applying the proposed approximation method. The currents distribution was given by the Physical Optics (PO). The result by the approximation, localization method is in good agreement with the results by the usual, non-localization methods. The right vertical axis indicates a ratio of local area to the whole and corresponds to a dashed line. However the ratio changes depending on an observation angle, surface-integrating at most $43 \%$ areas of the whole can reproduce a scattering field. The picture below the graph shows the local areas and values of a weight. A shape and the number of local areas change depending on an observation angle.

Figure 8 shows the frequency dependence of the ratio of local area to the whole area. The frequency is normalized by the frequency assumed for the case of Figure 5. The ratios become small in proportion to the first power of the frequency. The fact suggests that at most $4.3 \%$ areas of the whole are large enough to reproduce a scattering field in 10 times frequency. Local areas in 10 times frequency are visualized below the graph. That is to say that computational load of radiation integrals becomes smaller at higher frequencies by this method.

## 5. Conclusion

In calculating a scattering field from a rectangular plate, the approximation method based upon the locality of scattering phenomena was proposed. The Fresnel zone was used as a criterion for determining local areas and EYE function was applied to these areas. We showed that suggestion technique was the good approximation method. Furthermore, this method has the effect of computational load reduction at the high frequency. MoM analysis for currents on reduced scatterer is left for the future works.

## References

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Figure 2: Steps in approximation based upon the locality and the location of this paper


|  | 2-D Structure | 3-D Structure |
| :---: | :---: | :---: |
| Step1: Localization of Integration Area | - - - - ' ${ }^{\text {Previous }}$ | This Paper |
| Step2: MoM for Currents on Reduced Scatterer | I Works. | Future Works |

Figure 1: Visualization of scattering phenomena from plate illuminated by a dipole


Figure 3: Concept of stationary phase property


$$
\begin{array}{rlrl}
|\overline{S I O}-\overline{S P O}| \leq n_{0} \cdot \frac{\lambda}{2} & \Delta n & =\left|n_{I}-n_{P}\right| \\
\Leftrightarrow\left|n_{I}-n_{P}\right| & \leq n_{0} & n_{0} & =\left|n_{B}-n_{P}\right|
\end{array}
$$

Figure 4: Determination of local area for the stationary phase point


Figure 5: Analysis model (a source and a rectangular)


Figure 6: Determining local area for the diffraction points


Figure 7: Computation Result (top) and visualization of local areas and weights (bottom)


Figure 8: Frequency characteristic graph of reduction ratio (top) and visualization (bottom)

