2008 International Symposium on Nonlinear Theory and its Applications
NOLTA'08, Budapest, Hungary, September 7-10, 2008

Code acquisition for asynchronous multi-user chaos-based DS-CDMA systems

Georges Kaddoum†‡, Daniel Roviras†, Pascal Chargé‡,Danièle Fournier-Prunaret‡

†Signal and Communication team , IRIT-ENSEEIHT University of Toulouse
‡Nonlinear dynamical systems team , LATTIS-INSA University of Toulouse
Email: : Georges.Kaddoum@enseeiht.fr, Pascal.Charge@insa-toulouse.fr

Abstract—In this paper we present a new way for achieving chaos synchronization in asynchronous multi-user chaos-based DS-CDMA systems. Synchronization process is realized thanks to a binary code used as a pilot sequence. Performance of the system is studied and evaluated. An additive white Gaussian noise channel is assumed. Gold sequences are used as pilot signals for the different users to accomplish the synchronization. In addition, the Chebyshev chaotic map is used as spreading signal for data sequences. The synchronization process is evaluated in terms of probability of detection and probability of false alarm

1. Introduction

In the last past decade chaotic signals have been widely applied to communication systems [2]. One of these applications is the use of chaotic signal instead of conventional binary codes for spread spectrum techniques. Chaotic signals can offer robustness in multipath environments, resistance to jamming, low probability of interception [2]. One of the key problems for chaotic signals in digital communications remains the synchronization at the receiver side. The synchronization process has been studied by Pecora and Carrol in [1]. Due to the weak efficiency in noisy transmissions, this method cannot be applied for practical chaotic communication systems [2]. An alternative synchronization technique can be used based on the classical synchronization method used for spread spectrum communication systems. Synchronization is achieved in two distinct phases called the code acquisition and the code tracking phases [3]. The acquisition process is first activated in order to align the local sequence to the received sequence. This phase is followed by a tracking phase based on the code loop [3]. The acquisition procedure can be performed if the spreading sequence exhibits some kind of periodicity. This classical synchronization technique has been applied for chaos-based DS-CDMA systems in [7]-[10]. In [8], [9] the authors have studied the performance of the acquisition process when the Markov chaotic sequence is used as a spreading code for DS-CDMA. It has been shown in [8] that Markov codes outperform independent and identically distributed codes concerning acquisition and bit error rate performance. The authors have shown in [7] that the Bernoulli and Tailed Shift map give a better performance in the acquisition phase than Gold sequences. Noise perturbations have not been evaluated in the acquisition and performance study in [7] and [10]. In most cases, when evaluating the sequence synchronization of chaos-based DS-CDMA systems only the code acquisition is analyzed [7]-[10]. Jovic et *al* presented in [4] a new method for achieving and maintaining synchronization for synchronous multi-user chaos-based DS-CDMA. This method called Code Aided Synchronization (CAS) is proposed and evaluated in the presence of additive white Gaussian noise and multi-user interferences. The code acquisition and tracking phase are studied and analyzed in [4] . The synchronization system proposed in [4] uses one single pseudo random binary sequence as pilot signal to achieve and main-tain the synchronization. The advantage in using a binary pilot signal instead the chaotic sequence for chaos-based DS-CDMA synchronization system is mentioned in [4]. The novelty of our paper is the extension of the synchronous CAS method of [4] to an asynchronous multi-user case. In the paper we will focus on the initial synchronization phase (code acquisition) and referring to [4] we assume that the system can achieve correctly code tracking after this first synchronization phase. Gold sequences are used as pilot signals to achieve synchronization in our system. The choice of Gold sequences relies on the good correlation properties of these sequences [5]. We assume an Additive White Gaussian Noise (AWGN) channel.

The paper is organized as follows. In section 2 the chaotic generator used for spreading data signals and the transmission system are presented. In section 3 the receiver structure is studied. Synchronization is presented and analyzed in term of probability of detection (P_d) and probability of false alarm (P_{fa}) in section 4. Section 5 shows simulation results. The final section reports some conclusive remarks.

2. Transmitter structure

Throughout the paper, a Chebyshev polynomial function of order 2 (CPF) is chosen : $x_k = 1 - 2x_{k-1}^2$
This map has been chosen because its simplicity and because it has been shown in [6], that it allows better performance than many other maps for chaos-based DS-CDMA systems.

We assume that M asynchronous users are emitting their signal with the same power (ideal power control assumed). For each user m , a stream of binary data symbols $S_i^{(m)} = \pm 1$

with period T_s are spreaded by a chaotic signal $x^{(m)}(t)$ at the emitter side. Chaotic signal $x^{(m)}(t)$ is generated by the CPF map. A new chaotic sample is thus generated every T_c time interval ($x_k^{(m)} = x^{(m)}(kT_c)$). Chaotic signals are generated by the same chaotic map for all users with different initial conditions. The chaotic sequences are normalized such that their mean values are all zero and their mean squared values are unity, i.e., $E[x_k] = 0$ and $E[x_k^2] = 1$. As shown in Figure 1(a), a Gold code $p^{(m)}(t)$ is added to each user, and is used as the Periodic Pilot Signal (PPS) with Gold period equal to T. PPS and chaotic sequence are generated with synchronous generators every time interval T_c. These generators are using the same master clock. The period of the PPS is equal to $T = NT_c$. The pilot signal and the spreaded data of each user are summed and transmitted through an AWGN channel. As shown in Figure 1 (b), chaotic generators are initialized every LT time interval in order to let the receiver know the starting and the ending samples of every spreading data frame. In addition, the power of the pilot signal is lower than the power of the spreading data signal to avoid degradations in term of bit error rate (BER) coming from the PPS. The PPS power level must be sufficiently high for detection and synchronization purposes but sufficiently low for the BER performance of the chaotic transmission. The emitted signal by the user m , $s^{(m)}(t)$ is given by :

$$s^{(m)}(t) = d^{(m)}(t) + p^{(m)}(t) \quad (1)$$

$$d^{(m)}(t) = \sqrt{P_1} \sum_i \sum_{k=0}^{\beta-1} S_i^{(m)} x_{i\beta+k}^{(m)} g(t - (i\beta + k)T_c)$$
$$p^{(m)}(t) = \sqrt{P_2} \sum_j \sum_{l=0}^{N-1} p_l^{(m)} g(t - (jN + l)T_c)$$

where $x_{i\beta+k}^{(m)}$ are the chaotic samples corresponding to data symbol $S_i^{(m)}$, $g(t)$ is the rectangular pulse of unit amplitude on $[0, T_c]$, L is an integer, and P_1 (respectively P_2) are the power of the spreaded data signal (respectively the PPS). Parameter β is equal to the number of chaotic samples in a symbol duration $\beta = T_s/T_c$ and, by analogy with DS-CDMA, we have called this parameter the spreading factor.

3. Receiver structure

The additive noise has a Power Spectral Density, equal to $N_0/2$. In order to recover transmitted symbols, an exact replica of chaotic sequences must be generated in the receiver. We assume that the initial conditions of the M chaotic generators used at the receiver side are known and equal to the ones used at the transmitter side. In this paper, we are mainly interested in the code acquisition process, thus, despreading and demodulation process will not be explained. Spreaded data signals can be seem as interferences during the synchronization process, since only PPS are of interest here. The received signal can be written as :

$$r(t) = \sum_{n=1}^{M} s^{(n)}(t - \tau^{(n)}) + n(t) \quad (2)$$

where $\tau^{(n)}$ is the delay associated to user n, and $n(t)$ is the additive white Gaussian noise.

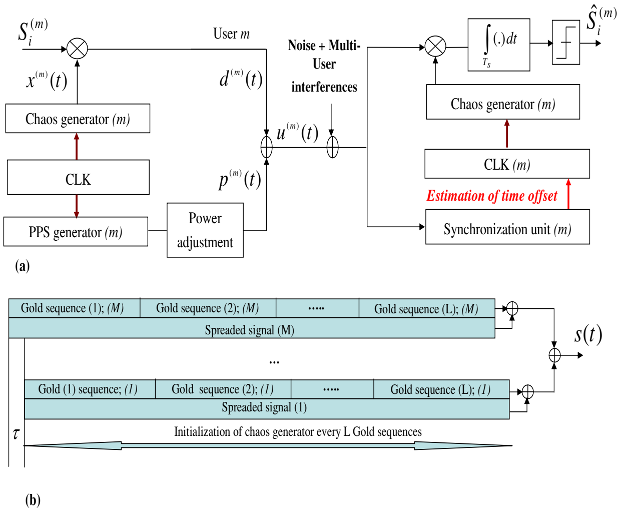

Figure 1: (a) chaotic communication system with the synchronization unit, (b) structure of the transmitted signal

Acquisition of the PPS signal within the chaotic communication system is possible due to the fact that the various PPS and the CPF chaotic signal are non correlated with low magnitude peaks of their cross-correlation functions [4].

As shown in Figure 1(a), the PPS and the chaotic sequence are generated synchronously every time interval T_c using the same master clock. To establish and maintain synchronisation for user m at the receiver side, the local clock must be synchronized. When the PPS sequence offset of user m is known at the receiver (end of acquisition phase) the chaos offset is also de-facto found and the synchronous detection of chaos spreaded signal can take place. The chaotic sequence is aided by the PPS acquisition to achieve clock synchronization. This is the CAS method.

The classical CDMA acquisition method is presented in [3]. In our paper we have applied classical serial search mode. The received signal of a given user m is multiplied by a locally generated pilot signal ($p^{(m)}(t - \delta)$), where δ is an arbitrary delay. We integrate the product of $r(t)$ by $p^{(m)}(t-\delta)$ over a period named acquisition time integration. Without loss of generality, we have taken the acquisition time integration equal to $T = NT_c$. Then, we compare the output decision variable to a predetermined threshold θ in order to know if the acquisition is accomplished or not. The term of interest in the decision variable is :

$$R^{(m)}(\tau^{(m)} - \delta) = \int_{NT_c} p^{(m)}(t - \tau^{(m)}) p^{(m)}(t - \delta) dt \quad (3)$$

4. Theoretical expression of P_d and P_{fa}

In this section, theoretical expressions of the probability of detection (P_d) and the probability of false alarm (P_{fa}) are evaluated. The received signal multiplied by the local PPS of user m is expressed, after integration, by :

$$D^{(m)} = R^{(m)}(\tau^{(m)} - \delta) + \eta^{(m)} + \gamma^{(m)} + \psi^{(m)} \tag{4}$$

$$\begin{aligned} \eta^{(m)} &= \int_{NT_c} n(t) p^{(m)}(t-\delta)dt \\ \gamma^{(m)} &= \int_{NT_c} \sum_{n=1}^{M} d^{(n)}(t-\tau^{(n)}) p^{(m)}(t-\delta)dt \\ \psi^{(m)} &= \int_{NT_c} \sum_{\substack{n=1 \\ n\neq m}}^{M} p^{(n)}(t-\tau^{(n)}) p^{(m)}(t-\delta) \end{aligned}$$

where $R^{(m)}(\tau^{(m)} - \delta)$ is the term of interest for the code acquisition given by (3).
$\eta^{(m)}$ is a zero mean Gaussian noise because noise $n(t)$ and PPS are independent. The variance of $\eta^{(m)}$ is :

$$\sigma^2_{\eta^{(m)}} = \frac{1}{2} P_2 N_0 N T_c \qquad with \quad \sigma^2_{p^{(m)}} = P_2 \tag{5}$$

$\gamma^{(m)}$ is the multi-user noise. Thanks to the central limit theorem, this noise is zero mean and Gaussian with variance:

$$\sigma^2_{\gamma^{(m)}} = P_1 P_2 M N T_c^2 \tag{6}$$

Expression (6) relies on the fact that chaotic sequences and the PPS of user m are uncorrelated. Furthermore, for a given chaotic sequence, all samples have a low correlation value [11].

Referring to Gold codes properties [5], the integral $\alpha = \int_{NT_c} p^{(n)}(t-\tau^{(n)}) p^{(m)}(t-\delta)dt$ can only take three values μ_1, μ_2, μ_3 . A probability of appearance is associated to each of these values, $\xi \in \{\xi_1,\ \xi_2,\ \xi_3\}$ for a given set of delays. The mean value of α is given by :

$$E(\alpha) = \sum_{i=1}^{3} \mu_i \xi_i \tag{7}$$

For $M-1$ interfering and independent users, the mean value of $\psi^{(m)}$ is :

$$E\left(\psi^{(m)}\right) = (M-1)E(\alpha) \tag{8}$$

The variance of α and $\psi^{(m)}$ are given by :

$$\begin{aligned} Var(\alpha) &= E\left(\alpha^2\right) - E(\alpha)^2 \\ Var\left(\psi^{(m)}\right) &= (M-1) Var(\alpha) \end{aligned} \tag{9}$$

Thanks to the central limit theorem $\psi^{(m)}$ follows a Gaussian distribution.

Finally we have

$$\begin{aligned} E\left[D^{(m)}\right] &= R^{(m)}(\tau^{(m)} - \delta) + (M-1)E[\alpha] \\ \sigma^2_{D^{(m)}} &= \sigma^2_{\delta^{(m)}} + \sigma^2_{\gamma^{(m)}} + \sigma^2_{\psi^{(m)}} \end{aligned}$$

4.1. Probability of detection and probability of false alarm

The main motivation of this paper is to present a new method for robust synchronization for asynchronous chaos-based DS-CDMA. For this reason we are interested in computing the P_d and the P_{fa} to demonstrate the feasibility of this method.

Probability of detection for user m is given by $\Pr\left(D^{(m)} \geq \theta\right)$ when $\delta = \tau^{(m)}$ (θ is the threshold) :

$$P_d^{(m)} = Q\left(\frac{\theta - R^{(m)}(0) - E\left(\psi^{(m)}\right)}{\sigma_{D^{(m)}}}\right) \tag{10}$$

where $Q(x) = \int_x^{+\infty} \frac{1}{\sqrt{2\pi}} e^{\left(\frac{-u^2}{2}\right)} du$

P_{fa} for user m and for all offsets δ_0 is given by : $\Pr\left(D^{(m)} \geq \theta\right)$ when $T_c < \left|\tau^{(m)} - \delta_0\right| < (N-1)T_c$

The interest term can take three values [5] with probability of appearance ξ_j, $R_{p^{(m)}}\left(\tau^{(m)} - \delta_0\right) = \mu_j$.

The P_{fa} for this system is given by :

$$P_{fa}^{(m)} = \sum_{i=1}^{3} \xi_j Q\left(\frac{\theta - E\left(\psi^{(m)}\right) - \mu_j}{\sigma_{D^{(m)}}}\right) \tag{11}$$

5. Simulation results

For our simulations, the spreading factor is $\beta = 16$, the number of users $M = 3$ and each transmitted signal contains $L = 256$ sequences of PPS with $N = 127$. The respective emitted powers are $P_1 = 1\ W$ and $P_2 = 0.2\,W$. First of all, we have compared in Figure 2 the theoretical expression of the P_d given by (10) with simulation results for different thresholds (θ = 30, 20, 10). It is clear, looking at Figure 2, that we have a perfect fit between the theoretical probability of detection and simulation results.

In Figure 3 we have plotted the probability of false alarm for fixed probabilities of detection. The threshold is computed from expression (10) for a given P_d. If we need to increase the synchronization performance for fixed values of E_c/N_0 and P_d , we must increase the power of the PPS. On the other hand, increasing the power of the PPS will degrade the BER performance. After simulation of various PPS powers, we have seen that P_2 equal to 20% of P_1 can give a good compromise between BER and synchronization performance.

6. Conclusions

In this paper an extended synchronization method of [4] applied for the asynchronous multi-user chaos based DS-

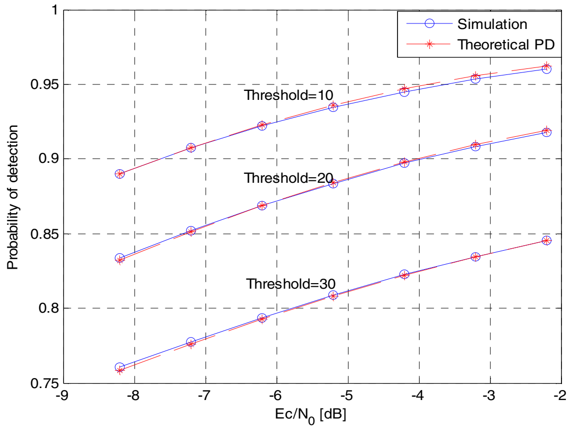

Figure 2: Theoretical and simulation performance of the probability of detection for a fixed threshold $\theta = 10,\ 20,\ 30$

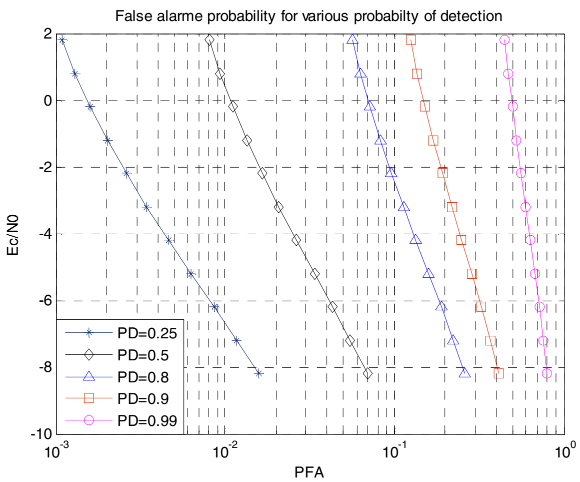

Figure 3: False alarm probabilities for a fixed probability of detection

CDMA system is presented. The synchronization performance of the chaotic communication system is evaluated in presence of noise and multi-user interferences. Conventional ideas of CDMA synchronization process have been applied to the chaos based DS-CDMA using a PPS signal. Using Gold sequences, the code acquisition phase has been evaluated in terms of probability of detection and probability of false alarm. Theoretical expressions of the probability of detection and the probability of false alarm have been proposed with very good agreement with simulation results. Extension of this method applied to multipath channels is currently under study.

References

[1] L. M. Pecora and T. L. Carroll, "Synchronization in chaotic systems," *Phys. Rev. Lett.*, vol. 64, 1990

[2] M. P. Kennedy, R. Rovatti and G. Setti, " Chaotic Electronics in Telecommunications," *London, U.K.: CRC*, 2000.

[3] J. K. Holmes, "Coherent Spread Spectrum Systems," *Krieger publishing company*, 1982.

[4] B. Jovic, C.P. Unsworth, G.S. Sandhu and S. M. Berber, "A robust synchronization unit for multi-user DS-CDMA chaos based communication systems," *Signal Process.*, val. 87, pp. 1692?1708, 2007.

[5] J.G. Proakis, "Digital Communications," *McGraw-Hill*, Boston, 2001.

[6] G. Kaddoum, P. Chargé, D. Roviras and D. Fournier-Prunaret, "Comparaison of chaotic sequences in a chaos based DS-CDMA system," *NOLTA 2007*, Vancouver, Canada, September 2007.

[7] G. Setti, R. Rovatti and G. Mazzini, "Synchronization Mechanism and Optimization of Spreading Sequences in Chaos-Based DS-CDMA Systems," *IEICE Trans. on Fundam. of Electronics, Comm and Computer Sciences* E82-A (9) pp.1737-1746, 1999

[8] N. Eshima, Y. Jitsumatsu and T. Kohda, "Markovian SS Codes Imply Inversion-Free Code Acquisition in Asynchronous DS/CDMA Systems," *ISCAS?04*, pp. 617 ? 620, Bangkok, Thailand, 2004

[9] N. Eshima and T. Kohda, "Statistical approach to the code acquisition problem in direct-sequence spread-spectrum communication systems," *IMA Journal of Mathematical Control and Information* vol. 23, pp. 149-163, 2006

[10] G. Setti , G. Mazzini and R. Rovatti, "Gaussian characterization of self-interference during synchronization of chaos based DS-CDMA systems," *ICECS?98* Lisboa, Portugal, september 1998

[11] W. M. Tam, F. C. M. Lau and C. K. Tse, "An approach to calculating the bit error rate of a coherent chaos-shift-keying digital communication system under a noisy multiuser environment," *IEEE Trans. On circuits and systems-I*, Vol. 49, pp 473-481,2002.