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Interstate switching induced by non-Gaussian noise in nonlinear micromechanical oscillators

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Abstract– We study interstate switchings induced by noise in nonlinear micromechanical oscillators. Under sufficiently strong periodic excitation, nonlinear micromechanical oscillators possess multiple oscillation states that have different amplitudes and phases. The presence of noise makes it possible for the system to switch between these states. Our data demonstrate that the dependence of the interstate switching rate on device parameters is qualitatively different for Gaussian noise and Poisson pulses.

1. Introduction

In micro- and nano-systems, the interplay of noise and nonlinearity often yields novel phenomena. These phenomena are of both fundamental and practical interest because they can potentially offer new functionalities and improve the performance of sensors. For instance, when nonlinear resonators are subjected to sufficiently strong periodic driving, multistability develops. Fluctuations can induce the system to escape from one metastable oscillation state into the other, with the escape rate given by $W = C\, exp(-Q)$, where C is the prefactor that is largely independent of noise intensity and Q is the switching exponent. For the most common case of Gaussian noise, the switching has been shown to follow Kramer's equation so that $Q = R/D$, where R is the activation barrier and D is the noise intensity [1-7].

Here, we explore activated switching out of nonlinear micromechanical oscillators that are periodically driven into bistability [8]. Between the bifurcation frequencies ω_{b1} and ω_{b2}, two stable oscillation states coexist [5]. As the driving frequency ω_d is tuned towards the bifurcation value ω_b, one of the two stable oscillation states merges with the unstable state and the system becomes monostable. We focus on revealing the differences in switching induced by Gaussian noise and Poisson pulses. For Gaussian white noise, the switching rate obeys the Arrhenius relation, with $Q \propto 1/D_G$, where D_G is the intensity of the Gaussian noise. In contrast, for Poisson pulses, we observe a logarithmic dependence of Q on the Poisson noise intensity D_p, when we vary D_p by tuning the mean rate ν of the pulses. Theoretical analysis [9] predicted that Gaussian and Poisson pulses yield different dependence of Q on frequency detuning $\Delta\omega = \omega_d - \omega_b$. We observe that Q exhibits a power law dependence $Q \propto (\Delta\omega)^\eta$ for Gaussian noise, with the measured value of η consistent with the predicted value of 3/2 for saddle-node bifurcations, as verified by a number of other experiments [5,8]. For Poisson noise, it is expected that instead of a simple power law dependence, Q depends on the square root of $\Delta\omega$ with an additional logarithmic factor [9].

2. Experimental Setup

2.1. Micromechanical resonator

Figure 1(a) shows a scanning electron micrograph of our device that consists of a polycrystalline silicon beam 100 μm by 1.2 μm by 1.5 μm suspended above the substrate [8]. A close-up of one end of the beam is shown in Fig. 1(b). Both ends of the beam are anchored to the substrate. When an ac current is passed through the beam in a perpendicular magnetic field of 5T, vibrations of the beam in its in-plane fundamental mode can be excited by the Lorentz force. Motion of the beam generates an electromotive force that changes the transmitted ac power.

2.1. Duffing resonator: Nonlinearity and hysteresis

The beam can be modeled as a Duffing oscillator of the form

$$\ddot{q} + 2\Gamma\dot{q} + \omega_o^2 q + \beta q^3 = h\cos(\omega_d t) + f(t) \quad (1)$$

where q is the normalized displacement, Γ = 96 rad s^{-1} is the damping coefficient, ω_0 = 7,133,339 rad s^{-1} is the resonant frequency, $\beta = 2.3\text{x}10^9$ $m^{-1}s^{-2}$ is the coefficient of the cubic nonlinearity, h and ω_d are respectively the amplitude and frequency of the external driving force, and $f(t)$ is the noise force.

As shown in Fig. 1(c), at small oscillation amplitudes, the resonator behaves as a simple harmonic oscillator. As the driving field increases, the resonance peak tilts towards high frequencies. When the oscillation amplitude exceeds the critical value, the resonator develops bistability in its frequency response due to the cubic term in Eq. (1). Within a certain range of driving frequencies (between ω_{b1} and ω_{b2}), there are two stable dynamic states with different oscillation amplitude and phase [Fig. 1(c)].

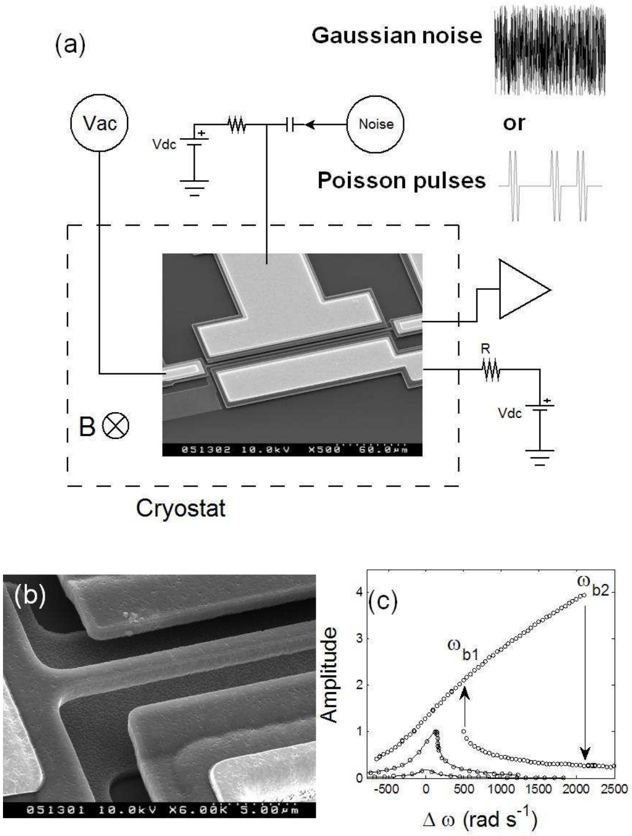

Fig. 1 (a) Schematic of the experimental setup (b) Scanning electron micrograph of one end of the doubly-clamped silicon beam. (c) The dependence of oscillation amplitude on driving frequency detuning $\Delta\omega$. In order of ascending amplitude, the curves correspond to linear, critical and nonlinear responses. All 3 curves are normalized to the amplitude of the critical response.

2.2. Generating the noise voltages

Next, we fixed the driving frequency at a value in the bistable region close to the lower bifurcation frequency ω_{b1}, with frequency detuning $\Delta\omega = \omega_d - \omega_{b1}$. By injecting noise in the driving force, the oscillator can be induced to escape from the low-amplitude state into the high-amplitude state. In our experiment, we apply two different types of noise: Gaussian or Poisson. To create the Gaussian noise voltage, we amplify the Johnson noise of a 50 ohm resistor. When this voltage is applied to one of the electrodes next to the beam [Figs. 1(a) and 1(b)], a random electrostatic noise force is exerted on the oscillating beam.

Generating Poisson noise requires additional circuitry [8]. The Gaussian noise is used to trigger a pulse generator. Whenever the Gaussian noise voltage exceeds a predetermined threshold, the pulse generator outputs a square pulse of fixed height and duration. For each single pulse, the width (t_g = 400 μs) is much smaller than the mean time between successive pulses (from 30 ms to 200 ms). This train of voltage pulses is then used to amplitude-modulate a sinusoidal rf voltage at the driving frequency of the resonator. This Poisson noise voltage is then applied to the electrode to create the noise force electrostatically.

3. Interstate switching

Next, we apply the Gaussian noise and Poisson pulses to the resonator and measure the switching rate W for different noise intensities at a fixed frequency detuning $\Delta\omega$. For Gaussian noise, Fig. 2(a) shows that $-\log W$ depends linearly on the inverse noise intensity $1/D_G$, where D_G is given by:

$$2D_G\partial(t-t') = \langle f(t) f(t')\rangle \qquad (2)$$

Figure 2(a) confirms that interstate switching of our resonator under Gaussian noise obeys Arrhenius relations and is activated in nature.

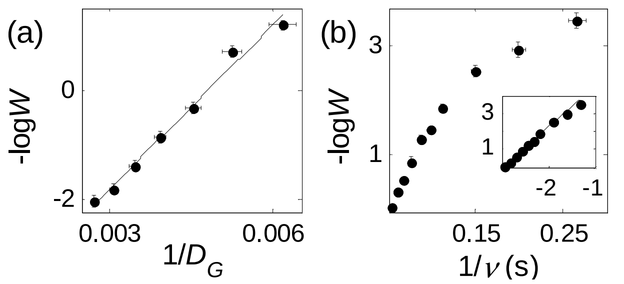

Fig. 2. (a) The dependence of $-\log W$ on $1/D_G$ for Gaussian noise. The frequency detuning $\Delta\omega$ is 3.14 rad s^{-1}. The solid line is a linear fit. (b) $-\log W$ as a function of $1/\nu$ at $\Delta\omega$ = 12.56 rad s^{-1} for rf-modulated Poisson pulses. The noise intensity is proportional to the mean pulse rate ν, provided that the height of the pulses is kept constant. Inset: the same data plotted vs $\log(1/\nu)$.

The intensity of Gaussian noise is characterized by a single quantity D_G according to Eq. (2). For Poisson noise, however, the intensity D_P depends on both the mean rate of pulses ν and the area under each pulse g. In our experiment, we change ν and maintain both the height and the duration of the pulses fixed so that g remains constant. Figure 2(b) plots $-\log W$ as a function of $1/\nu$ for Poisson rf pulses. The dependence is clearly sub-linear.

Instead, if we replace $1/\nu$ by $\log(1/\nu)$ on the x axis, a much improved linear fit [inset in Fig. 2(b)] is obtained. Since the Poisson noise intensity D_P is proportional to ν, Fig. 2(b) shows that $-logW$ is proportional to the logarithm of reciprocal of the noise intensity $log(1/D_P)$ as the pulse rate is varied. Our observation indicates that there are qualitative differences in the interstate switchings induced by Poisson pulses and Gaussian noise.

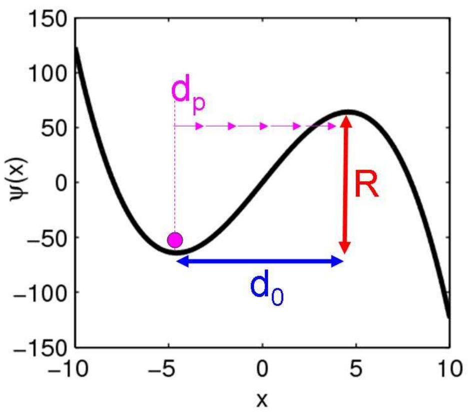

Fig. 3. Near a bifurcation point, the motion in the 2D phase space of slow variables can be mapped onto a 1D potential of the form $\psi(x) = -x^3/3 + \eta x$. For Gaussian noise, switching is induced when a large outburst of noise overcomes the deterministic force from the barrier $R = 4\eta^{3/2}/3$. For Poisson noise, each pulse translates the system by a distance d_p along x. The system can reach the saddle point if $n_o = d_o/d_p$ pulses occur during the relaxation time.

Near bifurcation points, the motion of our resonator can be mapped onto that of a Brownian particle in a 1D potential of the form $\psi(x) = -x^3/3 + \eta x$, where η is the system parameter that decreases to zero at the bifurcation point (Fig. 3) [10]. Gaussian noise and Poisson noise induce switching out of the metastable state through qualitatively different mechanisms [8,9]. For Gaussian noise, switching is induced when a large outburst of noise overcomes the deterministic force from the barrier. For the effective potential $\psi(x)$, the barrier height scales as $\eta^{3/2}$. The switching rate is given by $\Gamma \propto \exp(-Q)$. For Gaussian noise, $Q_G = 4\eta^{3/2}/3D_G$ where D_G is the noise intensity. For Poisson noise, provided that the duration of a pulse is much shorter than the relaxation time of the system, prior theoretical and experimental works have shown that each pulse translates the system by a fixed distance d_p in the X-Y phase space [9,11]. For potential $\psi(x)$, the separation between the metastable state and the saddle point is $d_o = 2\eta^{1/2}$. The system can reach the saddle point if $n_o = d_o/d_p$ pulses occur during the relaxation time t_r before the system relaxes back to the metastable state. Such probability can be obtained using Poisson's distribution $P(n_o, t_r) = (\upsilon t_r)^{n_o} \exp(-\upsilon t_r)/n_o!$, yielding an estimate for the switching exponent:

$$Q_P = (2\eta^{1/2}/\tilde{g})\log(\kappa\eta/\tilde{g}\upsilon), \qquad (3)$$

where υ is the mean rate of pulses, $\tilde{g}$ is the effective pulse area in phase space and κ is a constant. Here, the $\eta^{1/2}$ factor originates from the square root dependence of d_o on η. Figure 3 illustrates the different mechanisms for switching induced by Gaussian and Poisson noise. Q_P and Q_G exhibit different dependence on η and the noise intensity, indicating that the noise statistics affect the dependence of the switching rates on device parameters.

4. Summary

We study noise induced switchings in nonlinear micromechanical oscillators periodically driven into bistabiliy. Our data demonstrate that the dependence of the interstate switching rate on device parameters is qualitatively different for Gaussian noise and Poisson pulses. The findings could create new opportunities for using driven nonlinear systems as detectors for non-Gaussian noise.

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