Application of An Approximate Fresnel Function to Discrete Ray Tracing Method

Yuki Kimura, Junichi Honda and Kazunori Uchida
Department of Communication and Information Engineering, Fukuoka Institute of Technology
3-30-1 Wajiro-Higashi, Higashi-ku, Fukuoka, 811-0295 Japan
Email: k-uchida@fit.ac.jp

Abstract

This paper is concerned with reducing computation time of discrete ray tracing method (DRTM) by using an approximate Fresnel function. By applying approximate Fresnel function to DRTM, we consider the accuracy and the computation time of the numerical examples for electromagnetic field along random rough surfaces.

Key words: approximate Fresnel function, computation time, diffraction, DRTM, random rough surface

1. Introduction

Recently, the discrete ray tracing method (DRTM) has been proposed in order to analyze electromagnetic wave propagation along random rough surfaces (RRSs) in relation to sensor networks [1]. Two types of discretizations are carried out in the DRTM. One is discretization of RRS, and the other is discretization of searching ray. By these discretizations, the DRTM enables us to reduce much computation time in comparison with the conventional ray tracing method (RTM). In the DRTM computation, we assume that we need N rays with M diffraction points to evaluate an electromagnetic field at one observation point and the number of observation points is L. Then the total number of computation times of diffraction function composed of the Fresnel function is given by $M \times N \times L$. Thus it is concluded that the computation time of diffraction function dominates the total computation time of a field analysis based on the DRTM. In this context, it is significant to propose an approximation for the diffraction function to reduce the computation time of the numerical analyses based on the DRTM.

An approximate expression for Fresnel function has been proposed by authors, and we checked its accuracy and computation time. Comparing with accurate expression, the error of proposed approximations was within 0.2%, and the reduction rate of computation time was about 13% [2]. In this paper, we introduce an approximation for the diffraction function which is simple but effective to DRTM analyses. It is shown that the approximate function is in good agreement with the accurate Fresnel function. We apply the approximation for Fresnel function to DRTM analyses, and we numerically compute field distribution along RRSs. It is shown that the computation time is faster than the conventional DRTM with the accurate Fresnel function, and the accuracy is almost the same between the two methods.

2. Approximation for Diffraction function

2.1 Approximate Function

The diffraction function plays an important role in the analytical investigation of electromagnetic wave diffraction by edges. Consequently our knowledge about the analytical properties for the diffraction function is very useful when we tackle a more complicated diffraction problem such as propagation in an urban area or along RRS [1].

The complex type of Fresnel function is expressed in terms of the real type of Fresnel functions as follows:

$$F(X) = \frac{1}{2} - \frac{e^{\frac{\pi}{4}j}}{\sqrt{\pi}} \left[C(X) - jS(X) \right] \tag{1}$$

where the real type of Fresnel functions are defined by

$$\begin{aligned} C(X) &= \int_0^X \cos(u^2)du \\ S(X) &= \int_0^X \sin(u^2)du \ . \end{aligned} \tag{2}$$

These real type of Fresnel functions are expanded in terms of Taylor series [3] as follows:

$$\begin{aligned} C(X) &= \sum_{n=0}^{\infty}(-1)^n \frac{X^{4n+1}}{(4n+1)(2n)!} \\ S(X) &= \sum_{n=0}^{\infty}(-1)^n \frac{X^{4n+3}}{(4n+3)(2n+1)!} \ . \end{aligned} \tag{3}$$

On the other hand, the asymptotic forms of the above functions are expressed in the following form:

$$\begin{aligned} C(X) \simeq & \sqrt{\frac{\pi}{8}} - \cos x^2 \sum_{n=0}^{\infty} \frac{(-1)^n(4n-1)!!}{2^{2n+1}X^{4n+1}} \\ & + \sin x^2 \sum_{n=0}^{\infty} \frac{(-1)^n(4n+1)!!}{2^{2n+2}X^{4n+3}} \\ S(X) \simeq & \sqrt{\frac{\pi}{8}} - \sin x^2 \sum_{n=0}^{\infty} \frac{(-1)^n(4n-1)!!}{2^{2n+1}X^{4n+1}} \\ & + \cos x^2 \sum_{n=0}^{\infty} \frac{(-1)^n(4n+1)!!}{2^{2n+2}X^{4n+3}} \ . \end{aligned} \tag{4}$$

For numerical computations, Eq.(3) is effective for relatively small X, while Eq.(4) is effective for relatively large X. Thus the numerical recipes for the real type of Fresnel functions are devised by combining these two relations [4]. Although numerical results of these recipes are accurate, we cannot ignore computation time when we use these functions many times. This is the case when we apply DRTM to an electromagnetic diffraction or propagation problem in a complicated natural environment. So we need an efficient algorithm to provide numerical data for the complex type of Fresnel function faster than the accurate recipes [4] do.

Mathematics for diffraction is not the Fresnel function $F(X)$ itself but the diffraction function $D(X)$ whose relationship has been given by $D(X) = e^{jX^2}F(X)$. The diffraction function $D(X)$ exhibits a monotonic property as shown in Figure 1 which plots its real part in x-axis and its imaginary part in y-axis with a parameter X ranging from 0 to infinity. The curve is very similar to a parabola given by a square polynomial. Thus we can approximate the imaginary part $\mathtt{Im}D(X)$ by the real part $\mathtt{Re}D(X)$ by using a polynomial function as follows:

$$\mathtt{Im}D(X) \simeq H(\mathtt{Re}D(X)) \tag{5}$$

where the polynomial function is given by

$$H(x) = 2x(x-0.5)[1-38x(x-0.125)(x-0.5)^2] \ . \tag{6}$$

Figure 1 shows an approximated curve compared with an accurate one obtained by the numerical recipes [4]. It is demonstrated that two curves are in good agreement with rms error of 0.55%. When X is small ($X < 0.55$), combining Eqs.(1) and (3) leads to an approximate perturbed expression as follows:

$$F(X) \simeq \frac{1}{2} - \frac{X}{\sqrt{2\pi}}\left[(1+\frac{X^2}{3}-\frac{X^4}{10}) + j(1-\frac{X^2}{3}-\frac{X^4}{10})\right] \ . \tag{7}$$

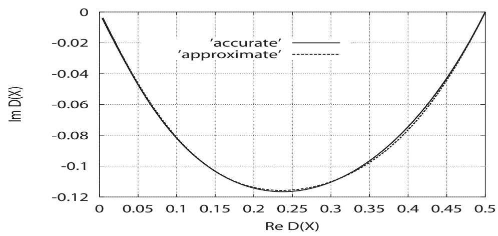

Figure 1: Real and imaginary parts of the diffraction coefficient $D(X)$.

When the argument X is large ($X > 1.55$), combining Eqs.(1) and (4) leads to an approximate asymptotic expression as follows:

$$F(X) \simeq \frac{e^{-j(X^2-\frac{\pi}{4})}}{2\sqrt{\pi}X}\left[\frac{1}{2X^2} - j(1-\frac{3}{4X^4})\right] . \tag{8}$$

Based on the above polynomial expressions, we can introduce an approximate diffraction function in a simplified form as follows:

$$\mathrm{Re}D(X) \simeq \begin{cases} (1-\frac{X^4}{2})[0.5 - \frac{X}{\sqrt{2\pi}}(1+\frac{X^2}{3}) + \frac{X^3}{\sqrt{2\pi}}(1-\frac{X^2}{3}-\frac{X^4}{10})] \\ \quad X < 0.55 \\ 0.6152434X_2X_3 - 0.7984184X_1X_3 + 0.2775040X_1X_2 \\ \quad 0.55 < X < 1.55 \\ \frac{1}{2\sqrt{2\pi}X}(1+\frac{1}{2X^2}-\frac{3}{4X^4}) \quad X > 1.55 \end{cases} \tag{9}$$

where we have used the following relations:

$$X_1 = X - 0.55, \quad X_2 = X - 1.05, \quad X_3 = X = 1.55 . \tag{10}$$

Eq.(9) shows that approximation has been made for the real part of the diffraction function in a perturbed form for $X < 0.55$, in an asymptotic form for $X > 1.55$ and in a square polynomial form in other region.

Needless to say, the imaginary part of the diffraction function can be computed by the polynomial given by Eq.(6). Thus the proposed approximation enables us to save computation time, since it is enough to use only the lower order of polynomials. We have checked the computation time for the accurate and approximate diffraction functions $D(X)$ with step size 0.001 of X in the range from 0 to 1000. The computation time using the proposed approximation was about 13.2% of the computation time using the accurate Fresnel function.

2.2 DRTM Computation

DRTM has been proposed by authors recently. In the DRTM, two types of discretizations are carried out: one is discretization of RRSs, and the other is discretization of searching rays between source and receiver. These discretizations enable us to rapidly compute electromagnetic fields above RRSs [1]. We omit here detailed discussions of the DRTM algorithm. The electric field $\boldsymbol{E}$ at the receiver is formally expressed in the following diadic and vector form [1]:

$$\boldsymbol{E} = \sum_{n=1}^{N}\left[\prod_{m=1}^{m=M_n^i}(\boldsymbol{D^i_{nm}}) \cdot \prod_{k=1}^{k=M_n^s}(\boldsymbol{D^s_{nk}}) \cdot \boldsymbol{E_0}\right]\frac{e^{-\kappa r_n}}{r_n} \tag{11}$$

where $\boldsymbol{E_0}$ is the electric field of the incident wave at the first reflection or diffraction point, and κ is the wavenumber in the free space. N is the total number of rays considered, M_n^s is the number of times of its source diffractions, and M_n^i is the number of times of its image diffractions. Reflection coefficients are included into a diadic form $\boldsymbol{D^i}$ which means an image diffraction, and the proposed approximate function discussed in previous section is also included into $\boldsymbol{D^i}$. $\boldsymbol{D^s}$ is a diadic of source diffraction. Based on the ray data, r_n is the distance of n-th ray from source to receiver.

3. Numerical examples

Figure 2 shows geometry of the problem. RRSs are strongly related to three parameters. First parameter is spectrum function, second parameter is deviation of rough surfaces height (dv) and third parameter is correlation length (cl). We select RRS's parameters as $dv = 10.0[m]$ and $cl = 50.0[m]$. Spectrum type of the RRS is assumed to be Gaussian. Source point is placed at $x = 1.0[m]$ and its height is $H = 30.0[m]$ from the average height of RRSs. Operating frequency is selected as $f = 1.0[GHz]$, and material constants are chosen as $\epsilon_r = 5.0$ and $\sigma = 0.0023[S/m]$. Figure 3 shows two curves: these curves are the ensemble average of electric field intensities computed by using 30 generated samples of rough surfaces. Red line is a field distribution computed by the accurate diffraction function, and green line is a field distribution computed by the proposed approximate function. Comparing two curves, we can see that two curves are in good agreement with each other, and the error is within 1.0%. Also, we consider computation time between two results. Computation time using approximate function is reduced about 5.0% in comparison with that of rigorous one. As a result, it is concluded that the approximation proposed in this paper is accurate and we can reduce the computation time for field intensity a little.

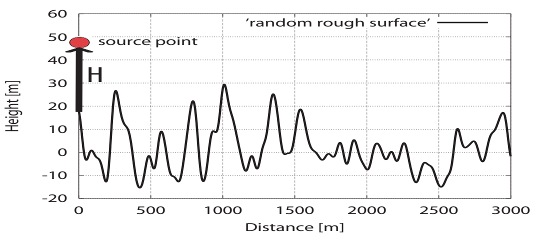

Figure 2: Geometry of the problem.

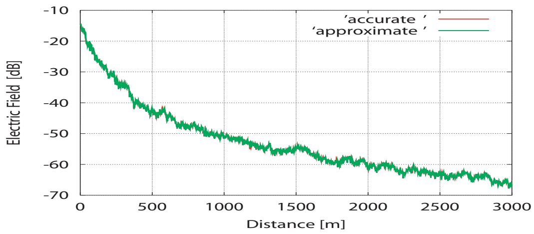

Figure 3: Comparison between accurate and approximate fields on a random rough surface.

4. Conclusion

In this paper, we have introduced an approximate function for the Fresnel function to reduce computation time. An approximate function enables us to reduce 13.2% of the computation time in comparison with accurate one in the rang $0 \leqq D(X) \leqq 1000$ with step size 0.001 of X. In case of the numerical analyses based on DRTM above RRSs, we have compared the computation time of the accurate Fresnel function with that of approximate function. It is found that computation time using approximation has been reduced about 5.0% in comparison with that of accurate Fresnel function. We have also found that the error of approximation is within 1.0%.

We need another better approximate function than the proposed one to reduce computation time. This is a future problem.

Acknowledgments

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