A Uniform Asymptotic Solution for Transmitted Waves through a Plane Dielectric Interface from a Denser to a Rarer Mediums

#Dinh T. Quang, Keiji Goto, Toru Kawano, and Toyohiko Ishihara
Department of Communication Engineering, National Defense Academy
Hashirimizu 1-10-20, Yokosuka, Kanagawa, 239-8686 Japan
Tel: +81-46-841-3810, Fax: +81-46-844-5903, Email: ed10002@nda.ac.jp

Abstract

We have derived a novel uniform asymptotic solution for the transmitted and scattered waves when the cylindrical wave is incident on a plane dielectric interface from the denser dielectric medium. We have confirmed the validity of the uniform asymptotic solution by comparing with the reference solution obtained numerically.

Keywords: Uniform asymptotic solution Plane dielectric interface Transmitted wave

1. Introduction

When the cylindrical wave or the spherical wave is incident on the plane dielectric interface from a denser side of medium, the observation point placed in the rarer medium may receive only the transmitted geometrical ray in the short distance from the source and both the transmitted geometrically ray and the evanescent wave in the long distance [1]-[6]. Thus the solution in the short distance is different from the solution in the long distance. Therefore, it is necessary to find out the solution which can connect smoothly the two different solutions.

In the present study, we shall derive a novel uniform asymptotic solution for the transmitted and scattered waves observed in the rarer medium when the cylindrical wave radiated from the line source is incident on a plane dielectric interface from the side of the denser dielectric medium [6]-[9]. We will derive the asymptotic solution for the transition wave which plays an important role to connect the one solution in the near region to the other solution in the far region smoothly through the transition region [2], [6]-[9]. Also shown is the physical interpretation of the asymptotic solution.

2. Uniform Asymptotic Solution for Transmitted Waves

2.1 Integral Representation for Transmitted Waves

In Fig.1, we have shown the Cartesian coordinate system (x, y, z), plane dielectric interface $z = 0$ consisting of denser upper medium (ε_1, μ_0) and rarer lower medium (ε_2, μ_0), $(\varepsilon_1 > \varepsilon_2)$, electric line source $Q(0, h)$ directing y direction, totally reflected ray, transmitted ray, and evanescent wave. Area of the lower medium is divided into three regions, i.e., the near region, the transition region, and the far region along the *x*-direction and into two regions (shallow region and deep region) along the $(-z)$-direction.

When the electromagnetic wave is radiated from the electric line source $Q(x, z) = Q(0, h)$, the electric field $E_y(x, z)$ observed in the lower medium (ε_2, μ_0) may be given by [2], [6]-[10]

$$E_y = i\omega\mu_0 IG, \quad G = \frac{i}{4\pi}\int_{P_\theta} T(\theta)e^{ik_1 q(\theta)}\, d\theta \tag{1}$$

$$T(\theta) = \frac{2\cos\theta}{\cos\theta + \sqrt{n^2 - \sin^2\theta}}, \quad q(\theta) = R\cos(\theta - \theta_s) + |z|\sqrt{n^2 - \sin^2\theta}, \quad n = \sqrt{\varepsilon_2/\varepsilon_1} \tag{2}$$

where R and θ_s in (2) are defined geometrically in Fig.1.

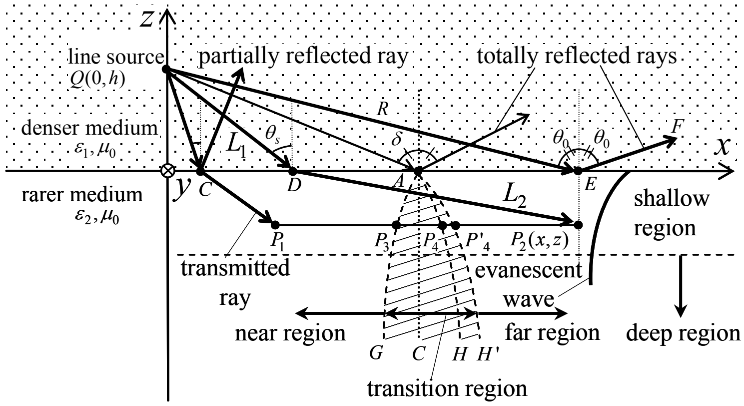

Fig.1 Coordinate system (x, y, z), plane dielectric interface at $z = 0$, and schematic figures for transmitted waves.

In Fig.2, we have shown the branch cuts associated with the branch points at $\theta = \pm\delta$ $(\delta = \sin^{-1} n)$ of the integrand and the original integration path P_θ in (1) in the complex θ-plane.

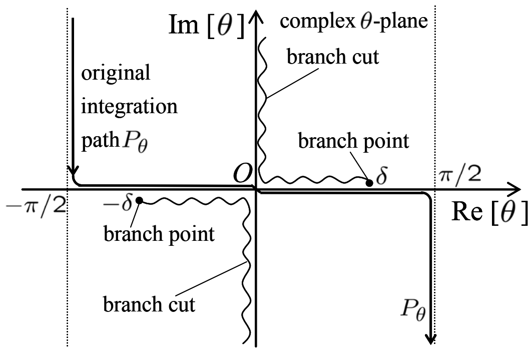

Fig.2 Branch cuts associated with branch points at $\theta = \pm\delta$, $\delta = \sin^{-1} n$ and original integration path P_θ in the complex θ-plane.

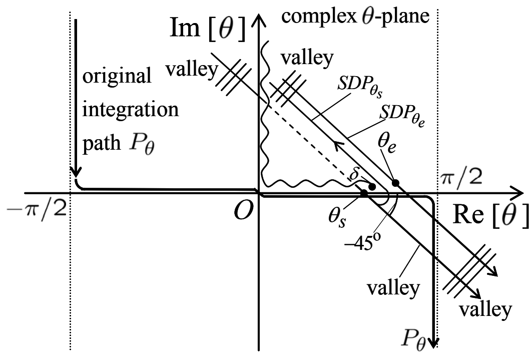

Fig.3 Steepest descent path SDP_{θ_s} and SDP_{θ_e} passing through saddle points at θ_s and θ_e, respectively.

2.2 **Uniform Asymptotic Solution in Far and Shallow Regions**

When the observation point $P(x, z)$ is placed in the far region from the source $Q(0, h)$ and in the shallow region from the dielectric interface $z = 0$ the integrand in (1) possesses two saddle point θ_s and θ_e satisfying the saddle point equation $(d/d\theta)q(\theta) = 0$ or

$$R\sin(\theta - \theta_0) + |z|\frac{\sin\theta\cos\theta}{\sqrt{n^2 - \sin^2\theta}} = 0 \tag{3}$$

We have shown in Fig.3 the steepest descent paths SDP_{θ_s} and SDP_{θ_e} passing through the saddle points θ_s and θ_e, respectively. The integral in (1) may be represented as

$$G = G_{eva} + \hat{G},\ G_{eva} = \frac{i}{4\pi}\int_{SDP_{\theta_e}} T(\theta)e^{ik_1 q(\theta)}\,d\theta\,,\ \hat{G} = \frac{i}{4\pi}\int_{SDP_{\theta_s}} T(\theta)e^{ik_1 q(\theta)}\,d\theta \tag{4}$$

The integral G_{eva} along the steepest descent path SDP_{θ_e} can be evaluated asymptotically by applying the isolated point technique [10]. The asymptotic solution may be given by

$$G_{eva} = \frac{i}{4\pi}\sqrt{\frac{2}{\pi k_1 R}}\,e^{ik_1 R - i\frac{\pi}{4}}T(\theta_0)e^{-k_1|z|\sqrt{n^2-\sin^2\theta}},\quad \theta_0 = \mathrm{Re}[\theta_e] \tag{5}$$

By using (5), we have drawn the picture for the evanescent wave $Q \to E \to P_2$ excited by the incident geometrical ray $Q \to E$ in Fig.1. While the first-order asymptotic solution for the integral $\hat{G}$ may be given by

$$\hat{G} \sim G_{lat.go} = \frac{i}{2k_1}\sqrt{\frac{2n}{\pi k_1}}\frac{\exp\,(i3\pi/4)}{1-n^2}\frac{1}{L_2^{3/2}}\,e^{ik_1 L_1 + ik_2 L_2} \tag{6}$$

This wave propagates along $Q \to D \to P_2$ as shown in Fig.1. It is very interesting to observe that the amplitude of $G_{lat.go}$ in (6) behaves like a lateral wave [1]-[5], [7], [9]-[11] which is observed in the denser (upper) medium. Therefore, we define the first-order asymptotic solution $G_{lat.go}$ in (6) as the lateral wave type geometrical ray.

One may note that when the observation point moves along $P_1 \to P_2$ parallel to the interface $z = 0$, the first-order solution $G_{lat.go}$ in (6) produces relatively large errors in the transition region between P_3 and P_4' (see Fig.1). Therefore, in order to derive the accurate solution we will represent the integral $\hat{G}$ in (4) as follows

$$\hat{G} = G_{lat.go} + G_{tran}, \qquad G_{tran} = \hat{G} - G_{lat.go} \tag{7}$$

Here, $\hat{G}$ approaches $G_{lat.go}$ ($\hat{G} \to G_{lat.go}$) or G_{tran} approaches zero ($G_{tran} \to 0$) as the observation point moves away from the transition region. Thus, we define G_{tran} as the transition wave which plays an important role only in the transition region. Upon substituting (7) into (4), one may obtain the following novel uniform asymptotic solution for the transmitted wave.

$$G = G_{eva} + G_{lat.go} + G_{tran} \tag{8}$$

It is remained to derive the asymptotic solution for $\hat{G}$ defined in (4). Since both the functions $T(\theta)$, corresponding to the reflection coefficient, and $q(\theta)$ possess the branch points at $\theta = \pm\delta$, $\delta = \sin^{-1}n$ in the complex θ-plane, lengthy calculation is required. However, since the paper space is limited here, we will give only the final result for $\hat{G}$ in (4). The solution will be given by

$$\hat{G} = \hat{G}_{go.1} + \hat{G}_{go.2} + \hat{G}_{go.3} \tag{9}$$

$$\hat{G}_{go.1} \sim \frac{i}{4\pi} T(\theta_s) e^{ik_1 q(\theta_s)} (I_1^- + I_1^+) \tag{10}$$

$$I_1^{\mp} \sim \pm \frac{1}{\sqrt{k_1 R\cos(\theta_s - \theta_0)}} e^{-i\pi/4} \sum_{k=0}^{\infty} \frac{(\alpha^{\mp})^k}{k!} \int_0^{\infty} \left(\sqrt{\beta^{\mp} \pm w} - \sqrt{\beta^{\mp}}\right)^k e^{-\frac{w^2}{2} + \gamma w} dw \tag{11}$$

$$\hat{G}_{go.2} \sim -\frac{i}{4\pi} T^{(2)}(\theta_s) \sqrt{n^2 - \sin^2\theta_s}\, e^{ik_1 q(\theta_s)} (I_2^- + I_2^+), \qquad I_2^{\mp} = I_1^{\mp} \tag{12}$$

$$\hat{G}_{go.3} \sim -\frac{i}{4\pi} T^{(2)}(\theta_s) \sqrt{\sin(\delta + \theta_s)}\, e^{ik_1 q(\theta_s)} (I_3^- + I_3^+) \tag{13}$$

$$I_3^{\mp} \sim \pm \frac{e^{-\frac{i\pi}{4}} \eta^{\mp}}{\sqrt{k_1 R\cos(\theta_s - \theta_0)}} \sum_{k=0}^{\infty} \frac{(\alpha^{\mp})^k}{k!} \int_0^{\infty} \sqrt{\beta^{\mp} \pm w} \left(\sqrt{\beta^{\mp} \pm w} - \sqrt{\beta^{\mp}}\right)^k e^{-\frac{w^2}{2} + \gamma w} dw \tag{14}$$

where $\alpha^{\mp}$, $\beta^{\mp}$, γ, and $\eta^{\mp}$ used in the above equations are defined as follows.

$$\alpha^{\mp} = ik_1|z| \frac{\sqrt{\sin(\delta + \theta_s)\cos(\delta - \theta_s)}}{\{k_1 R\cos(\theta_s - \theta_0)\}^{1/4}} e^{i\pi/8 \mp i\pi/4} \tag{15a}$$

$$\beta^{\mp} = \mp\tan(\delta - \theta_s)\sqrt{k_1 R\cos(\theta_s - \theta_0)}\, e^{-i3\pi/4} \tag{15b}$$

$$\gamma = e^{i\pi/4} \frac{k_1 R\sin(\theta_s - \theta_0)}{\sqrt{k_1 R\cos(\theta_s - \theta_0)}}, \qquad \eta^- = e^{-i\pi/8}, \ \eta^- = e^{-i3\pi/8} \tag{15c}$$

As shown in (11), (12), and (14), $I_1^{\mp}$, $I_2^{\mp}$, and $I_3^{\mp}$ are given by the integrals. However, these integrals for $k = 1,2,3, \ldots$ are expressed by using the parabolic cylinder function [12]. By using $\hat{G}$ in (9), one may obtain the transition wave G_{tran} in (8) (see (7)).

3. Numerical Result and Discussions

In Fig.4, we have shown electric field magnitude vs. distance $x[\lambda]$ curve. The solid curve (——) is calculated by using the uniform asymptotic solution proposed in the present study. Note that the uniform asymptotic solution for the near region $x < P_4$ including the transition region has been considered elsewhere [9].

It is shown in Fig.4 that the uniform asymptotic solution (—— : solid curve) agrees excellently with the reference solution (∘ ∘ ∘: open circles) calculated numerically from (1). Also shown in Fig.4 is the solution (–·–· : chain-dot curve) calculated from the transition wave G_{tran} derived in (7) associated with (9). It is clarified that the transition wave plays an important role in the transition region $P_3 < x < P_4'$ shown in Fig.1 and Fig.4. It is observed that the electric field magnitude oscillates as the function of $x[\lambda]$ in the range $x[\lambda] > P_4$ due to the interference of the evanescent wave G_{eva}, the lateral wave type geometrical ray $G_{lat.go}$, and the transition wave G_{tran}.

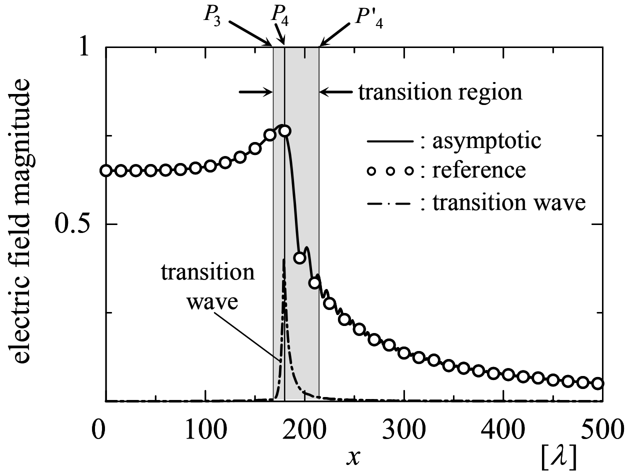

Fig.4 Comparison of asymptotic solution with reference solution. Numerical parameters used in the calculation: $h = 200\lambda,\ z = -0.3\lambda,\ f = 3\mathrm{GHz},\ \varepsilon_1 = 2.3\varepsilon_0,\ \varepsilon_2 = \varepsilon_0,\ n = 0.659.$

4. Conclusion

We have derived the uniform asymptotic solution for the transmitted and scattered waves observed in the rarer medium when the cylindrical wave is incident on the dielectric interface from the denser medium. We have shown the validity and utility of the uniform asymptotic solution by comparing with the reference solution calculated by the numerical integration. It is clarified that the transition wave plays an important role in the transition region. Also shown is the physical interpretation of the asymptotic solution proposed in the present study.

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Acknowledgments

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